Refinement of Weights using Attribute Support for Multiple Attribute Decision Making

Hengshan Zhang\textsuperscript{a,d}, Yimin Zhou\textsuperscript{b}, Tianhua Chen \textsuperscript{c}, Richard Hill\textsuperscript{c}, Zhongmin Wang\textsuperscript{a,d}, Yanping Chen\textsuperscript{a,d}

\textsuperscript{a}School of Computer Science and Technology, Xi’an University of Posts and Telecommunications, Xi’an, China
\textsuperscript{b}Shenzhen Institutes of Advanced Technology, Chinese Academy of Sciences, Shenzhen, China
\textsuperscript{c}Department of Computer Science, School of Computing and Engineering, University of Huddersfield, Huddersfield, United Kingdom
\textsuperscript{d}Shanxi Key Laboratory of Network Data Analysis and Intelligent Processing, Xi’an, China

\section*{Abstract}

A number of approaches have been proposed to determine the weights for multiple attribute decision making. However, the resultant weights are usually assumed to be fixed, making it lack of tolerance to accommodate variation if the patterns of the subsequent data are subject to change. This article proposes a method to facilitate the adjustment of attribute weights, which accommodates a number of relevant characteristics. A model is first constructed that is able to express the requirements of a particular application. The concept of attribute support and consensus are then proposed for subsequent weight modification. A full algorithm is finally presented for the attribute weight adjustment. The effectiveness of the proposed method is validated by way of a case study in the tax credit domain with a sensitivity analysis of the method further evaluated.

\textbf{Keywords:}

Attribute Weight Adjustment, Attribute Support, Strategic Weight Manipulation, Consensus.

\textsuperscript{*}Corresponding author: Yimin Zhou

\textit{Email address: hengshzhang@foxmail.com} (Hengshan Zhang\textsuperscript{a,d})

\textit{Preprint submitted to Elsevier} January 3, 2021
Table 1: Sampled tax data

<table>
<thead>
<tr>
<th>No.</th>
<th>$In.1$</th>
<th>$In.2$</th>
<th>$In.3$</th>
<th>$In.4$</th>
<th>$In.5$</th>
<th>$In.6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80</td>
<td>90</td>
<td>75</td>
<td>81</td>
<td>80</td>
<td>73</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>89</td>
<td>75</td>
<td>78</td>
<td>80</td>
<td>74</td>
</tr>
<tr>
<td>3</td>
<td>78</td>
<td>90</td>
<td>75</td>
<td>79</td>
<td>80</td>
<td>72</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>91</td>
<td>75</td>
<td>82</td>
<td>80</td>
<td>75</td>
</tr>
<tr>
<td>5</td>
<td>79</td>
<td>90</td>
<td>75</td>
<td>80</td>
<td>80</td>
<td>74</td>
</tr>
<tr>
<td>6</td>
<td>78</td>
<td>92</td>
<td>75</td>
<td>76</td>
<td>80</td>
<td>72</td>
</tr>
<tr>
<td>7</td>
<td>80</td>
<td>87</td>
<td>75</td>
<td>78</td>
<td>80</td>
<td>73</td>
</tr>
</tbody>
</table>

1. Introduction

The calculation of weights, which determine the contributions of individual arguments, is important to make decision for a number of real-world applications [1–3]. Weighted Arithmetic Mean (WAM), among a serious of advanced approaches [4–8], has been used to reflect the relative significance of attributes in the multiple attribute decision making problems. A fundamental challenge with these methods is the lack of tolerance to accommodate changes in the underlying data, especially when the pattern of subsequent data differs significantly from that has previously been utilized for weight calculation, making the existing weights no longer representative.

For instance, the tax data for some micro-enterprises in the 1st quarter of 2019 is presented in Table 1, where “No.” denotes the serial number of the enterprise, “$In.i$” ($i = 1, 2, \ldots, 6$) represents the attribute of the data, and the weight vector of the attributes is $\{0.08, 0.4, 0.05, 0.12, 0.05, 0.3\}$. As the economy changes for the 2nd quarter of 2019, the attribute “$In.6$” denotes “Scale of the tax arrears”, and we observe that the values have decreased for micro-enterprises, as $\{63, 62, 62, 64, 63, 63, 61\}$.

By using a weighted arithmetic mean, the re-calculated results of the tax credit rating for an enterprise may change, as per Table 2 (The rows corresponding to “1st”, “2nd”), where “$i$” represents the serial number of the enterprise. Each micro-enterprise can then be classified as “excellent”, “good”, “medium”, “general” or “poor”, and the relation between these five levels is displayed in Table 3, where “$s$” is the calculated score of the micro-enterprise.

For the micro-enterprises in Table 1, the tax credit rating in the 2nd quarter is “medium” rather than “good” as they were in the 1st quarter of 2019. We observe that the production capacities of micro-enterprises may
Table 2: Calculated tax credit rating for each enterprise

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
<td>81.8</td>
<td>81.3</td>
<td>81.1</td>
<td>82.9</td>
<td>81.9</td>
<td>81.5</td>
<td>80.2</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>78.7</td>
<td>77.8</td>
<td>78.1</td>
<td>79.6</td>
<td>78.5</td>
<td>78.7</td>
<td>76.7</td>
</tr>
<tr>
<td>Adj.</td>
<td>81.7</td>
<td>80.7</td>
<td>81.1</td>
<td>82.5</td>
<td>81.5</td>
<td>81.7</td>
<td>79.6</td>
</tr>
</tbody>
</table>

Table 3: Relations between the five levels and the scores of enterprise

<table>
<thead>
<tr>
<th>Tax credit rating</th>
<th>Score range</th>
</tr>
</thead>
<tbody>
<tr>
<td>excellent</td>
<td>90 ≤ s ≤ 100</td>
</tr>
<tr>
<td>good</td>
<td>80 ≤ s &lt; 90</td>
</tr>
<tr>
<td>medium</td>
<td>70 ≤ s &lt; 80</td>
</tr>
<tr>
<td>general</td>
<td>60 ≤ s &lt; 70</td>
</tr>
<tr>
<td>poor</td>
<td>s &lt; 60</td>
</tr>
</tbody>
</table>

have increased, while it is not necessary with a corresponding increase in productivity.

The attribute weight of the tax data does not represent the significance of the attribute, while the tax data reflects changes in the data. If the weight of the attribute is calculated by utilizing the current method [8], the tax credit rating of the enterprise obtained subsequently will not be directly comparable to the prior calculation.

Furthermore, it is not possible to guarantee the scores of the alternatives, while the relative difference is larger, between the calculated score of the alternatives via the new attribute weights, and the original score. As a result, these differences cannot exceed \( \alpha \), where \( \alpha \) is a threshold. We therefore must adjust the attribute weight such that a meaningful representation of the altered data can be presented.

If the attribute weights are adjusted by the vector \( \{0.10, 0.47, 0.08, 0.15, 0.06, 0.16\} \), the calculated results for enterprises, in row “Adj.”, Table 2, are not obviously reduced when compared with the results in the 1<sup>st</sup> quarter of 2019, and the tax credit rating is also “good” except for enterprise “7”. Additionally, the “consensus” is increased post attribute weight adjustment, from 0.90 to 0.92. The “consensus” represents the degree of closeness between the computed tax credit rating and tax data; the larger the value of “consensus”, the higher the degree of closeness.

Little work exists that explores the adjustment of attribute weights in relation to changes in data. In the literature, the integrated method aims
to combine weights to obtain a result [9–15], where the arithmetic mean of the subjective and objectively-determined weights has been considered as the final result [11]. Methods of this sort are developed to obtain a suitable weight so that the default determination of the weight can be released. However, weight adjustment in the application is not considered, and the ranking of certain alternatives can not be changed. The integrated method for the determination of the weight is not suitable to adjust the attribute weight.

Alternatively, the dynamic method is used to obtain attribute weight in the situation where the attribute weight varies with time [16]. Most of the dynamic methods assume that attribute weights are subject to some mathematical distributions over time [17–24]. Despite such assumptions may ease the calculation of the weights, but the distribution of the underlying model can not be practically obtained in real world situations. For other dynamic methods, the attribute weight remains static when the data has changed, and the dynamic period weight substitutes the static attribute weight. For instance, the Dombi operations, which have an advantage of flexibility within the working behaviour of parameter [25], have been combined with prioritized aggregation operators in fuzzy environment [26–29], which have been widely applied in Multiple Attribute Decision Making (MADM). However, existing dynamic methods cannot generally be used in the situation where the attribute weight is required to make the adjustment for the requirements of the application.

More recently, Dong et al. propose Strategic Weight Manipulation (SWM) in multiple attributes decision making [30, 31]. If the ranking of the alternatives is given, the proposed SWM can obtain the new attribute weights to guarantee the given ranking of the alternatives. In [32], the authors proposed SWM with minimum cost in the group decision making, considering the partially known attribute weights. They are presented as numerical intervals, where the attribute weight is guaranteed for a given ranking of the alternatives. However, these proposed SWM approaches cannot be used to the attribute weight adjustment with varied data due to the following reasons:

1. SWMs do not consider the relations between the calculated scores and the used data;
2. The ranking of the alternatives is obtained based on the corresponding data in this study, rather than given by the experts in SWMs;
3. If the changes of the calculated scores for certain alternatives are small,
SWMs cannot function well.

Based on the attribute relations of the data and the requirements of the applications, we propose a method for attribute weight adjustment. The attribute weight can adapt to changes in data, and the calculated scores of alternatives based on the weighted arithmetic mean can guarantee the requirements of the application, together with a higher consensus with the changed data. The key contributions are as follows:

1. A model of attribute weight adjustment is constructed, which presents the objective requirements for attribute weight adjustment;

2. The concept of attribute support is introduced to describe the closeness of two attributes, based on a data vector which is associated with each corresponding attribute;

3. Using the proposed attribute support and model together, a method is developed to enable attribute weight adjustment to take account for changes in data and to guarantee an application’s requirements;

4. We verify, by way of experiment, the effectiveness of the resulting algorithm, and we examine the sensitiveness of various parameters of the approach. Furthermore, the limits of the parameter values are determined.

The remainder of this paper is organized as follows. First, the model of attribute weight adjustment is constructed in section II. The concept of attribute support is then proposed and developed, and subsequently, the method for attribute weight adjustment which can adapt to changes in data, is described in Section III. Section IV presents an experiment in the domain of tax credit ratings for enterprises and individuals, and an analysis of the results is also conducted. Concluding remarks and further work are proposed in Section V.

2. Attribute Weight Adjustment Modeling

We first present the requirements of attribute weight adjustment, for a common business scenario where tax credit ratings are calculated for both enterprises and individuals. For a tax credit rating that is calculated with an attribute weighted mean, the requirements for enhanced attribute weight adjustment can be summarized as:
1. The calculated scores of alternatives based on the adjusted attribute weights have minimum differences with respect to the data vector associated with the attribute;

2. Attribute weight adjustment is based on existing data, and the changes for the existing attribute weights are as minimal. Furthermore, the adjusted attribute weights can integrate the relations of the data vector, which is associated with each attribute, and can adapt to changes in data;

3. The calculated scores of alternatives based on adjusted attribute weights are required to satisfy the following constraint:

$$\frac{|y^* - y^0|}{y^0} \leq \alpha$$

where $y^0$ is the score of the alternative which is calculated based on the original attribute weights, $y^*$ is the score which is obtained by using the adjusted attribute weights, and $\alpha$ is a threshold which constrains the changes for the score of the alternative;

4. There are some special cases where the attribute weight adjustment can not alter the existing ranking of alternatives.

$$\min \sum_{j=1}^{n} w_j^* \left( \sum_{i=1}^{N} (y_i^* - x_{ij})^2 \right)^{1/2}, \min \sum_{j=1}^{n} |w_i - w_i^*|$$

$$\left\{ \begin{array}{l}
\mathbf{r}_{W_k} \{O\} = \mathbf{r}_{W_k^*} \{O\}, \ k = 1, 2, \ldots, m \\
\frac{|y^*_i - y^0_i|}{y^0_i} \leq \alpha, \ i = 1, 2, \ldots, N \\
y_i^* = F(x_{i1}, x_{i2}, \ldots, x_{in}, w_{1}^*, w_{2}^*, \ldots, w_{n}^*) \\
y_i^0 = F(x_{i1}, x_{i2}, \ldots, x_{in}, w_{1}^0, w_{2}^0, \ldots, w_{n}^0) \\
0 \leq w_j^* \leq 1, \ j = 1, 2, \ldots, n \\
0 \leq w_j^0 \leq 1, \ j = 1, 2, \ldots, n \\
\sum_{j=1}^{n} w_j^* = 1 \\
\sum_{j=1}^{n} w_j^0 = 1
\end{array} \right. \quad (1)$$
Table 4: Notation used in this article

<table>
<thead>
<tr>
<th>Notation</th>
<th>Meanings</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y = { y_i } )</td>
<td>Set of scores of the alternatives</td>
</tr>
<tr>
<td>( A = { a_j } )</td>
<td>Set of attributes</td>
</tr>
<tr>
<td>( O = { o_i } )</td>
<td>Set of scores of alternatives whose ranking cannot be changed</td>
</tr>
<tr>
<td>( N )</td>
<td>The number of the alternative ( y_i )</td>
</tr>
<tr>
<td>( n )</td>
<td>The number of the attribute ( a_j )</td>
</tr>
<tr>
<td>( m )</td>
<td>The number of the alternative ( o_i )</td>
</tr>
<tr>
<td>( r_W(O) )</td>
<td>The ranking of the alternatives in set ( O ) with the weight sector ( W )</td>
</tr>
<tr>
<td>( X = [x_{ij}]_{N \times n} )</td>
<td>Data corresponding to the alternatives and attributes</td>
</tr>
<tr>
<td>( w_j )</td>
<td>Existing weight of attribute ( a_j )</td>
</tr>
<tr>
<td>( w_j^* )</td>
<td>Adjusted weight of attribute ( a_j )</td>
</tr>
<tr>
<td>( \bar{a}_j )</td>
<td>Data vector associated to attribute ( a_j )</td>
</tr>
<tr>
<td>( \bar{y}_i )</td>
<td>Data used to calculate ( y_i )</td>
</tr>
</tbody>
</table>

Based on the summarized requirements, we propose an optimal model for attribute weight adjustment. Notation for representing the proposed model is shown in Table 4, and the proposed model is presented as (1). In model (1), the data vector associated to attribute \( a_j \) is represented as vector \( \bar{a}_j = \{x_{1j}, x_{2j}, \ldots, x_{N_j}\}^T (j = 1, 2, \ldots, n) \), and the data used to computed \( y_i \) is vector \( \bar{y}_i = \{x_{i1}, x_{i2}, \ldots, x_{in}\} (i = 1, 2, \ldots, N) \), where \( x_{ij} \in X \). \( F \) may be the function which can combine vector \( \bar{y}_i \) and attribute weight vector \( W = \{w_1, w_2, \ldots, w_n\} \) to obtain \( y_i \), \( W_0 \) and \( W^* \) are the original and adjusted weight vectors, respectively. The objective of the model guarantees the requirements 1) and 2), constrains \( r_{W_0}(O) = r_{W^*}(O) \) and \( \frac{y_i^* - y_i^0}{y_i^0} \leq \alpha \) correspond to the requirements 3) and 4).

Where, \( y_i^* (i \in \{1, 2, \ldots, N\}) \) is the obtained results based on the adjusted attribute weight, and \( r_{W_0}(O) = r_{W^*}(O) \) means that the ranking of the alternatives cannot be changed in the attribute weight adjustment. Let \( O = \{o_1, o_2, \ldots, o_m\} \) be the initial scores of the alternatives whose ranking can not be changed in the attribute weight adjustment, the corresponding scores which are calculated based on adjusted attribute weights denote as \( O^* = \{o_1^*, o_2^*, \ldots, o_m^*\} \), \( r_{W_0}(O) = r_{W^*}(O) \) can be presented as \( o_1 \leq o_2 \leq \ldots, \leq \)
$o_m \Leftarrow o_1^* \leq o_2^* \leq \ldots \leq o_m^*$. Furthermore, let $M = \{r_i | r_i = o_{i+1}^* - o_i^*, i = 1, 2, \ldots, m-1\}$, $o_1^* \leq o_2^* \leq \ldots \leq o_m^*$ can be presented as $r_i \geq 0, \forall r_i \in M$.

The weighted arithmetic mean is often used in applications as it is simple to comprehend and easy to use. Function $F$ in model (1) is replaced by a weighted arithmetic mean in this study. In applications, attribute weight adjustment is calculated using the base of the existing attribute weight vector. In this study, the process of attribute weight adjustment is as follows.

First, the factor to adjust the attribute weight is calculated for each attribute, which can modify the existing attribute weight. Second, the factor and existing attribute weight is calculated for each attribute, giving the adjusted attribute weight. An iterative process is conducted to guarantee that changes to the existing attribute weight are minimised. Furthermore, the selection of the suitable $\alpha$ in the constraint $|y_i^* - y_0^*| \leq \alpha$ can also guarantee minimal changes to the existing attribute weight. As a result, model (1) is improved as in model (2).

$$
\begin{align*}
\min & \sum_{j=1}^{n} w_j^* \left( \sum_{i=1}^{N} (y_i^* - x_{ij})^2 \right)^{1/2} \\
\text{s.t.} & \begin{cases} 
  0_1 \leq o_2 \leq \ldots \leq o_m \Leftrightarrow r \geq 0, \forall r \in M \\
  \frac{|y_i^* - y_i^0|}{y_i^0} \leq \alpha, \quad i = 1, 2, \ldots, N \\
  y_i^* = w_1^* x_{i1} + w_2^* x_{i2} + \ldots + w_n^* x_{in}, \quad i = 1, 2, \ldots, N \\
  y_i^0 = w_1^0 x_{i1} + w_2^0 x_{i2} + \ldots + w_n^0 x_{in}, \quad i = 1, 2, \ldots, N \\
  0 \leq w_j^* \leq 1, \quad j = 1, 2, \ldots, n \\
  0 \leq w_j^0 \leq 1, \quad j = 1, 2, \ldots, n \\
  \sum_{j=1}^{n} w_j^* = 1 \\
  \sum_{j=1}^{n} w_j^0 = 1 
\end{cases}
\end{align*}
$$

3. Attribute Weight Adjustment Based on Attribute Support

Based on our proposed model for attribute weight adjustment, we describe the method to achieve attribute weight adjustment. First, the concept of attribute support is proposed, which can represent the relations of attributes based on the associated data. Second, the concept of the consensus
is proposed to measure the closeness between the data vectors and scores of the alternative. Third, an algorithm for attribute weight adjustment is developed based on the proposed attribute support and model, with the result being a guarantee that the requirements of the applications can have a higher consensus with the changed data.

3.1. Attribute Support

Attribute support represents the closeness between attributes according to the associated data vectors. When the distance between two attributes is higher, the attribute support for the attributes is correspondingly larger. Using attribute support, the factor to adjust the existing attribute weight can be calculated.

Definition 1. Let $Y = \{y_i | y_i \in [0, 1], i = 1, 2, \ldots, N\}$ denote the scores of the alternatives, $A = \{a_j | j = 1, 2, \ldots, n\}$ be the set of the attributes, $X = \{x_{ij} | x_{ij} \in [0, 1], i = 1, 2, \ldots, N, j = 1, 2, \ldots, n\}$ represent the data set associated with the alternatives and attributes. The distance between attributes $a_i$ and $a_j$ is defined as the distance between $\bar{a}_i$ and $\bar{a}_j$, i.e., $d(a_i, a_j) = d(\bar{a}_i, \bar{a}_j)$, where $\bar{a}_i$ and $\bar{a}_j$ are the data vectors associated to attributes $a_i$ and $a_j$. The support for the two attributes $a_i$ and $a_j (i \neq j)$ is the function of the distance between the attributes, it is denoted as $Sup(a_i, a_j) = F(d(\bar{a}_i, \bar{a}_j))$ and the function $F$ has the following properties:

1. $F(d(\bar{a}_i, \bar{a}_j)) \in [0, 1]$ is a single variable function about $d(\bar{a}_i, \bar{a}_j)$;
2. $F(d(\bar{a}_i, \bar{a}_j))$ is the monotone non-increasing function about $d(\bar{a}_i, a_j)$;
3. $F(d(\bar{a}_i, \bar{a}_j)) = 1$.

Based on the definition 1, attribute support for the two attributes $a_i$ and $a_j (i \neq j)$ has the following properties:

1. $Sup(a_i, a_i) = 1$;
2. $Sup(a_i, a_j) = Sup(a_j, a_i)$;
3. $Sup(a_i, a_j) \geq Sup(c, d)$ if $d(a_i, a_j) < d(c, d)$, where $c$ and $d$ are the other two attributes in set $A$. 

9
**Definition 2.** Let $A = \{a_i| i = 1, 2, \ldots, n\}$ be the set of the attributes, the support of the attribute $a_i$ is defined as follows:

$$S(a_i) = \sum_{j=1}^{n} \text{Sup}(a_i, a_j)$$

If the existing distance between two vectors is closer, their distance is smaller, and it can be used to calculate the attribute support for two attributes. We consider two types of the distance.

**Typical Distance**

Typical distance measures include Euclidean distance, Manhattan distance, Mahalanobis distance, Minkowski distance etc. In fact, Manhattan distance, Euclidean distance and Chebyshev distance can be considered as special cases of the Minkowski distance. As such, the Minkowski distance is used to calculate the attribute support in this work as the typical distance.

Using Minkowski distance, we can obtain, $d_N(a_i, a_j) = \frac{1}{\phi N \left( \sum_{k=1}^{N} |x_{ki} - x_{kj}|^p \right)^\frac{1}{p}}$ is the normalized distance.

**Distance via Similarity**

The similarity can reflect the closeness between 2 vectors, we define the distance via similarity as follows.

**Definition 3.** With the set of attributes $A$ and $a_1, a_2 \in A$ being two arbitrary attributes, $\text{sim}(a_1, a_2)$ represents the similarity between $a_1$ and $a_2$. If $\phi(\text{sim}(a_1, a_2))$ is the monotonic decreasing function on $\text{sim}(\alpha_1, \alpha_2)$, $\phi(\text{sim}(\alpha_1, \alpha_2))$ is defined as distance via similarity, and denoted as $d_{\text{sim}}(a_1, a_2)$.

For example, $d_{\text{sim}}(a_1, a_2) = \frac{1}{\text{sim}_N(a_1, a_2)} - 1$, and $d_{\text{sim}}(a_1, a_2) = 1 - \text{sim}_N(a_1, a_2)$ are the distances via similarity, where $\text{sim}_N(a_1, a_2)$ represents the normalized similarity. In the rest of paper, we utilize $d_{\text{sim}}(a_1, a_2) = 1 - \text{sim}_N(a_1, a_2)$, and compute the normalization of distance via similarity as

$$d_N(a_i, a_j) = \frac{d(a_i, a_j) - \min_{i,j} \{d(a_i, a_j)\}}{\max_{i,j} \{d(a_i, a_j)\} - \min_{i,j} \{d(a_i, a_j)\}}$$

(3)

Without loss of generality, we use cosine similarity and Pearson coefficient to demonstrate the computation of distance via similarity.
Let $a_i$, $a_j$ be two attributes, the cosine similarity and Pearson coefficient are respectively written as,

$$d(a_i, a_j) = 1 - Sim_{cosine}(a_i, a_j) = 1 - \frac{a_i \cdot a_j}{\|a_i\| \times \|a_i\|}$$  \hspace{1cm} (4)$$

$$d(a_i, a_j) = 1 - sim_{Pearson}(a_i, a_j) = 1 - \left|\frac{Cov(a_i, a_j)}{\sigma_{a_i} \times \sigma_{a_j}}\right|$$  \hspace{1cm} (5)$$

**Forms to Compute the Attribute Support**

In the following, we introduce 3 forms to compute the attribute support.

1. Let $a_i$ and $a_j (i \neq j)$ be the two attributes, their attribute support can be computed by using the following equation:

$$Sup(a_i, a_j) = (1 - d_N(a_i, a_j))^k, k > 0$$  \hspace{1cm} (6)$$

2. Attribute support for $a_i$ and $a_j$ can be computed based on the Eq.(7):

$$Sup(a_i, a_j) = e^{-kd_N(a_i, a_j)}, k > 0$$  \hspace{1cm} (7)$$

3. Eq.(8) also supports the computation of the attribute support.

$$Sup(a_i, a_j) = (1+d_N(a_i, a_j))^{-k}, k > 0$$  \hspace{1cm} (8)$$

The **Proposition 1** represents the main properties of the forms for the calculation of attribute support.

**Proposition 1.** Let $x = d_N(a_i, a_j)$, be the normalized distance between attributes $a_i$ and $a_j$, and $y_1 = (1 - x)^k$, $y_2 = e^{-kx}$, $y_3 = (1 + x)^{-k}$, $k > 0$, the following conclusions can be obtained:

1. If $a_i = a_j$, then $y_i = 1 (i = 1, 2, 3)$
2. $y_i (i = 1, 2, 3)$ is the monotonic decreasing function with respect to $x$;
3. $y_i (i = 1, 2, 3)$ decreases monotonously with the increase of $k (k > 0)$.

**Proof.** 1. If $a_i = a_j$, then $x = 0$, it naturally follows that $y_i = 1$ for $i = 1, 2, 3$. 

2. Let \( F_1(x,k) = \ln(y_1) = k\ln(1-x) \), \( F_2(x,k) = \ln(y_2) = -kx \), \( F_3(x,k) = \ln(y_3) = -k\ln(1+x) \), we can obtain, \( F'_1(x) = -\frac{k}{1-x} < 0 \), \( F'_2(x) = -k < 0 \), and \( F'_3(x) = -\frac{k}{1+x} < 0 \). These show that \( F_i(i=1,2,3) \) decreases monotonously with \( x \) increasing; therefore, \( y_i(i=1,2,3) \) is the monotonically decreasing function of \( x \).

3. We can also observe, \( F'_1(k) = \ln(1-x) < 0 \), \( F'_2(k) = -x < 0 \), and \( F'_3(k) = -\ln(1+x) < 0 \), following the same principles as the above discussions, thus 3rd property in Proposition 1 can also be validated.

Property (1) and (2) in Proposition 1 guarantee that Eqs. (6), (7) and (8) can be used to calculate attribute support for a given data set. Furthermore, they can be used to develop the method which makes the adjustment for attribute weight. Property (3) facilitates the selection of appropriate parameter \( k \) for the 3 computation forms of attribute support. In the following experiments, the details for selecting a appropriate \( k \) are empirically studied for the 3 computation forms of attribute support.

3.2. Consensus Between Scores of Alternatives and Attributes

The main objective of attribute weight adjustment is that the computed scores of the alternatives should have minimum differences with data vectors associated with the attributes. In this research, consensus based on weighted distance is proposed to measure the differences between the scores of alternatives and the attributes.

Definition 4. Let \( Y \) be the scores of the alternatives and \( \bar{a}_i \) be the data vector associated to the attribute \( a_i \) for \( i = 1,2,\ldots,n \), and \( X = \{ \bar{a}_1, \bar{a}_2, \ldots, \bar{a}_n \} \), the consensus between the scores of the alternatives and the attributes can be defined as follows:

\[
Cons(Y,X) = 1 - \sum_{i=1}^{n} w_i d(Y, \bar{a}_i)
\]  

(9)

where \( d(Y, \bar{a}_i) \) denotes the distance between \( Y \) and \( \bar{a}_i \), which is calculated using Minkowski distance and distance via similarity in this study, and \( w_i \) is the weight for attribute \( a_i \), for \( i = 1,2,\ldots,n \).
3.3. An Approach for Attribute Weight Adjustment

The proposed attribute weight adjustment model (2) is a non-linear optimization problem. Given the potentially complicated calculation to solve model (2), we adopt the strategy of “finding satisfactory solutions for a more realistic world” from [33], with an aim to acquire a satisfactory approximation solution based on the attribute support. The method comprises 3 parts as follows:

3.3.1. Support Calculation Between the Scores of Alternatives and Attributes

For model (2), based on the weighted arithmetic mean and initial weight vector, the initial scores of the alternatives are calculated as \( Y_0 = \{y^0_1, y^0_2, \ldots, y^0_N\} \), where \( y^0_i = \sum_{k=1}^{n} w^0_k x_{ik} \). One objective of the model (2) is that the distance between the calculated scores of the alternatives and data vectors associated with the attributes is minimized, and the strategy for attribute weight adjustment is to adjust the weights of the certain attributes which have the smaller distances with the scores of the alternatives to become larger, and the others are the contrary. As a result, the calculated scores of the alternatives based on the adjusted weights have smaller distances with the data vectors. Attribute support \( sup(a_i, Y_0)(i = 1, 2, \ldots, n) \) represents the degree of the closeness between \( a_i \) and \( Y_0 \), i.e., the larger of \( sup(a_i, Y_0) \), the closer it indicates for \( a_i \) and \( Y_0 \). In our proposed method, attribute weight is adjusted based on the attribute support \( sup(a_i, Y_0)(i = 1, 2, \ldots, n) \).

If Minkowski distance is applied, the 3 forms of attribute support calculation can be obtained, and denoted as,

\[
sup_{Mink_1}(a_i, Y_0) = \left( 1 - \frac{1}{\sqrt[1-p]{N}} \left( \sum_{k=1}^{N} |x_{ki} - y^0_{ki}|^p \right)^{\frac{1}{p}} \right)^k
\]

\[
sup_{Mink_2}(a_i, Y_0) = \exp \left( -k \frac{1}{\sqrt[1-p]{N}} \left( \sum_{k=1}^{N} |x_{ki} - y^0_{ki}|^p \right)^{\frac{1}{p}} \right)
\]

\[
sup_{Mink_3}(a_i, Y_0) = \left( 1 + \frac{1}{\sqrt[1-p]{N}} \left( \sum_{k=1}^{N} |x_{ki} - y^0_{ki}|^p \right)^{\frac{1}{p}} \right)^{-k}
\]

where \( p, k > 0 \).
For the distance via similarity, which applied the cosine similarity and Pearson correlation, we obtain the 3 forms of attribute support $sup(a_i, Y_0)(i = 1, 2, \ldots, n)$, and respectively presented as

$$sup_{Cosf_1}(a_i, Y_0) = \left( \frac{a_i \cdot a_j}{\|a_i\| \times \|a_i\|} \right)^k$$  \hspace{1cm} (13)

$$sup_{Cosf_2}(a_i, Y_0) = \exp \left( -k + \frac{k a_i \cdot a_j}{\|a_i\| \times \|a_i\|} \right)$$  \hspace{1cm} (14)

$$sup_{Cosf_3}(a_i, Y_0) = \left( 2 - \frac{a_i \cdot a_j}{\|a_i\| \times \|a_i\|} \right)^{-k}$$  \hspace{1cm} (15)

$$sup_{Pearf_1}(a_i, Y_0) = \left( \frac{\text{Cov}(\alpha_i, \alpha_j)}{\sigma_{\alpha_i} \times \sigma_{\alpha_j}} \right)^k$$  \hspace{1cm} (16)

$$sup_{Pearf_2}(a_i, Y_0) = \exp \left( -k + k \left| \frac{\text{Cov}(\alpha_i, \alpha_j)}{\sigma_{\alpha_i} \times \sigma_{\alpha_j}} \right| \right)$$  \hspace{1cm} (17)

$$sup_{Pearf_3}(a_i, Y_0) = \left( 2 - \left| \frac{\text{Cov}(\alpha_i, \alpha_j)}{\sigma_{\alpha_i} \times \sigma_{\alpha_j}} \right| \right)^{-k}$$  \hspace{1cm} (18)

where $k > 0$.

3.3.2. Adjustment for Attribute Weights

Based on model (2), adjustment for attribute weights is required to make the minimum changes on the initial weight vector $W_0$. Here, factors which are used to adjust the attribute weights are calculated based on support $sup(a_i, Y_0)(i = 1, 2, \ldots, n)$, and the adjusted attribute weights are obtained by using the corresponding factors and weights.

Let $\tilde{W} = \{\tilde{w}_i|i = 1, 2, \ldots, n\}$ represent the factors are used to adjust the weights of the attributes, the following equation is used to calculate the factors,

$$\tilde{w}_i = \frac{sup(a_i, Y_0)}{\sum_{j=1}^{n} sup(a_j, Y_0)} \hspace{1cm} i = 1, 2, \ldots, n$$  \hspace{1cm} (19)

The adjusted attribute weights are denoted as $W^* = \{w_i^*|i = 1, 2, \ldots, n\}$, and $w_i^*$ is calculated by utilizing the factors which are based on Eq.(19), and
presented as the following equation.

\[ w_i^* = \frac{w_i \tilde{w}_i}{\sum_{j=1}^{n} w_j \tilde{w}_j} \quad i = 1, 2, \ldots, n \]  

(20)

3.3.3. Algorithm to Acquire Adjusted Attribute Weight

An iterative algorithm is developed to obtain the approximate solution for model (2), which satisfies the requirements of the applications. For each iteration, the adjusted attribute weights are updated based on Eq.(20) until the model constraints (2) are violated. The main steps of the algorithm are as follows:

Input: Data vector \( \bar{a}_j = \{x_{1j}, x_{2j}, \ldots, x_{Nj}\} \) \( (j = 1, 2, \ldots, n) \) and the initial weight vector \( \mathbf{W}^0 = \{w_0^0, w_2^0, \ldots, w_n^0\} \).

Step 1: Calculate the initial scores of alternatives, \( y_i^0 = \sum_{j=1}^{n} w_j^0 x_{ij} \) and \( Y^0 = \{y_i^0 | i = 1, 2, \ldots, N\} \), \( T = Y^0 \).

Step 2: Compute the attribute support \( sup(a_i, T) \) for \( a_i \), \( i = 1, 2, \ldots, n \) with the adjusting factor \( \tilde{w}_i \) for \( a_i \) obtained based on Eq.(19). Adjusted weights \( w_i^* \) for the attributes are obtained by using Eq.(20), \( \mathbf{W}^* = \{w_i^* | i = 1, 2, \ldots, n\} \).

Step 3: Compute \( y_i^* = \sum_{j=1}^{n} w_j^* x_{ij} \), and \( Y^* = \{y_i^* | i = 1, 2, \ldots, N\} \).

Step 4: Let \( O = \{o_1, o_2, \ldots, o_m\} \) and \( o_1 \leq o_2 \leq \ldots \leq o_m \), and compute \( r_i = o_{i+1} - o_i^* \) for \( i = 1, 2, \ldots, m \). If \( \exists r_{i_0} \in M = \{r_i | i = 1, 2, \ldots, m - 1\} \), \( r_{i_0} < 0 \), the algorithm terminates.

Step 5: Calculate \( C = \max_{i=1,2,\ldots,N} \left| \frac{y_i^* - y_i^0}{y_i^0} \right| \). If \( C \leq \alpha \), \( T = Y^* \), repeat Steps 2 to 5 until \( C > \alpha \), where \( \alpha \) is the threshold.

Output: \( \mathbf{W}^* \), which is the adjusted attribute weighting vector, and \( Y^* \), which is the final scores of the alternatives.

There are 3 forms for the algorithm which correspond to the 3 computing forms of attribute support. To demonstrate the convergence of the proposed
iterative algorithm, without losing generality, let $W^*_i$ and $Y^*_i$ be the adjusted weight vector and calculated scores of the alternatives at the $i^{th}$ iteration, respectively. Owing to the iterative nature of the algorithm, the adjusted weight vector $W^*_i$ is calculated based on the $W^*_{i-1}$ and $Y^*_{i-1}$ at a previous iteration. That is to say, the adjusted weight $w^*_j$ of attribute $a_j$ becomes larger when the distance between $\alpha_j$ and $Y^*_{i-1}$ gets large, and vice versa. As the difference between $W^0$ and $W^*$ becomes larger while iterating the attribute weight adjustment, the difference between $Y^0$ and $Y^*$ also tends to be larger. Finally, the termination condition of the algorithm set in Step 4 and 5 guarantees the order of alternatives based on the original scores while satisfying the requirement of the underlying application.

The time complexity for calculating $C$, $Y^0$, $sup(a_i,T)$ and $Y^*$ is $O(N)$, where $N$ is the size of available data. The time complexity is $O(m)$ for computing $r_i, i = 1, 2, \ldots, m$. Therefore, the overall complexity of the algorithm is $K \times O(N)$, where $K$ is the number of iterations.

4. Experimentation

In this section, the experimental study is conducted to illustrate the effectiveness of the proposed method with factors that might affect the proposed method further assessed through a sensitivity analysis.

4.1. Experimental Settings

There are two data sets which were obtained based on the practice of tax credit rating for enterprises and individuals. These data are randomly selected from the tax data of a province in China in the production environment. For the privacy protection, the data sets do not include the name of the attributes, enterprises and individuals. We represent the name of the attribute as $In.i(i = 1, 2, \ldots, n)$, and the name of the enterprises and individuals as the serial number. The first data set is tax data for 3000 enterprises in one country, with 20 attributes. The second data set is the tax data for 4500 individuals in one country, with 14 attributes. For each data set, the initial weights of the attributes are previously given and the alternatives in the data set are enterprises or individuals. Let $X = (x_{ij})_{N \times n}$ be the data set, where $x_{ij} \in [0, 100]$ is the data entry, and $n$ is the number of attribute, $N$ is the number of the enterprise or individual. Each $x_{ij}$ is normalized by using the following equation:
4.2. Experimental Results Analysis

Based on the data set, the initial weights of attributes are adjusted. Experimental results show that the proposed method can reach the objectives for the adjustments of the attribute weights.

4.2.1. Calculation and Analysis of the Consensus Between the Scores of Alternatives and Data Vectors

As the validation of the effectiveness of the proposed method, the consensus between the scores of the alternatives and data vectors was computed for each iteration of the proposed method. Three computing forms for attribute support are proposed in this research. Adjustments of attribute weights have 3 forms in the proposed method, and they correspond to Equation (6) - (8) and are denoted as “Form $i$” ($i = 1, 2, 3$). While using the typical distance, for the calculation of consensus, parameters $k = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\alpha = 20\%$ are given. For the Minkowski distance, 3 values $1, 2, 3$ for $p$ are accepted as representatives to obtain the experimental results. To the distance via similarity, we select $k = 1$ for distance via cosine similarity, and $k = \frac{1}{4}$ for distance via Pearson correlation, and $\alpha = 20\%$ still.

In this part of the experiment, we focus on the analysis of consensus between scores of the alternatives and data vectors. The in-variance of the order for the given alternatives is not required. The results are displayed in Fig.1. Number $i$ for the iterative order means the $i$th adjustment of the attribute weights. In each adjustment of the attribute weights, tax credit scores of the enterprises or individuals are calculated based on adjusted attribute weights. The consensus between computed scores and the data vectors associated with attributes are calculated (Eq.(9)) and presented in Fig.1. For the typical distance, Fig.1(a)-(c) displays the results that were obtained based on tax data for enterprises, and Fig.1(e)-(g) corresponds to the results for tax data of individuals. For the distance via cosine similarity, Fig.1(d) displays the obtained results based on tax data for enterprises. Fig.1(h) demonstrates the results based on the tax data for individuals to the distance via Pearson correlation.

According to the results shown in Fig.1, we observe that the computed consensus between scores of the alternatives and data vectors increases with respect to the attribute weight adjustments, without consideration of the
Figure 1: Consensus between the tax credit scores of enterprises & individuals and the associated data vectors
parameters, forms of attribute support computation, and type of distance. The degree of fitness between the calculated tax credit scores (based on the adjusted attribute weights) and tax data are improved, while adjustments of the attribute weights are continuously made.

4.2.2. Invariance of the Order for the Given Alternatives in the Attribute Weight Adjustments

In this part of the experiment, the analysis about the invariance of the order for the given alternatives is conducted. In practice, data is often non-uniform. For example, the initial tax credit scores of the enterprises are computed based on the initial attribute weights, and the distribution of initial tax credit scores is shown in Fig.2. Most of the tax credit scores are located in region [0.78, 0.90], and the difference between the data in this region is relatively small. Slight adjustment of the attribute weight may change the order of the data. In order to maintain the in-variance of the order for given alternatives, the selection of given alternatives is combined with the application requirements and distribution of the data.

If we analyse the variations of the order of tax credit scores for selected alternatives, they are sorted by using tax credit scores which are calculated
Based on initial attribute weights. The experimental results corresponding to typical distance and distance via similarity are shown in Fig. 3 and Fig. 4, respectively. Where the serial number represents the selected alternatives, and the offset values denote the tax credit scores of the selected alternatives. The “Initial” corresponds to the tax credit scores of the alternatives based on the initial attribute weights. “Form \( i \)” \((i = 1, 2, 3)\) corresponds to the tax credit scores of alternatives with the corresponding Form “\( i \)” of the attribute weight adjustment. As a representative, the parameters used in the algorithm to make the attribute weight adjustment are described as follows:

- **Typical distance:** \( \alpha = 0.2, \ p = 2, \ k = 1 \) for the tax data of the enterprises, and \( p = 2, \ k = \frac{1}{5} \) for the individuals.

- **Distance via similarity:** first to distance via cosine similarity, \( k = \frac{1}{9} \) (Form 1), \( k = \frac{1}{21} \) (Form 2, 3) for the individuals. Second, to distance via Pearson correlation, \( k = \frac{1}{5} \) (Form 1), \( k = \frac{1}{15} \) (Form 2), \( k = \frac{1}{7} \) (Form 3), for the enterprises, and \( k = \frac{1}{121} \) (Form 1), \( k = \frac{1}{15} \) (Form 2), \( k = \frac{1}{21} \) (Form 3), for the individuals. \( \alpha = 0.2 \) is used in these two circumstances.

It can be seen that the tax credit scores of the selected alternatives have the same increases, no matter what form (“Form \( i \)”, \( i = 1, 2, 3 \)) of the attribute weight adjustment and what type of distance (2 types of distance) are adopted, while the initial scores of the selected alternatives increases. Therefore, the order of the selected alternatives based on the initial tax credit scores has no influence on the attribute weight adjustment, while the value of \( k \) is selected correctly. Next, the effects of \( k \) to the attribute weight adjustment is analyzed.

**4.2.3. Variation of Attribute Weight during Adjustment Process**

In this group of the experiments, we analyse the variation of attribute weight through the standard deviation and maximum in the adjustment process. The obtained results are demonstrated in Fig.5, with \( \alpha \) set as 0.2 and the parameters summarized in Table 5. In particular, Fig.5 (a) and (b) correspond to the tax data for enterprises, and Fig.5 (c) and (d) display the results of individuals.

For the typical distances and distance via Pearson similarity, the standard deviation of the adjusted attribute weight rises as the iteration increases. For the distance via cosine similarity, the same results are obtained for tax data
Figure 3: The order of the tax credit scores for the given enterprises and individual unchanged (corresponding to the typical distance)
Figure 4: The order of the tax credit scores for the given enterprises and individual unchanged (corresponding to the distance via similarity)
of the enterprises. This demonstrates that the change of the attribute weight often becomes larger in the process of the attribute weight adjustment (except for tax data of individuals when the distance via cosine similarity used). Another observation made from Fig. 5 is that the maximum of the attribute weight can always be achieved with the iterative adjustment regardless of the parameter selection.

4.3. Sensitivity Analysis for Attribute Weight Adjustment

In order to make the attribute weight adjustment, the calculations of the attribute support have 2 parameters $p$ and $k$ (Eq. (10)-(12)) for typical distance and 1 parameter $k$ (Eq. (13)-(18)) for distance via similarity. Different attribute supports can be obtained for the same attribute when different values are given to $p$ and $k$ for typical distance and $p$ for distance via similarity. As a result, adjusted weights for the same attribute are different with respect to values $p$ and $k$. As the constraints are the same in attribute weight adjustment, the difference between various adjusted weights of the same attribute is quite small. So the influence of parameters $p$ and $k$ on the attribute weight adjustment is mainly reflected in the differences of the iteration times of the algorithm. As a result, analysis of the influences of the parameter upon the iteration times of the algorithm is conducted, showing that it includes 2 partitions for $p$ and $k$, respectively.

4.3.1. Sensitivity Analysis of Parameter $p$

The typical distance (Minkowski distance) between 2 attributes includes parameter $p$, and the different distances can be obtained for the 2 same attributes when the values given to $p$ are not the same. Let $p$ be the various values and $k$ be the certain value and $\alpha = 20\%$, the experiments are conducted to analyze the influences of $p$ upon the iteration times of the algorithm. The value of $k$ is selected from the set $\{\frac{1}{5}, \frac{1}{3}, \frac{1}{2}, 1\}$ and $p \in \{\frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1, 2, 3, 4, 5, \infty\}$. The experimental results are displayed in Fig.6.
Figure 5: Variation of the standard deviation and maximum value of the adjusted attribute weight
Fig.6(a) shows the results based on tax data for enterprises, Fig.6(b) shows the results for tax data of the individuals. It can be deduced that the iteration times of the algorithm are slightly changed when $p$ has a variety of values, in spite of the different values of $k$, and forms of attribute weight adjustment. Only for situation $p = \infty$, the iteration times of the algorithm have a clear change. As a result, the selection of $p$ in attribute weight adjustment can be made flexibly based on the applications.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6}
\caption{Variances of iteration number with parameter $p$}
\end{figure}

### 4.3.2. Sensitivity Analysis of Parameter $k$

The 3 forms of the attribute weight adjustment are based on the calculation of attribute support, and $k$ is an important parameter for the computation of the attribute support (Eq.(10)-(18)). For typical distance, let $p = 2$, $\alpha = 20\%$, and $k \in \{1, 1.5, 2, 2.5, 3, 3.5, 4, 5\}$, the experimental results for the variance of the iteration times with various values of $k$ are displayed in Fig.7. Fig.7 (a) and (b) correspond to the experimental results of tax data for enterprises and individuals, respectively. Iteration times of the algorithm are decreased when values of $k$ are increased, in spite of the forms for the computation of the attribute support. For distance via similarity, Fig.8 (a) corresponds results for enterprises to the Form 2 with distance via cosine similarity, and Fig.8 (b) corresponds results for individuals to the Form 1 with distance via Pearson correlation. The same conclusion is observed. As a result, $k$ should be selected carefully and appropriately with specific application environments.
Figure 7: Iteration number with different values of k for typical distance

Figure 8: Iteration number with different values of k for distance via similarity
For attribute weight adjustment with a requirement where the ranking of the selected alternatives requires to be maintained, if \( k \) is given with varied values, different results are thus obtained. That means the attribute weight adjustment can be completed without violating the ranking only for some values of \( k \). Experimental outcomes are shown in Fig.9, where the cases corresponding to “Form 1” and “Form 2” are selected to make attribute weight adjustment for enterprises tax data, with results displayed in Fig.9 (a) and (b), respectively. For the tax data of individuals, “Form 2” and “Form 3” are selected with the results shown in Fig.9.(d) and (e) respectively. For distance via cosine similarity, “Form 1” is selected with the experimental results based on the enterprises data demonstrated in Fig.9.(c); For distance via Pearson correlation, the experimental results of individuals tax data are presented in Fig.9.(f) via “Form 2”.

In all these cases, we can observe that for the larger values of \( k \), such as 1, 3 and 5, even with one attribute weight adjustment, the ranking of the selected alternatives based on initial scores is changed. For example, in Fig.9(a), in case of, \( k = 3 \), the calculated scores based on the adjusted weight have no increase as the initial scores escalate. Similar results can be observed for other figures when the value of \( k \) is generally becoming large. However, if the value of \( k \) is small, the existing ranking of the selected alternatives can be reserved in attribute weight adjustment. For example, based on the adjusted attribute weight, the calculated tax credit scores of the selected individuals rise along with the initial scores, when \( k = \frac{1}{7} \), in Fig.9 (c). The same results can be obtained for other figures, when the value of \( k \) is among the smallest, such as \( k = \frac{1}{5} \) or \( k = \frac{1}{7} \).

Based on the above analysis, it can be concluded that the iteration number of the algorithm is large when the value of \( k \) is small, where more computation is required for the attribute weight adjustment. However, the ranking of the selected alternatives can still be preserved with attribute weight adjustment.

5. Conclusion

Attribute weight determination is an important issue in multi-attribute decision making. However, in case the pattern of data that associate with an attribute may have changed, the literature is sparse to discuss the attribute weight adjustment. Existing approaches such as the Strategic Weight Manipulation (\(SWM\)) [30, 31] can not function well in this situation, especially
Figure 9: The different rankings of the selected alternatives for the different $k$ values
when there is no solution for SWM to utilize attribute weight adjustment with the requirement of protecting the ranking of alternatives. This article proposes such a method for the attribute weight adjustment, so that the scores of the alternative based on adjusted attribute weight can adapt to changes in the data. Through a three-step approach, the effectiveness of the proposed algorithm is demonstrated based on real-world tax data of the enterprises and individuals. A sensitivity analysis of the proposed method is also conducted through empirical experiments which demonstrate the robustness of the proposed method.

Whilst promising, this research opens up an avenue for significant further investigation. For instance, it would be useful to extend the work to deal with the scenarios where the underlying data are subject to continuous change. Future work also involves incorporating the weight adjustment method into fuzzy models [34–37] that have commonly been adopted to work with linguistic uncertainty arising from group decision making.

**Acknowledgements** This work is sponsored under the National Science Foundation of China under Grant No.61973296, No.61702414, No.61373116, and No.61602369. This work is also supported under the Shenzhen Basic Research Program Ref.JCYJ20170818153635759 and Science and Technology Planning Project of Guangdong Province Ref.2017B010117009; Science and Technology Project in Shaanxi Province of China (Program No.2019ZDLGY07-08); Science and Technology Co-ordination and Innovation Project (Program No.2016KTZDGY04-01); Xi’an Project for College Talent Providing Services to Enterprise (No.GXYD17.15).

**References**


[32] Y. Liu, Y. Dong, and H. Liang et.al., “Multiple attribute strategic weight manipulation with minimum cost in a group decision making context with


