On-machine Calibration Method for In-situ Stereo Deflectometry System

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Abstract—Deflectometry is a promising method for the in-situ measurement of specular surfaces. However, the accuracy of this method largely depends on the geometrical calibration, which suffers from a low convergence rate and local minima during the optimization. Off-machine calibration results in a change in geometry parameters owing to the repeated disassembly of the measurement system and change of ambient temperature, which largely affects the measurement accuracy. This paper presents a flexible on-machine calibration method with fewer intermediate parameters to accelerate the optimization for the in-situ stereo deflectometry system. An inclined mirror with a fixed angle was mounted on the spindle of an ultraprecision machine tool and used as a reference plane reflecting the fringe displayed on the display screen. The reflected fringe captured by the cameras was encoded by an inclined mirror moving along the C-axis and Z-axis of the machine tool. The preknowledge of the movement distance of the ultraprecision machine tool along the Z-axis and rotation angle of the spindle with a nanoscale accuracy was used to largely reduce the number of intermediate variables involved in the optimization, thus improving the convergence speed of the global optimization method and calibration reliability. The experiment results show that this calibration method has higher calibration accuracy, measurement accuracy and reliability compared to the conventional off-machine calibration method.

Index Terms—Calibration, Deflectometry, In-situ measurement, Specular surface

I. INTRODUCTION

THE demand for in-situ measurements of complex specular surfaces increases [1], [2]. The topology and roughness [3] of the specular surface should be measured during the in-situ measurement. In-situ measurements can avoid repetitive clamping errors of the workpiece, and thus improve the efficiency and stability of the manufacturing process. The deflectometry measurement system [4], [5] is a competitive method for the in-situ measurement of specular or transparent surfaces [6] because of its high measurement accuracy and convenience compared to interferometry. However, the “height slope ambiguity” [7] must be addressed because the measured

slope of the surface depends on the height of the surface relative to the camera, as shown in Fig. 1(a). An infinite number of surfaces can be reconstructed using the same corresponding information. Thus, an extra screen [8] or preknowledge of the surface topography and position [9], [10] are used to address this problem [11], as depicted in Figs. 1(b) and 1(c). Stereo vision is also used to overcome this ambiguity problem, including an extra camera [12], multi-view measurement [13], or optical flow algorithm [14], as shown in Figs. 1(d)(e).

In the deflectometry measurement system, the slope of the measurement surface and reconstruction of the surface using the integration process largely depend on the accuracy of the calibration parameters. Therefore, a high-precision calibration algorithm is required. Numerous methods for the calibration of the deflectometry system have been considered. Additional measurement equipment, including laser tracker or three-coordinate measuring machines [15], [16], has been used to obtain the geometric relationship between the screen and camera. The geometric parameters are calculated directly through the measurement of marker points attached to the screen and camera. However, these methods are inconvenient and expensive. Zhang [17] proposed a calibration method for a camera that can provide the internal and external parameters through only three or more pictures of the pattern. As the screen is out of the field of view (FOV) of the camera in the deflectometry system, a flat mirror was introduced to reflect the pattern shown on the liquid-crystal display (LCD) screen to address this problem. Petz [18] proposed the use of a flat mirror with marker points stuck on the mirror to calibrate the geometric relationship between the camera and screen out of the FOV. The accuracy of the calibration results depends mainly on the measurement accuracy of the marker points. Xiao [19] proposed a calibration method using a flat mirror without a marker located at more than three different poses with different normal vectors. The normal vectors of the flat mirror are obtained through the orthogonality of the reflection matrix instead of the marker points. However, the singular solution of normal vectors may be caused by the small movement of the flat mirror. The wrong solution of the normal vector will cause

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Fig. 1. Relationship between the camera, screen, and measured surface. (a) “Height slope ambiguity” problem. (b) Introduction of an extra screen. (c) Preknowledge of the reference surface. (d) Introduction of an extra camera. (e) Multi-view measurement.

wrong calibration parameters. Ren [20] proposed a system optimization algorithm for a stereo deflectometry system using the normal vector of a flat mirror as the intermediate variable during the optimization process. However, a large number of intermediate variables causes a low convergence rate and locally optimal minima. Xu [12] proposed a distortion compensation algorithm to overcome the nonlinearity distortion problem in the imaging of deflectometry. A distortion map for every pixel was generated during the calibration process, instead of using the traditional distortion model. Although these calibration methods are convenient for use in off-line deflectometry measurement systems, it is inconvenient to use them in an in-situ deflectometry measurement system. Repeated disassembly of the measurement system and the influence of the ambient temperature result in a change in the geometric parameters, which largely influences the measurement accuracy.

In this paper, a flexible on-machine calibration method for in-situ stereo deflectometry is proposed to overcome the repeated disassembly problem and the influence of the ambient temperature. An inclined markerless flat mirror fixed on the spindle of the machine tool was utilized as the reference plane that reflected the fringes on the screen. The movement and rotation of the spindle with a nanoscale accuracy are fully used to generate the initial estimate of the system parameters. The normal vector of the flat mirror can be estimated directly through the movement of the spindle, which overcomes the singular solution problem. A system optimization method with fewer intermediate parameters is proposed to refine the initial results. The reliability of the calibration results and convergence rate of the optimization process are largely improved owing to the fewer intermediate parameters. The calibration and measurement can be processed in same working condition, which will guarantee of the effectiveness of the system parameters. The frequent calibration can be performed to overcome the influence of vibration in the machining process due to the convenience of this method. The calibration method proposed in this paper can also be realized using a moving stage and a rotation stage if the ultraprecision machine is unavailable. The remainder of this paper is organized as follows. Section 2 introduces the basic principles of stereo deflectometry. Section 3 presents the calibration method, including the initial value estimation and following refinement through numerical optimization. Section 4 summarizes the experimental results to

verify the effectiveness of the proposed calibration method. Section 5 summarizes the study.

II. PRINCIPLE

An in-situ stereo deflectometry system is utilized to illustrate the on-machine calibration method, as shown in Fig. 2. The in-situ deflectometry system was mounted on the machine tool to realize the in-situ measurement of the workpiece machined on the spindle of the machine tool. The compensated path of the machining tool can be generated based on the in-situ measurement results without repeated disassembly of the workpiece. Stereo deflectometry overcomes the “height slope ambiguity” by introducing a reference camera. In Fig. 2, the machine coordinate system is defined as $\{M\}$, while the screen coordinate system is defined as $\{S\}$. The coordinate systems of the two cameras are defined as $\{C1\}$ and $\{C2\}$.

A synoptic schema of measurement process is shown in Fig.3. The fringe pattern is displayed on the screen, which is reflected by the measured surface fixed on the spindle of the machine tool. The deformed fringe was captured by two cameras. As the imaging process of the two cameras is the same, the following introduction of the imaging process is provided for only one camera. The normal vector of the surface is calculated according to the law of reflection,

Fig. 2. In-situ stereo deflectometry system.

Fig. 3. The synoptic schema of measurement process.

$$\begin{cases} \mathbf{n}_r = \frac{\mathbf{R}_{SC}(\mathbf{p}_{screen}^{[S]} - \mathbf{p}_{surface}^{[S]})}{\|\mathbf{R}_{SC}(\mathbf{p}_{screen}^{[S]} - \mathbf{p}_{surface}^{[S]})\|} \\ \mathbf{n}_i = \frac{\mathbf{K}^{-1}\mathbf{p}_{image}^{[C]}}{\|\mathbf{K}^{-1}\mathbf{p}_{image}^{[C]}\|} \\ \mathbf{n} = \frac{\mathbf{n}_r + \mathbf{n}_i}{2} \end{cases} \quad (1)$$

where $\mathbf{p}_{image}^{[C]}$ and $\mathbf{p}_{screen}^{[S]}$ are the corresponding points of the image and screen obtained through the absolute phase encoding by the fringe shown on the screen. An eight-step phase shift and three-frequency heterodyne algorithm were used to obtain the absolute phase of every pixel in order to generate the mapping relationship of $\mathbf{p}_{image}^{[C]}$ and $\mathbf{p}_{screen}^{[S]}$. The superscript $\{S\}$ indicates that the coordinate is under the $\{S\}$ frame (similarly for $\{C\}$). The $\mathbf{p}_{surface}^{[S]}$ is the point on the measurement surface in the $\{S\}$ frame. $\mathbf{K}$ and $\mathbf{R}_{SC}$ are intrinsic parameters of the camera and geometric relationship between the camera and LCD screen, which need to be determined through the calibration process, respectively.

Usually, a one-dimensional linear search method is used in stereo deflectometry to overcome the “height slope ambiguity” problem [21], [11]. By assuming the position of the measured point on the incident light direction of the main camera, the normal vector is calculated using Eq. 1 in the two camera coordinate systems. As shown in Fig. 2, the red arrow represents the normal vector at the assumed point calculated from $\{C1\}$, while the blue arrow is calculated from $\{C2\}$. The position of the measured point is determined by minimizing the angle of the two normal vectors calculated under the two camera coordinate systems, as shown by the black arrow in Fig. 2. The real normal vector of the surface was also obtained during the above searching process. The final surface form can be generated using a surface integration algorithm [22], [23].

III. CALIBRATION PROCESS

To realize the on-machine calibration, an inclined flat mirror instead of the measured workpiece is fixed on the spindle of the machine tool to reflect the fringe displayed on the screen. The calibration process involves the following steps, as shown in Fig. 4.

Step 1: The cameras of the in-situ stereo deflectometry system collect the fringe reflected by the flat mirror.

Step 2: The spindle of the machine tool moves a distance

along the Z-axis, captures the fringe, and then rotates at an angle of 30° around the C-axis of the machine tool.

Step 3: Move the Z-axis back to the original position and transfer to step 1 until the total rotation angle reaches 360° .

The machine coordinate system is defined as $\{M\}$, while the screen coordinate system is defined as $\{S\}$. The coordinate systems of the two cameras are defined as $\{C1\}$ and $\{C2\}$, respectively, while the virtual screen coordinate system is defined as $\{VS\}$, which is fixed to the virtual screen and is the mirror of the screen coordinate system reflected by the flat mirror. The intrinsic matrix and distortion coefficient of the cameras and geometric relationship between $\{S\}$ and $\{C1\}$ $\{C2\}$ are system parameters that should be obtained through calibration.

The calibration algorithm includes the following steps.

Step 1: Initial value calculation for a single camera twice.

Step 2: Single camera system parameter refinement twice via numeric optimization.

Step 3: System parameter refinement via numeric optimization.

At each step, an iteration process is performed to remove the invalid marker points through the reprojection error criterion to improve the accuracy of the calibration results. The entire algorithm is described in the following subsections.

Fig. 4. Three steps of the calibration.

A. Initial value calculation

During the entire calibration process, the inclined flat mirror was placed at different angles with the rotation of the C-axis of the machine tool. Thus, the virtual screen reflected by the flat mirror was viewed by the two cameras at different positions. The Zhang’s camera calibration method [17] can be used to obtain the intrinsic and external parameters of the cameras; the latter are the geometric relationships between $\{VS\}$ and $\{C1\}$ $\{C2\}$. As the initial value calculation process of the two cameras is the same, the following introduction of the initial value generation is for only one camera. The optimization function is expressed in the form

$$[\mathbf{K}, \mathbf{R}_{VSC}, \mathbf{T}_{VSC}] = \min \sum \|m - f(p')\|^2 \quad (2)$$

where p' represents the marker point in $\{VS\}$ and m represents the image point of p' . The function f represents the nonlinear relationship of the imaging process. $\|m - f(p')\|$ represents the reprojection error of one marker point. The internal parameter $\mathbf{K}$ and external parameter $\mathbf{R}_{VSC}, \mathbf{T}_{VSC}$ of each camera can be estimated using Eq. 2 by minimizing the optimization function.

According to the reflection transformation, the geometric relationship between $\{S\}$ and $\{C\}$ can be generated through the geometric relationship between $\{VS\}$, $\{C\}$ and normal vector of the flat mirror,

$$\begin{cases} \mathbf{R}_{VSC}(I_3 - 2e_3e_3^T) = (I_3 - 2\mathbf{n}\mathbf{n}^T)\mathbf{R}_{SC} \\ \mathbf{T}_{VSC} = (I_3 - 2\mathbf{n}\mathbf{n}^T)\mathbf{T}_{SC} + 2d\mathbf{n} \end{cases} \quad (3)$$

where $\mathbf{n}$ represents the normal vector of the flat mirror and d represents the distance between the flat mirror and camera main point along the normal vector $\mathbf{n}$. $\mathbf{R}_{SC}$, $\mathbf{T}_{SC}$ are the geometric relationships between $\{S\}$ and $\{C\}$. I_3 is a 3×3 identity matrix. e_3 is $[0 \ 0 \ 1]^T$ used to switch $\{VS\}$ from the right-hand frame to the left-hand frame. The detailed relationship is shown in Fig. 5.

Fig. 5. Schematic of the reflection transformation.

In step 2 of the calibration process, the flat mirror moves along the Z-axis of the machine tool, as shown in Fig. 6. The normal vector $\mathbf{n}$ of the flat mirror was unchanged during the movement of the flat mirror. The relation between $\{S\}$ and $\{C1\}$ was also unchanged because the screen and camera were fixed together. The $\{VS1\}$ coordinate system is fixed on the virtual screen before the z-axis movement while $\{VS2\}$ is fixed on the virtual screen after the movement of the Z-axis. The relationship between $\{VS1\}$, $\{VS2\}$ and $\{C1\}$ can be expressed in the same form using the same normal vector $\mathbf{n}$ but different distance d . Thus, the reflection transformation of $\{VS1\}$ and $\{VS2\}$ can be expressed by the following equations independently,

$$\begin{cases} \mathbf{R}_{VS1C1}(I_3 - 2e_3e_3^T) = (I_3 - 2\mathbf{n}\mathbf{n}^T)\mathbf{R}_{SC1} \\ \mathbf{T}_{VS1C1} = (I_3 - 2\mathbf{n}\mathbf{n}^T)\mathbf{T}_{SC1} + 2d_1\mathbf{n} \end{cases} \quad (4)$$

Fig. 6. Flat mirror moving along the Z-axis

$$\begin{cases} \mathbf{R}_{VS2C1}(I_3 - 2e_3e_3^T) = (I_3 - 2\mathbf{n}\mathbf{n}^T)\mathbf{R}_{SC1} \\ \mathbf{T}_{VS2C1} = (I_3 - 2\mathbf{n}\mathbf{n}^T)\mathbf{T}_{SC1} + 2d_2\mathbf{n} \end{cases} \quad (5)$$

Through the translation vector of the two virtual screens calculated through the Zhang's calibration, the normal vector $\mathbf{n}$ of the flat mirror can be calculated by

$$\mathbf{n} = \frac{\mathbf{T}_{VS1C1} - \mathbf{T}_{VS2C1}}{\|\mathbf{T}_{VS1C1} - \mathbf{T}_{VS2C1}\|} \quad (6)$$

where $\mathbf{T}_{VS1C1}$ and $\mathbf{T}_{VS2C1}$ are two translation vectors of the two $\{VS\}$, as shown in Fig. 6, before and after the movement of the Z-axis. Thus, the normal vector of the flat mirror can be calculated directly through the movement of the Z-axis of the machine tool to avoid the singular solution problem.

In step 2 of the calibration process, the inclined flat mirror rotates around the C-axis of the machine tool to every 30° . The normal vector of the flat mirror rotates around the Z-axis of the $\{M\}$ Z_M , as shown in Fig. 7. The angle between Z_M and normal vector of the flat mirror was always α during the rotation of the spindle. Using the normal vectors of the flat mirror at different poses, the relation between the screen and camera can also be calculated through reflection transformation,

$$\mathbf{R}_{SC1}^i = (I_3 - 2\mathbf{n}^i(\mathbf{n}^i)^T)\mathbf{R}_{VSC1}^i, \quad i = 1, 2, \dots, k \quad (7)$$

$$\begin{bmatrix} (I_3 - 2\mathbf{n}^1(\mathbf{n}^1)^T) & 2\mathbf{n}^1 & 0 & 0 & \dots & 0 \\ (I_3 - 2\mathbf{n}^2(\mathbf{n}^2)^T) & 0 & 2\mathbf{n}^2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ (I_3 - 2\mathbf{n}^k(\mathbf{n}^k)^T) & 0 & 0 & 0 & \dots & 2\mathbf{n}^k \end{bmatrix} \begin{bmatrix} \mathbf{T}_{SC1}^1 \\ d^1 \\ \mathbf{T}_{SC1}^2 \\ d^2 \\ \vdots \\ \mathbf{T}_{SC1}^k \\ d^k \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{VSC1}^1 \\ \mathbf{T}_{VSC1}^2 \\ \vdots \\ \mathbf{T}_{VSC1}^k \end{bmatrix} \quad (8)$$

where k is the number of image sequences captured during the rotation process. If the C-axis rotates to every 30° , k is 12. The average rotation matrix between $\{S\}$ and $\{C1\}$ is $\mathbf{R}_{SC1} = \exp(\frac{1}{k} \sum_{i=1}^k \log(\mathbf{R}_{SC1}^i))$, which is used as the initial value for the refinement process below. $\exp$ and $\log$ represent the matrix exponent mapping and matrix logarithm mapping, respectively. $\mathbf{T}_{SC1}$ and d^i can be solved using the least-squares method.

As shown in Fig. 7, the angle between Z_M and normal vector is always α , and thus the average of the normal vector can be regarded as the initial value of Z_M . The

Fig. 7. Flat mirror rotating along the C-axis.

geometric relationship between $\{M\}$ and $\{C1\}$ can be calculated using

$$\begin{cases} \mathbf{Z}_M = \sum_{i=1}^k \mathbf{n}^i / \|\sum_{i=1}^k \mathbf{n}^i\| \\ \mathbf{X}_M = \mathbf{Z}_M \times \mathbf{n}^1 \\ \mathbf{Y}_M = \mathbf{X}_M \times \mathbf{Z}_M \end{cases} \quad (9)$$

$$\begin{cases} \mathbf{R}_{MC1} = [\mathbf{X}_M & \mathbf{Y}_M & \mathbf{Z}_M] \\ [n^1 & n^2 & \dots & n^k]^T \mathbf{T}_{MC1} = [d^1 & d^2 & \dots & d^k]^T \end{cases} \quad (10)$$

The geometric relationship between $\{M\}$ and $\{S\}$ can be calculated using two rigid transforms,

$$\begin{bmatrix} \mathbf{R}_{MS} & \mathbf{T}_{MS} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{SC1} & \mathbf{T}_{SC1} \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{R}_{MC1} & \mathbf{T}_{MC1} \\ 0 & 1 \end{bmatrix} \quad (11)$$

The initial value of camera 1 parameters and geometric relationship between $\{C1\}$, $\{S\}$, and $\{M\}$ has been estimated. The acquisition of the initial value of camera 2 is exactly the same as that of camera 1, while the parameters involved in $\{C1\}$ are independent on those in $\{C2\}$. The geometric relationship between $\{S\}$ and $\{M\}$ and angle α of the inclined flat mirror should be the same for the two camera systems $\{C1\}$ and $\{C2\}$. This is considered in the following calibration procedure.

B. System parameter refinement

We define $\mathbf{R}_{MS}$, $\mathbf{T}_{MS}$, and angle α of the inclined flat mirror as intermediate variables during the refinement because these parameters are useless in the surface reconstruction, but should be considered in the refinement. After acquisition of the initial value, the parameters of each camera should be optimized independently by considering all images captured at different positions of the machine tool. The cost function for refinement is the sum of the reprojection errors of all screen-to-camera point pairs to be traced from all positions of the inclined mirror, which is constructed as

$$[\mathbf{K}_1^*, \mathbf{R}_{SC1}^*, \mathbf{T}_{SC1}^*, \mathbf{R}_{MS}^*, \mathbf{T}_{MS}^*, \alpha^*] = \min \sum_{i=1}^k \sum \|m - f(p)\|^2 \quad (12)$$

$$\begin{cases} f(p) = F((\mathbf{I} - 2\mathbf{n}^i(\mathbf{n}^i)^T)(\mathbf{R}_{SC1}p + \mathbf{T}_{SC1}) + 2d^i\mathbf{n}^i) \\ \mathbf{n}^i = \mathbf{R}_{SC1}\mathbf{R}_{MS}\text{Rotz}(2\pi(i-1)/k)\text{Rotx}(\alpha)[0 \ 0 \ 1]^T \\ d^i = (\mathbf{R}_{SC1}\mathbf{T}_{MS} + \mathbf{T}_{SC1})^T \mathbf{n}^i \\ i = 1, 2, 3, \dots, k \end{cases} \quad (13)$$

where the function F represents the nonlinear relationship of the imaging process with distortion and k is the number of image sequences captured at different positions of the machine tool. Rotz and Rotx denote the rotation matrices rotating around the z -axis and x -axis, respectively. The parameters with stars are refined parameters after minimizing the cost function.

The geometric relationship between $\{S\}$ and $\{M\}$ and angle α of the inclined flat mirror should be the same after the independent calibration of the two cameras. The initial values of $\mathbf{R}_{MS}$, $\mathbf{T}_{MS}$, α during the system refinement process were set as the averages of the two independent values. The system optimization cost function can be constructed as

$$[\mathbf{K}_1^*, \mathbf{R}_{SC1}^*, \mathbf{T}_{SC1}^*, \mathbf{K}_2^*, \mathbf{R}_{SC2}^*, \mathbf{T}_{SC2}^*, \mathbf{R}_{MS}^*, \mathbf{T}_{MS}^*, \alpha^*] = \min(\sum_{i=1}^k \sum \|m_1 - f(p)\|^2 + \sum_{i=1}^k \sum \|m_2 - f(p)\|^2) \quad (14)$$

The Levenberg–Marquardt algorithm is used as an optimization method to refine the parameters of the entire system. The complete calibration is illustrated in Fig. 8.

Fig. 8. Calibration flowchart

IV. EXPERIMENT RESULTS

To fully verify the validity of the proposed calibration method, two different experiments have been conducted to test the calibration accuracy of the system and the measurement accuracy obtained by the calibrated in-situ deflectometry prototype.

A. Calibration accuracy verification

In order to verify the calibration accuracy of the proposed method, a virtual system is performed with known system parameters, as shown in Fig.9. An inclined flat mirror with a diameter of 100 mm and inclination angle of 2° was fixed on the $\{M\}$ coordinate system. The $\{M\}$ coordinate system can move and rotate along Z -axis. The true-value of system parameters are shown in Tab.1.

During the calibration process, the fringes with frequencies of 225, 224, and 210 are displayed on the screen and reflected by the flat mirror. The fringes were captured at every 30° of $\{M\}$ rotation. The simulation pictures are generated through the ray tracing process. Gauss noise with $\sigma=5$ is added on the virtual pictures in order to approach the real calibration process.

The absolute phase of the fringe was used as the target to obtain the control points. An eight-step phase shift and three-

Fig. 9. Setup of the simulation deflectometry system

frequency heterodyne algorithm were used to generate the absolute phase map of the fringe. To improve the calibration accuracy, the control points of each mirror position were chosen at every 20 pixels and a region of interest (ROI; 1200×1100 pixels) was selected. Thus, 60×55 points were chosen as control points during the calibration process. The coordinates of the control points in {S} can be expressed by $(\varphi_x p_x / 2\pi, \varphi_y p_y / 2\pi, 0)$, where p_x and p_y are the physical fringe widths on the display screen and φ_x and φ_y are the absolute phases of the fringe. The system parameters are acquired through the method proposed in this paper and conventional calibration method proposed by Ren [20] using the same control points. Results are shown in Tab.1.

The result shows that the on-machine calibration method has higher calibration accuracy compare with conventional calibration method, especially in the estimation of focal length of camera and system translation vector, which has nearly 10 times higher accuracy.

TABLE I
SIMULATION RESULT

Parameters	True-value	New method (Error)	Conventional (Error)
Focal length(cam1)	(7500, 7500)	(0.11, 0.37)	(2.70, 2.83)
Main point(cam1)	(1232, 1028)	(1.98, 1.67)	(1.32, 1.31)
Focal length(cam2)	(7500, 7500)	(1.79, 2.06)	(16.44, 16.43)
Main point(cam2)	(1232, 1028)	(0.06, 1.03)	(2.63, 0.83)
Euler angle {S}->{C1}/degree	(0, 0, 15)	(9.1e-4, 3.0e-2, 3.0e-2)	(1.5e-3, 6.2e-2 2.3e-2)
Translation {S}->{C1}/mm	(0, 150, 0)	(3.0e-3, 1.3e-3, 7.8e-2)	(2.7e-2, 6.9e-1 3.3)
Euler angle {C2}->{C1}/degree	(0, -4, 0)	(1.2e-4, 2.9e-2, 8.6e-3)	(5.2e-3, 1.5e-2, 5.8e-2)
Translation {C2}->{C1}/mm	(20, 0, 0)	(4.0e-3, 1.7e-2, 9.7e-2)	(2.7e-2, 1.22 4.69)

B. Measurement accuracy verification

To verify the measurement accuracy of above method, an in-situ stereo deflectometry prototype was built and integrated on an ultra-precision machine tool Nanotech650, as shown in Figs. 10(a)–(c). Two AVT 2460 cameras with a 25-mm lens and resolution of 2465×2056 were used. An iPad Pro (12.9 inch, resolution: 2732×2048) was used as

the display screen. An inclined flat mirror with a diameter of 100 mm and inclination angle of 2° was fixed on the spindle of Nanotech650. The flat mirror can be moved along the Z-axis with a positional accuracy of 10 nm or rotate along the C-axis with a positional accuracy of 1.0 arcsec. Thus, the positional error of the machine tool can be ignored during the calibration process.

The absolute phase of the fringe was used as the target to obtain the control points the same as the virtual system calibration process. The fringes with frequencies of 225, 224, and 210 are displayed on the screen. The control points of each mirror position were chosen at every 20 pixels and a region of interest (ROI; 1200×1100 pixels) was selected. The fringes were captured at every 30° of spindle rotation.

The calibration was carried out as shown in Fig. 8. Figure 11 shows the reprojection error and its histogram at each step. After system calibration, the average of the reprojection error was reduced to $(0.20 \times 10^{-10}, -0.11 \times 10^{-10})$, while the root mean square (RMS) was reduced to (0.12, 0.15). The results are presented in Table 2. The histogram shows that the reprojection error can be expressed as a normal distribution.

Using the calibration result, a flat mirror turning by Nanotech650 with a diameter of 40 mm was measured using the in-situ stereo deflectometry system, as illustrated in Fig. 12(a). The zonal reconstruction method [23] was used to generate the surface form. The conventional calibration

Fig. 11. Reprojection error (a) Camera 1 before optimization. (b) Camera 2 after optimization. (c) Histogram of the reprojection error for camera 1 (d) Camera 2 before optimization. (e) Camera 2 after optimization (f) Histogram of the reprojection error for camera 2 (g) System before optimization. (h) System after optimization (i) Histogram of the reprojection error for the whole system.

TABLE II
REPROJECTION ERROR

Error	Before system optimization	After system optimization
Average	(-0.23, -0.03)	$(-0.20 \times 10^{-10}, -0.11 \times 10^{-10})$
RMS	(0.24, 1.71)	(0.12, 0.15)

Fig. 10. Setup of the in-situ stereo deflectometry system. (a) Experiment system. (b) Vertical fringe. (c) Horizontal fringe.

method proposed by Ren [20] was used for comparison, which utilizes the same control points. The reconstruction result is shown in Fig. 12(b). To verify the measurement result, the surface was also measured by a Zygo verifier interferometer for the true value; the form error is shown in Fig. 12(c). The PV and RMS values of the measured surface are shown in Table 3. To demonstrate the stability of the calibration and measurement results, the turning surface was measured three times at different poses and positions. The average and standard deviation (STD) of the repetitive measurement results is shown in Table 3. Compared to the conventional calibration method, the calibration method presented above has lower PV and RMS errors compared to the true value obtained by the interferometer. The STD of the PV and RMS shows that the measurement result obtained using the on-machine calibration method is more stable than that of the conventional method.

V. CONCLUSION

This paper presents a flexible on-machine calibration method for a stereo deflectometry system. A global optimization method using fewer intermediate parameters is proposed to reduce the reprojection error and improve the accuracy, stability, and convergence speed of the calibrated system parameters. The movement of the Z-axis (with a positional accuracy of 10 nm) and rotation along the C-axis (with a positional accuracy of 1.0 arcsec) of the Nanotech650 machine tool were fully utilized and largely reduced the number of intermediate variables from 48 to 7, which were the geometric relationships between the display screen and machine tool, R_{MS} , T_{MS} , and angle α of the inclined flat mirror. The normal vector of the flat mirror can be calculated directly through the movement of the spindle along the z-axis to avoid the singular solution problem. The experiment results shows that the on-machine calibration method has higher calibration accuracy, measurement accuracy and reliability compare with the conventional method, and has comparable measurement accuracy to that of high-precision interferometry.

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Fig. 12. Flat mirror measurement results obtained using the (a) new calibration method, (b) conventional calibration method, and (c) interferometry.

TABLE III
MEASUREMENT FORM ERROR OF THE SURFACE

Error		PV	RMS
Interferometry/nm		253.5	47.9
New method	Average (μ)/nm	365.5	74.1
	STD (σ)/nm	9.3	4.6
Conventional method	Average (μ)/nm	468.6	98.9
	STD (σ)/nm	34.6	4.9

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