Effects of Idiosyncratic Jumps and Co-jumps on Oil, Gold, and Copper Markets

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Abstract
Using one-minute oil, gold and copper futures price from September 27, 2009, to July 1, 2020, this paper examines the effects of systematic and idiosyncratic (market-specific risk) jumps on intraday correlations, portfolio allocation decisions, and diversification benefits. We identify that these commodities contain high proportions of market-specific price discontinuities, which do not translate into systematic jumps. Co-jumps in the same direction lead to higher correlations and imply reduction in diversification benefits, while co-jumps in the opposite direction reduce correlations and positively affect diversification, similar to the idiosyncratic jumps. The results also demonstrate that the risk-averse investor’s gold portfolio allocations are not affected by co-jumps and are free from the non-diversifiable risks in oil and copper markets. However, idiosyncratic jumps in oil and copper markets increase allocations to gold. In contrast, allocations to copper and oil are significantly affected by the systematic risks outlined in copper-gold and oil-gold pairs, pushing risk-averse investors to oil from copper-gold and copper from oil-gold systematic risks. Finally, diversification benefits from price discontinuities are overall positive and driven by the idiosyncratic jumps in oil and copper markets when the minimum variance portfolio allocations are used.

Keywords: oil market; gold market; copper market; portfolio allocations; jumps and co-jumps; wavelets; COVID-19.
1 Introduction

It is well known that commodity price fluctuations can be sharp and sudden. There are also endogenous concerning the global macroeconomic conditions and large swings in prices can seriously harm both importing and exporting economies (Barsky & Kilian, 2004; Kilian, 2009). Researchers have expended significant effort on identifying key determinants that affect commodities pricing. For instance, supply shocks raised from production disruptions; demand shocks due to unexpected strong economic growth and rapid recovery from the Global Financial Crisis of 2008-9 (e.g., Abhyankar et al., 2013; Frankel, 2014). In addition, monetary policy and uncertainty are also believed to have played a critical role in driving the price of commodities (see e.g., Beck, 2001; Gozgor et al., 2016; Xu et al., 2021). While earlier works on the determinants of commodity prices assumed that the impact of macroeconomic shocks on commodities do not vary over time, there is increased evidence to show that the relationship may be unstable (Byrne et al., 2020). In particular, during periods of severe uncertainty, commodity prices may not be fully reflecting macroeconomic fundamentals due to informational frictions (e.g., Byrne et al., 2019; Sockin & Xiong, 2015).

The World Health Organization (WHO) declared the outbreak of COVID-19 as a Public Health Emergency of International Concern on January 30, 2020,1 and the situation escalated rapidly into a global pandemic on March 11, 2020.2 The pandemic has triggered a massive spike in uncertainty with significant impacts on demand, reductions in firms’ profitability and growth, income, employment and productivity (Altig et al., 2020). The oil market has also suffered a substantial plunge due to a combination of weak demand triggered by the COVID-19 pandemic and Saudi authorities’ unexpected decision to offer price discounts of $6 to $8 to their main customers in Europe, Asia, and the US in 2020. Thus, the ongoing COVID-19 pandemic is not only a health crisis, but a social, economic, and political catastrophe, affecting all populations worldwide. It is not surprising that researchers have begun to link the impact of COVID-19 on financial markets and commodity markets, as

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1https://www.who.int/emergencies/diseases/novel-coronavirus-2019/events-as-they-happen
the repercussions of this crisis greatly affect countries throughout the globe (e.g., Amar et al., 2021; Salisu et al., 2020; Sharif et al., 2020; Apergis et al., 2021). The interest of including non-conventional asset classes in the portfolio was triggered by the financialisation of commodity markets (Basak & Pavlova, 2016).

Motivated by the unique and sharp impact of the COVID-19 pandemic on commodity markets, our paper aims to analyse market-specific and systematic shocks on correlations, portfolio allocation decisions and diversification benefits in the high-frequency prices of oil, gold and copper continuous futures contracts by using the high-frequency (one-minute) price data. We have focused on oil, gold and copper given their importance in the global economy, high trading volumes, heterogeneous characteristics for investors in developed and developing markets. To be more specific, oil is an important source of energy for the global economy and is essential for economic activity (Xu & Ouenniche, 2011); gold stores wealth and is regarded as the safe haven (Bilgin et al., 2018); and copper is a general health indicator for the global economy given its wide use in the economy (Basak & Pavlova, 2016). These features attract hedge funds, insurance companies and other financial institutions to consider oil, copper and gold as viable alternative investments to reduce exposure of portfolios containing only conventional assets to harvest higher returns (e.g., Bonato et al., 2020; Y. Li et al., 2021).

Despite oil-gold-copper markets interdependencies have been extensively studied in the literature (e.g., Sadorsky, 2014; Barunik et al., 2016; Bonato et al., 2020), to the best of our knowledge, there is no study analysing the impact of market-specific (idiosyncratic jumps) and systematic risks (co-jumps) on interdependencies, portfolio allocations and diversification benefits in these markets, especially in connection with high-frequency data on commodities and the employed methodology in our investigation. Indeed, there is limited attention on investigating the impact of co-jumps on correlations, diversification benefits or an investor’s allocation decisions. For instance, Aït-Sahalia & Xiu (2016) investigate the impact of jumps and co-jumps on correlations for oil and US major stock index futures and conclude that co-jumps lead to increased intraday correlations in these markets using visual assessment of the obtained measures. In contrast, Barunik & Vacha (2018) perform a more formal analysis
of co-jumps impact on correlations. The authors demonstrate the statistical significance of co-jumps for correlations in currency markets during the European Union (EU) and US trading sessions. Barunik & Fiser (2019) extend the correlation analysis of Barunik & Vacha (2018) to the interrelationships in EU and US Treasury markets. Oliva & Renò (2018) demonstrate the significance of jumps and co-jumps in the hedge fund indices’ portfolio allocation problem using the monthly data. Arouri et al. (2019) investigate interrelationships of co-jumps, portfolio allocation decisions and diversification benefits in the developed and emerging stock markets relying on the high-frequency ETF data. More recently, Laurini et al. (2020) have investigated the portfolio allocation problem under co-jumps in the oil sector (oil and major oil-producing companies); however, their analysis is based on weekly data.

The portfolio theory suggests that diversification is an effective tool to minimise investors’ risks, given that the portfolio is composed of low correlated assets driven by different economic fundamentals. However, periods of high uncertainty such as the Global Financial Crisis of 2008-9 or the COVID-19 pandemic have shown that often uncorrelated assets demonstrate higher dependencies accompanied by sharp and simultaneous appreciations and depreciations across financial markets (e.g., Aït-Sahalia & Xiu, 2016). If a portfolio is well-diversified, market-specific jumps shall reduce assets’ correlation and thus improve diversification benefits. Hence, isolated jumps are diversifiable. Inversely, suppose there is a simultaneous price jump in more than one market. In that case, co-jumps shall increase correlations and reduce diversification benefits given that prices co-jump in the same direction. Hence, co-jumps represent systematic risk and are not diversifiable. Investors holding commodities in their portfolios are exposed to both systematic and market-specific risks (e.g., H. Li et al., 2020; Han et al., 2021). Therefore, it is valuable to explore how risks are originating in the high-frequency data impact asset pricing, hedging strategies and other investors’ decisions. Furthermore, identifying the systematic and market-specific risks and understanding their impact on investors is also valuable for policymakers aiming to stabilise energy or other commodity markets.

Several authors highlight the hedging value of gold and copper for oil investors in the
context of volatility spillovers and the growing interconnectedness of energy and commodity markets (e.g., Apergis et al., 2020; Guhathakurta et al., 2020). Our study extends this strand of literature in several ways. First, we employ wavelet decomposition of the stochastic processes by Barunik & Vacha (2018) to one-minute oil, gold and copper continuous futures contracts and obtain microstructure noise-free and bootstrap robust jumps and co-jumps components of the 3x3 Realised Variation-Covariation (VCOV) matrix. Second, we study the impact of the idiosyncratic jumps and co-jumps in these markets on correlations, minimum variance portfolio allocation decisions and diversification benefits. Our findings demonstrate that the risk-averse investor’s gold portfolio allocations are not affected by the studied commodities’ systematic risks. Besides, such an investor responds to the oil and copper market isolated shocks with increased allocations to gold. The only deterrent to the risk-averse investor’s gold allocations is the gold market idiosyncratic risk, a natural deterrent for a risk-averse investor. This evidence indicates gold’s safe haven properties against the risks outlined in the high-frequency commodity data. We also demonstrate that risk-averse investors respond with higher oil portfolio allocations to gold-copper systematic risk and higher copper allocations to oil-gold systematic risks. Their findings complement the S. H. Kang et al.’s (2017) argument on gold risk transmission properties in the commodities markets. Indeed, systematic risk in the oil-gold pair is a deterrent for oil investments, while the systematic risk of the gold-copper pair is a deterrent for copper investments. Finally, despite the statistically significant impact of co-jumps on correlations, we demonstrate that diversification benefits from price discontinuities for the oil-gold-copper investor are, overall, positive and driven by the idiosyncratic jumps in copper and oil markets when we account for portfolio allocation decisions. Since the statistically significant impact of co-jumps on correlation does not always translate into a reduction of diversification benefits, we suggest this evidence is a valuable empirical observation complimenting the findings of Barunik & Vacha (2018).

The rest of the paper is organised as follows. Section 2 provides a brief review of the literature, describing interrelationships of oil, gold and copper. Section 3 presents our testing
framework for the impact of jumps and co-jumps on correlations, portfolio allocations, and diversification benefits. Section 4 describes the data. Section 5 reports the findings and discusses the implications of the empirical results. Section 6 concludes the paper.

2 Literature Review

Our study relates to several strands of earlier literature that have examined the jumps, co-jumps, oil market, and interconnectedness of commodity markets. In this section, we provide a summary of the major studies along these lines.

2.1 Jumps, Co-jumps and Realised Correlations, Portfolio Allocations and Diversification Benefits

One strand of literature focuses on investigating the commodity markets’ interdependence and risk spillover (e.g., Maitra et al., 2021). However, such studies do not separate continuous and discontinuous parts of the VCOV matrices. Our paper employs the wavelet decomposition framework and overall testing philosophy of Barunik & Vacha (2018). To complement their correlation analysis, we take further steps and study the impact of jumps and co-jumps on portfolio allocations and diversification benefits similar to Arouri et al. (2019). However, Arouri et al. (2019) do not study the impact of price discontinuities on the international equity markets’ correlation structure. Besides, our approach to link the interrelationship of jumps and co-jumps with investor allocation decisions and diversification benefits is different. While Arouri et al. (2019) rely on jump and co-jump intensity measures and correlation analysis, we employ a logistic regression setting as in Barunik & Vacha (2018). In addition, Barunik & Vacha (2018) estimators allow determining the precise location of price discontinues and minimise identification of false systematic jumps due to the presence of large idiosyncratic jumps. This evidence provides robust inference in the volatile market environments, such as the early markets’ response to the COVID-19 pandemic outbreak.
2.2 What Drives Oil Prices?

Supply and demand fundamentals are important factors in driving oil price fluctuations (e.g., Abhyankar et al., 2013; Hamilton, 2003; Kilian, 2009). Dramatic swings in oil prices before and after the Global Financial Crisis have sparked discussions on the role of speculative trading activity and the share of “paper” oil markets. No obligation to store oil physically provides an alternative venue for trading based on expectations. Several authors point out that commodity markets’ financialisation and growth in index speculators’ positions significantly impacted commodity prices’ high volatilities between 2005 and 2008 (e.g., Cheng & Xiong, 2014; Singleton, 2014). An alternative empirical strategy to detect speculative effects is through the level of inventory. If momentum trading in financial markets is a primary driver of price trends in physical markets, one would expect to observe an upswing in inventories’ speculative holdings. However, there is a lack of consensus as to the effects of speculation on oil prices. For instance, several authors have found the level of inventories did not surge before the July 2008 oil price peaked (e.g., Fattouh et al., 2013; Kilian & Murphy, 2014), while others show that speculative shocks played an important role in explaining oil price fluctuations (e.g., Juvenal & Petrella, 2015; Kilian & Murphy, 2014).

On the other hand, Sockin & Xiong (2015) and Byrne et al. (2019) show that severe informational frictions could confuse market participants about the global economy’s strength and oil demand relative to supply. Several authors also explore the impact of uncertainty on oil prices and show that oil prices respond negatively to uncertainty indices such as economic policy uncertainty, energy market uncertainty (e.g., Antonakakis et al., 2014; Xu et al., 2021). In addition, a large body of studies focus on predicting the oil prices/volatility and evaluate the relative performance of competing forecasting models (e.g., Xu & Ouenniche, 2011; Ouenniche et al., 2014, 2017).

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3The composition of participants in commodity futures markets has changed dramatically since the 2000s. There has been a large inflow of investment capital from non-user speculators and passive investors in commodity futures markets.
2.3 Relationship between Oil and Gold Markets

The connectedness of oil and gold markets can be theoretically supported through the inflationary channel (Sari et al., 2010). An increase in oil prices is often linked to higher transportation and production costs; hence, higher inflationary pressures for oil-importing countries and, consequently, positive pressure on gold prices to hedge oil originated inflation. However, inflation should not serve as an ultimate theoretical foundation for the oil-gold relationship, and a set of financial, macroeconomic and geopolitical events shall be considered (Pindyck et al., 1990). In addition to increased financialisation of commodities (e.g., Adams & Glück, 2015), heterogeneous fundamentals of oil-gold interrelationship has led researchers to investigate integrated risk spillover frameworks in the commodity and other financial markets (Bonato et al., 2020). This strand of the literature is often based on the variations of the Vector Auto-Regressive (VAR) model forecast-error variance decomposition as in Diebold & Yılmaz (2014) and its spectral representation of financial cycles as in Baruník & Křehlík (2018). Risk spillover network representation advances literature analysing interdependence of oil and gold with variations of static or dynamic bivariate dependence structures (e.g., Bonato et al., 2020; Baruník et al., 2016; Junttila et al., 2018; Reboredo, 2013). However, there remains no uniform agreement on the shock origination in the commodities spillover literature. Awartani & Maghyereh (2013) argue that the oil market is the main transmitter of shocks in commodity markets after 2008. Diebold et al. (2017) confirm this view and show that energy commodities are closely connected with industrial and precious metals and are the main transmitters of shocks to other commodities.

On the other hand, motivated by the hinges of the commodity markets around growth in China’s industrial production and consumption levels, Organisation of Petroleum Exporting Countries (OPEC) production caps and import/export policy decisions scrutinised by W. Kang et al. (2017) for oil markets, Guhathakurta et al. (2020) show that gold is the main transmitter of shocks to oil and other commodities when spillover networks are accounted for structural breaks. This finding is also supported with frequency analysis of financial cycles in the commodity markets by Tiwari et al. (2021). Moreover, several authors employ time-varying
methodologies to capture the evolving relationship between oil prices and the macroeconomy over time. For instance, Byrne et al. (2019) use time-varying parameter VAR models and find that oil supply shocks impact on oil price were smaller during the 1980s, 1990s, and 2000s and intensified with the start of the US oil shale revolution.

### 2.4 Relationship between Oil and Copper Markets

Despite the importance of copper and its ability to capture stages in the world economy’s business cycles, less attention has been paid to studying the relationship between oil and copper than to oil and gold markets (Sadorsky, 2014). Karanasos et al. (2018) investigate time-varying volatility spillovers in gold and copper prices and highlight that supply factors of copper have been less elastic than supply factors for oil since the US oil shale revolution. Hence, oil and copper interrelationships may not always be fully reflected by the current macroeconomic cycle due to growing oil supply flexibility (Byrne et al., 2019). Sadorsky (2014) points out that differences in arbitrage activities, asymmetric information, and additional contract liquidity could also contribute to variation in oil and copper’s interdependencies. Yet, two commodities remain highly connected. For example, with a set of Heterogenous Auto-Regressive (HAR) models, Degiannakis & Filis (2017) demonstrate that information contained in copper realised volatility is valuable for oil intraday volatility forecasting. Ahmadi et al. (2016) and Y. Li et al. (2021) add that economic fluctuations and financial risks due to high uncertainty periods in the international financial markets, such as the Global Financial Crisis of 2008-9, the European Sovereign Debt Crisis in 2011, the US oil shale revolution, and China-US tensions, cause abnormal impact of gold and oil on copper markets. This evidence also highlights a consensus on the time-varying and heterogeneous nature of the commodities’ interrelationships, as empirically shown by Guhathakurta et al. (2020) and Tiwari et al. (2021) among others.
3 Methodology

Our computations rely on the set of realised variation-covariation (VCOV) matrices with and without discontinuous components of the log-price processes for oil, gold and copper continuous contracts. Steps to obtain VCOV matrices for computations rely on the wavelet decomposition of the stochastic processes outlined by Barunik & Vacha (2018) and are provided in the Appendix.

3.1 Realised Total and Continuous Correlations, Portfolio Allocations and Diversification Benefits

Once continuous elements of the VCOV matrix are obtained on the jumps adjusted data and validated with the bootstrap procedure, the bivariate total realised correlation measure is outlined by:

$$
corr_T^{(total)} = \frac{IC_{l_j,l_j} + CJ_{l_j,l_j}}{\sqrt{IC_{l_j,l_j} + CJ_{l_j,l_j}} \cdot \sqrt{IC_{l_j,l_j} + CJ_{l_j,l_j}}}
$$

and can be obtained with the noise robust two-scale (TSCV) estimators of Zhang (2011). That is,

$$
\hat{corr}_T^{(total)} = \frac{\hat{QV}_{TSCV}^{l_j,l_j}}{\sqrt{\hat{QV}_{TSCV}^{l_j,l_j} \cdot \sqrt{\hat{QV}_{TSCV}^{l_j,l_j}}}}.
$$

To achieve one of our main investigation targets, we are also required to construct continuous component of the total correlation measure as prescribed by Barunik & Vacha (2018). Continuous correlation is a jumps and co-jumps free measure formally defined as:

$$
\hat{corr}_T^{(cont)} = \frac{\hat{IC}_{JWC}^{l_j,l_j}}{\sqrt{\hat{IC}_{JWC}^{l_j,l_j} \cdot \sqrt{\hat{IC}_{JWC}^{l_j,l_j}}}}.
$$

Next, we rely on the parsimonious minimum variance portfolio strategy and obtain weights for the total realised VCOV matrix that minimises:

$$
\min_w \sigma_{T,P}^{(total)} = w^T \Sigma_{TSCV} w, \quad (3)
$$
where $w$ is a vector of optimal weights for the total VCOV matrix $\hat{\Sigma}^{TSCV}$ and is subject to the $w^T \cdot 1 = 1$ constraint. However, to investigate the impact of jumps and co-jumps on an investor’s portfolio allocation decisions we also require a vector of optimal weights for continuous components of the VCOV matrix. Similar to the weights in (3), it can be obtained by minimising:

$$\min_{w^*_s} \sigma^{(cont)}_{T,P} = w^*_s \hat{\Sigma}^{JWC} w^*_s,$$

(4)

where $w_s$ is a vector of optimal weights for the continuous VCOV matrix $\hat{\Sigma}^{JWC}$ and is subject to the $w_s^T \cdot 1 = 1$ constraint.

Finally, to study the impact of jumps and co-jumps on the diversification benefits in these markets for a risk averse investor, we compute the Conditional Diversification Benefits (CDB) measures of Christoffersen et al. (2012):

$$CDB^{(total)}_T(w, q) = \frac{w^T \sigma^{TSCV} - \sigma^{(total)}_P}{w^T \sigma^{TSCV} - q \cdot \sigma^{(total)}_P \cdot \Phi^{-1}(q)} \cdot \phi(\Phi^{-1}(q))$$

(5)

for the realised $\sigma^{(total)}_P$ portfolio variation defined in (3), its optimal weights vector $w$ and a vector of individual realised asset variations $\sigma^{TSCV}$, and:

$$CDB^{(cont)}_T(w^*_s, q) = \frac{w^*_s \sigma^{JWC} - \sigma^{(cont)}_P}{w^*_s \sigma^{JWC} - q \cdot \sigma^{(cont)}_P \cdot \Phi^{-1}(q)} \cdot \phi(\Phi^{-1}(q))$$

(6)

for the realised $\sigma^{(cont)}_P$ portfolio variation defined in (4), its optimal weights vector $w_s$ and a vector of individual realised asset continuous variations $\sigma^{JWC}$; where $\Phi$ and $\phi$ denote the standard normal distribution and density functions and $q = 0.05$ respectively. To enhance our understanding of the diversification benefits in the context of jumps and co-jumps, we also define a CDB measure based on the continuous VCOV matrix components and weights optimal for (3). It is given by:

$$CDB^{*}_T(w, q) = \frac{w^T \sigma^{JWC} - w^T \hat{\Sigma}^{JWC} w}{w^T \sigma^{JWC} - q \cdot w^T \hat{\Sigma}^{JWC} w \cdot \Phi^{-1}(q)} \cdot \phi(\Phi^{-1}(q))$$

(7)
Note that we make a rough normality assumption on the CDB and portfolio composition measures. It is convenient in operations with realised VCOV matrices and is consistent with assumptions on the data generating process Barunik & Vacha (2018) impose in the bootstrap procedure. Moreover, Arouri et al. (2019) do not find notable differences in portfolio compositions with VCOV and CVaR approaches in the similar investigation context. Therefore, normality assumption should be tolerable for our investigation.

3.2 Impact of Jumps and Co-jumps on Realised Correlations, Portfolio Allocations and Diversification Benefits

The impact of jumps and co-jumps on realised correlations can be established by obtaining total and continuous correlations with (1) and (2) for every trading session and running a simple univariate regression model for every data pair, such as:

\[
\hat{\text{corr}}^{(\text{total})}_T = \alpha + \beta \cdot \hat{\text{corr}}^{(\text{cont})}_T + \epsilon_T, \tag{8}
\]

where \(\epsilon_T\) denotes i.i.d. error term with zero mean and constant variance. Regression framework in (8) is straightforward to implement and it allows investigating the statistical significance of hypotheses:

\[H_0: \quad \alpha = 0 \cap \beta = 1\]
\[H_a: \quad \alpha \neq 0 \cap \beta \neq 1\]

If \(\alpha = 0\) and \(\beta = 1\) jointly, there is no statistical difference in correlations with and without jumps and co-jumps. Hence, \(\hat{\text{corr}}^{(\text{total})}_T - \hat{\text{corr}}^{(\text{cont})}_T = 0\) and impacts of jumps and co-jumps on correlations are negligible. However, Barunik & Vacha (2018) also suggest constructing an alternative test for the impact of jumps and/or co-jumps on correlations in the form of the parsimonious logistic regression. For impact of co-jumps on correlations it can be given by:
\[ P \left( \hat{\text{corr}}^{(\text{total})}_T \geq \hat{\text{corr}}^{(\text{cont})}_T \middle| \hat{C} \hat{J}_T \right) = \frac{1}{1 + e^{-(\theta + \beta_1 \cdot \hat{C} \hat{J}_T)}}. \] (9)

We also define idiosyncratic jumps (market specific or isolated jumps) as \( \hat{J} \hat{J}_T \) for \( \hat{C} \hat{J}_T = 0 \) and run a set of tests similar to the test setting in (9), such as:

\[ P \left( \hat{\text{corr}}^{(\text{total})}_T \geq \hat{\text{corr}}^{(\text{cont})}_T \middle| \hat{J} \hat{J}_T \right) = \frac{1}{1 + e^{-(\theta + \alpha_1 \cdot \hat{J} \hat{J}_T)}} \] (10)

to test specific impact of idiosyncratic jumps and:

\[ P \left( \hat{\text{corr}}^{(\text{total})}_T \geq \hat{\text{corr}}^{(\text{cont})}_T \middle| \hat{J} \hat{J}_T \cap \hat{C} \hat{J}_T \right) = \frac{1}{1 + e^{-(\theta + \alpha_1 \cdot \hat{J} \hat{J}_T + \beta_2 \cdot \hat{C} \hat{J}_T)}} \] (11)

for the joint assessment of the isolated jumps and co-jumps statistical significance for correlations. The test framework outlined in (9), (10) and (11) is straightforward to interpret and should be robust in the regression settings with low number of bootstrap validated co-jumps (Barunik & Vacha, 2018). We shall be relying on this framework to conduct assessment of allocation decisions and diversification benefits responses to idiosyncratic jumps and co-jumps in our three asset portfolio. For example, a joint test for the impact of jumps and co-jumps on realised minimum variance portfolio allocation decisions is given by

\[ P \left( \hat{w}^{(\text{total})}_T \geq \hat{w}^{(\text{cont})}_{T,*} \middle| \hat{J} \hat{J}_T \cap \hat{C} \hat{J}_T \right) = \frac{1}{1 + e^{-(\theta + \sum_n \alpha_n \cdot \hat{J} \hat{J}_T + \sum_m \beta_m \cdot \hat{C} \hat{J}_T)}}. \] (12)

Note that for bivariate correlations our testing is naturally restricted to the jump and co-jumps components relevant to the investigated correlation pair, while for the assessment of portfolio allocation decisions, we are not bound by a bivariate measure and can assess changes to portfolio allocations of each asset with all discontinuous components of our realised VCOV matrix. This shall provide a more comprehensive view on the risk-averse investor responses to the market specific and/or systematic risks contained in the high-frequency data. In addition to the context of our investigation and overall estimation methods, we believe that this is what differentiates our work from the analysis conducted by Arouri et al. (2019) the most. For the different data and portfolio allocation frequency, Arouri et al. (2019)
draw inference on investment decisions and diversification benefits based on the correlation analysis with respect to jump and co-jump intensity in the emerging and developed stock markets. However, we analyse differences in portfolio allocations and diversification benefits with and without discontinuous components, which shall provide robust and more stable results.

Finally, we outline two regression tests for impact of idiosyncratic jumps and co-jumps on CDB in oil, gold and copper markets given by:

\[
P\left( \hat{CDB}_T^{(total)} \leq \hat{CDB}_T^* | JJ_T \cap CJ_T \right) = \frac{1}{1 + e^{-(\theta + \sum_n^3 \alpha_n \cdot \hat{JJ}_T + \sum_m^3 \beta_m \cdot \hat{CJ}_T)}}
\]

and

\[
P\left( \hat{CDB}_T^{(total)} \leq \hat{CDB}_T^{(cont)} | JJ_T \cap CJ_T \right) = \frac{1}{1 + e^{-(\theta + \sum_n^3 \alpha_n \cdot \hat{JJ}_T + \sum_m^3 \beta_m \cdot \hat{CJ}_T)}}.
\]

Note that we assume CDB based on the continuous VCOV matrix to be higher or equal to the CDB with jumps and co-jumps following empirical results of Arouri et al. (2019) for diversification benefits and co-jump intensity. Since the impact of jumps and co-jumps on diversification benefits is conditional on portfolio weights, we also perform regressions with weighted jump and co-jump variations for weights in (3) and run separate regressions on the impact of co-jumps and jumps similar to (9) and (10) for correlation analysis. Moreover, dependence of CDB on a set weights explains CDB measure we introduce in (7) and test setting in (13). To clarify, with (7) and (13) we attempt to restrict the impact of weights optimal for the continuous VCOV matrix and enhance our understanding of the interrelationship between systematic and market specific risks with diversification benefits in the commodity markets.

4 Data

We investigate crude oil (henceforth CL), gold (hereafter GC) and copper (henceforth HG) futures contracts for the period beginning on September 27, 2009, to July 1, 2020. For
our investigation, we extract one-minute futures high-frequency data from the database in kibot.com. This evidence covers the start of the oil markets response to and recovery from the Global Financial Crisis of 2008-9. Up to the recent oil prices collapse due to the absence of OPEC+ agreement on production caps and fall in oil demand due to the COVID-19 pandemic. The data are synchronised and automatically converted to the New York Eastern Time Zone (ET) by the data provider.\textsuperscript{4} We concentrate on trading hours from 7:00 am to 4:00 pm ET for each contract. This evidence is the most liquid part of the trading session, covers trading activity for nine hours and includes times of release for major macroeconomic indicators. Similar to Barunik & Fiser (2019), we focus on this part of the continuous 23-hour futures trading session to reduce potential estimation bias. Overall, after cleaning every contract for non-trading days and performing necessary data adjustments, we obtain a high-frequency data set for 2775 trading days. Therefore, we have around 1.5 million observations for each market.

Table 1 reports basic descriptive statistics for log-returns of every contract at one-minute and five-minute frequencies. Table 1 provides expected statistics at these frequencies for every contract apart from the CL excess kurtosis values. CL high kurtosis values are driven by the unprecedented events for international oil markets in the absence of the OPEC+ deal and COVID-19 outbreak. For example, in Figure 1, we observe anomalies in the high number of jumps and idiosyncratic jump variations for CL contracts around April 2020 when the closest to maturity WTI contract was priced negatively.

Figure 1 describes when jumps (top panel) and co-jumps (bottom-middle panel) occur during the selected trading hours. We observe that most of the jumps and co-jumps occur before starting and at the typical trading session’s early stages. Although determinants of jumps and co-jumps in the studied markets are beyond our research scope, we observe jump and co-jump occurrences around the times of major macroeconomic indicator announcements (see e.g., Aït-Sahalia & Xiu, 2016, for a list of major macroeconomic releases and their

\textsuperscript{4}kibot.com rolls contracts as is and does not back-adjust previous contract’s data to generate continuous data series. Conditional on the highest trading volume, rollover is either on expiration date or 2 to 7 days before the expiration date. The reader is referred to kibot.com for further details on continuous futures used in our investigation.
announcement times). For example, jumps in CL contract and co-jumps in CL-HG contracts peak around oil inventory announcements. There is also a notable spike in jumps for GC and CL contracts around the Federal Open Market Committee (FOMC) announcements. The FOMC announcements can also be observed for co-jumps in all pairs. The co-jumps occurrences are less frequent and indicate that markets may not always respond to the FOMC announcements simultaneously or may interpret monetary policy surprises differently. It has been long established in the literature that jumps in the commodities markets may be linked to macroeconomic surprises (e.g., Alquist et al., 2020; Smales, 2015). However, the evidence on the joint surprises or co-jumps remains scarce in the literature. Aït-Sahalia & Xiu (2016) debate whether jumps and co-jumps lead to higher correlations in the CL, stock, currency and treasury futures. Nevertheless, our context, objectives and methods for investigation are different. Determinants of co-jumps in CL, GC and HG contracts may be a worthwhile path for future research. Next, Figure 1 demonstrates the frequency of jumps (top-middle panel) and co-jumps (bottom panel) over time. We observe that GC contains the highest number of jumps, while HG and CL display similar numbers of jumps over time, which are also bootstrap validated in Table 2.

Table 2 shows the highest number of bootstrap validated trading days with idiosyncratic jumps for GC, CL, and HG. CL is the only contract that depicts a spike in the number of jumps linked to the COVID-19 pandemic. Co-jumps are less frequent than jumps for all pairs, with the highest number of co-jumps observed in HG-GC contracts. This finding is also confirmed for the bootstrap validated co-jumps in Table 2. To ensure the robustness of our conclusions for the number bootstrap validated co-jumps in Table 2 and following Barunik & Vacha (2018), we will be relying on the logistic regression settings and will not be performing analysis for the linear regression in Eq. (8).

[Insert Table 1, 2 and Figure 1 around here.]
5 Empirical Results and Discussion

This section describes our estimation results and discusses the obtained dependency, portfolio allocations, and diversification benefits measures.

5.1 Realised Correlations and Diversification

Figure 2 provides a bivariate view on our high-frequency dataset with realised correlation, continuous and discontinuous components of the realised covariation, and differences in correlation due to the VCOV matrix’s discontinuous components. We observe time-evolving interdependencies of oil, gold and copper markets (bottom-middle panel). This evidence is also observed in the literature for these commodities (e.g., Barunik et al., 2016; Y. Li et al., 2021; Sadorsky, 2014). Dependency measures for all pairs peak around oil prices heights before the US oil shale revolution and the initial OPEC response to protect its market share from growing competition (see, e.g., Ansari, 2017; Bataa & Park, 2017; Fantazzini, 2016). After oil price heights, correlations in all pairs return to overall pre-financial crisis levels (Barunik et al., 2016; Sadorsky, 2014); however, dependency cycles that can be linked to economic fluctuations, such as the European Sovereign Debt Crisis, the US oil shale revolution, and the China-US tensions highlighted in Y. Li et al. (2021) are notable.

Observed evolution of the oil-gold and copper-gold correlations also implies the growing value of gold diversification properties for oil and copper markets over time. Oil-copper correlation has diminished from the oil prices heights before the US oil shale revolution; however, oil and copper dependency is relatively more stable and mostly positive for the period under investigation. This finding points out that both commodities have the potential to capture business cycle, inflation and strength of the US dollar (Wang & Chueh, 2013) under higher supply flexibility in the international oil markets after the shale revolution. Realised CDB for minimum variance in the oil-gold-copper portfolio in Figure 5 (top left panel) also confirms implications for diversification from our brief realised correlations analysis. Observed overall trend on growing diversification, compliments a strand of the literature pointing out a narrowing window for effective risk management across different sets of assets.
post-financial crisis (e.g., Avdulaj & Barunik, 2015; Barunik et al., 2016). To clarify, we observe lowering CDB in Figure 5 around periods of high economic uncertainties highlighted in Y. Li et al. (2021); however, a general trend and level of measured diversification potential is close or higher to the pre-financial crisis diversification levels reported in Avdulaj & Barunik (2015).

[Insert Figures 2, 3, 4 and 5 around here]

5.2 Realised Portfolio Allocations

Figure 4 (top panel) illustrates risk-averse investors’ realised portfolio allocations. We observe that portfolio allocations are heavily gold oriented. High reliance on the safe haven asset may be expected for a risk-averse investor and is consistent with the literature. For example, using daily data Guhathakurta et al. (2020) show that long oil positions hedged with gold contracts achieve higher risk reduction and hedging effectiveness than long and short oil positions with other commodities. However, the high-frequency setting allows us to complement a static and bivariate view provided in Guhathakurta et al. (2020). It is interesting to point out that the lowest gold allocations and highest oil allocations are achieved immediately before the oil price down-slide on the US shale supply shock. As oil markets stabilise, we observe that oil gradually regains positions in the risk-averse portfolio to its post-financial crisis levels; however, during high uncertainty periods on the oil markets oversupply and the COVID-19 pandemic outbreak, allocations are minimal. Allocations to copper are more stable than oil allocations and are less affected during oil prices collapse due to the US shale supply shock but are notably impacted by the Sino-US tensions.

Now we proceed analysing oil-gold-copper price discontinuities and their impact on the obtained realised dependency, portfolio allocations and diversification benefits measures that distinguish our contribution to the commodities interdependence literature.
5.3 Impact of Price Discontinuities on Realised Correlations and Diversification

Figure 2 also illustrates co-jump variation (top panel) and continuous covariation (top-middle panel) for the investigated pairs of commodities. We observe predominately positive continuous covariation for the oil and copper pair, while the systematic risk contained in its co-jump variation provides a less consistent outlook. To be specific, co-jump variation is mainly positive until the change in oil fundamentals by US shale suppliers. Continuous covariation indicates that oil and copper co-move together, reflecting the interdependence of commodities on the business cycle and overall macroeconomic outlook; however, from co-jump variation, we observe that information shocks are interpreted differently by oil and copper markets after the US shale revolution and during the period of COVID-19 shocks. Though still undiversifiable, such a heterogeneous response to the systematic risk is valuable for investors looking to benefit from the commodity cycles and minimise the adverse impact of simultaneous information shocks. On the other hand, this co-jump variation pattern is expected and observed for both gold pairs. It is given by the gold’s safe haven properties and for copper it indicates this metal’s growing potential as an oil risk diversifier in the current market conditions.

Figure 2 (bottom panel) correlation differences from price discontinuities are less profound than for the currency markets reported in Barunik & Vacha (2018). Moreover, we observe that correlation differences are mostly negative on average. This evidence can be explained by the dominance of idiosyncratic jump variation illustrated in Figure 3 and heterogeneous responses to simultaneous information shocks depicted in Figure 2 (top panel). Correlation difference is positive for oil and copper post-financial crisis until the oil prices decline on the US oil shale revolution supply shocks and is consistent with observation of the oil and copper co-jump variation. This positive difference in correlation implies lower diversification benefits for oil and copper investors and is empirically confirmed by the diversification benefits (CDB) differences for the oil-gold-copper minimum variance portfolio depicted in Figure 5 (bottom panel). Inversely, diversification gains from price discontinuities in the studied
commodities are, on average, positive during the US oil shale revolution and COVID-19 pandemic. This result is confirmed by the weighted total portfolio co-jump variation in Figure 5 (top right), especially regarding the period of unprecedented oil prices response to the COVID-19 pandemic outbreak. Therefore, we demonstrate that studied commodities could be a valuable addition to a well-diversified portfolio from the perspective of both systematic and market-specific risks contained in the high-frequency data.

Table 3 presents the impact of price discontinuities on realised correlations. We find that though bootstrap validated co-jumps are less common in these markets, they are statistically significant for all realised correlation pairs. We identify an overall positive relationship between systematic risk and correlation and empirically demonstrate that idiosyncratic jumps reduce the commodities market’s dependencies. From portfolio theory, this result has direct implications for diversification effectiveness and is consistent with Barunik & Vacha (2018) findings for currency and Barunik & Fiser (2019) for treasury markets. However, this bivariate correlation analysis neglects the potential impact of portfolio allocation decisions. Therefore, we report determinants of the realised CDB measures for the oil-gold-copper minimum variance portfolio in Table 5 for a more specific conclusion.

[Insert Tables 3, 4 and 5 around here]

From Table 5, we find that only systematic risk from the simultaneous jumps in copper and gold is positively associated with a reduction in diversification benefits when we account for minimum variance portfolio allocation weights. This finding can be explained by the highest number of trading days with bootstrap validated co-jumps in the gold-copper pair and the realised portfolio allocations’ findings. Accounting for portfolio weights, we also find that idiosyncratic jumps in oil and copper markets have a positive statistically significant impact on diversification benefits. In contrast, we do not obtain similar results for gold market-specific jumps. We offer the following explanation. Gold market-specific jumps as all price discontinuities in our CDB test settings, are accounted with weights optimal for the
total VCOV matrix in Eq. (3). They are statistically significant if we do not control portfolio allocations or in the test set with Eq. (14), which is also conditional on the set of weights optimal for the continuous VCOV matrix in Eq. (4). Therefore, statistically significant gold market-specific jumps may capture variation in the omitted continuous weights. However, suppose we neglect weights optimal for Eq. (4) in the test set with Eq. (13) and account only for portfolio weights optimal for Eq. (3). In that case, we do not find idiosyncratic gold jumps to be the statistically significant determinant for reduction in diversification benefits. This evidence could be an additional indication on gold safe haven properties for the commodities market.

5.4 Impact of Price Discontinuities on Realised Portfolio Allocations

From Figure 4 (bottom panel), it is interesting to point out that changes in gold allocations due to systematic and market-specific risks are negative. On the other hand, copper seems to gain from price discontinuities among studied commodities consistently. Oil neither depicts consistent gains nor losses from price discontinuities, with the most fluctuation in allocation differences observed before the US oil shale revolution. However, in the multivariate setting, conclusions drawn from the visual assessment of differences in portfolio allocations may be incomplete.

Table 4 provides determinants of portfolio allocations due to price discontinuities in oil, gold and copper markets to complement our visual assessment of investment decisions. We find that gold portfolio allocations are not affected by the systematic risk in the studied commodity markets.\(^5\) Besides, gold allocations increase in response to the idiosyncratic jumps in copper and oil markets. This evidence highlights the safe haven property of gold for commodities investors. Gold allocation is negatively affected only by its market-specific jumps. This result may be expected for a risk-averse investor and explains negative allocations to gold in Figure 4. Moreover, it is consistent with the number of trading days with bootstrap

\(^5\)We run additional estimations by substituting gold with silver continuous contract and do not observe similar properties for silver, while the majority of other implications hold. These results are provided in the Appendix.
statistically significant gold idiosyncratic jumps in Table 2 (highest among considered commodities). However, from Table 5, we do not find that systematic risk outlined in oil-gold and oil-copper pairs has a statistically significant impact on reducing diversification benefits.

Furthermore, by analysing allocation decision determinants for copper and oil, we observe that both commodities gain from gold market isolated jumps. However, the coefficient for oil allocation is consistently lower than for copper allocation. Though statistically significant, this indicates allocation to oil has a lower likelihood than allocation to copper due to the gold market-specific risks. Risk-averse investors reduce copper allocation due to its idiosyncratic jumps, with gold having a higher likelihood of acquiring portfolio rebalancing due to copper market-specific discontinuities than oil. It is also interesting to point out that the copper-gold and oil-gold pairs’ systematic risk pushes investors to oil and copper from copper and oil, respectively. Moreover, oil allocations are unaffected by its market-specific risks, indicating that the risk-averse investor prefers to hedge these risks with gold but not with copper. This evidence resonates well and complements general conclusions in Tiwari et al. (2021) on gold being the main transmitter of shocks in the commodity markets and gold’s effectiveness for hedging long positions in oil, as outlined by Guhathakurta et al. (2020).

6 Conclusion

Financialisation of commodity markets and instantaneous market reactions to external events, such as the COVID-19 pandemic, expose investors to adverse and simultaneous price movements in the commodity markets. Therefore, it is important to understand how systematic and market specific risks contained in the high-frequency data impact dependency measures and diversification benefits for profound investment decisions. Moreover, understanding investor behaviour in the presence of systematic and market-specific risks could be important for policy makers targeting stability in energy, commodities and other markets. Note that risks contained in the high-frequency commodities data often originate from news and policy surprises that can propagate beyond the commodities markets (e.g., see Alquist et al., 2020,
for oil inventories). Hence, knowledge of investor’s responses to sharp market diversifiable and non-diversifiable movements, allows policy markets to make more informed decisions on policy introductions to maintain market balance, efficiency and smooth transition to a new policy regime. With ongoing financialisation of oil, energy markets incorporate information faster and require a clear set of expectations from regulators to minimise such surprises through signalling prior potential policy announcements. However, Känzig’s (2021) argues that the information channel is less important for oil than for interest rate markets, since OPEC decisions may be driven by a political or market power agenda (e.g., similar to the OPEC+ failure to reach an agreement on the production caps in the early stages of the pandemic recently). Therefore, unless regulators abandon pursuing non-economic targets, understanding of risks contained in the high-frequency data may remain more valuable information for investors.

Our paper made a first attempt to fulfil this gap for commodities in the context of one-minute oil, gold and copper continuous futures data. To achieve this, we employed wavelet decomposition of the stochastic processes and focused on the data post-Global Financial Crisis 2008-9 period. We examined the effects of systematic and market-specific risks on intraday correlations, portfolio allocation, and diversification decisions. We demonstrated that a statistically significant impact of systematic risks on commodities correlations does not necessarily translate into reduction of diversification benefits and that investors can gain diversification from the market-specific price discontinuities in the copper and oil markets. We also found and highlighted safe haven properties of gold from the perspective of systematic risks in commodity markets as well as demonstrated portfolio rebalancing decisions under the different types of risk contained in the high-frequency data. Thus, our findings contain novel empirical observations that compliment Barunik & Vacha (2018) and enhance our understanding of the commodities market’s interdependence structure. Future papers can use high-frequency futures price data to analyse the effects of co-jumps and idiosyncratic jumps on intraday correlations, portfolio allocation decisions, and diversification benefits among other commodities, cryptocurrencies, and stock markets.
References


Table 1: Descriptive Statistics.

<table>
<thead>
<tr>
<th></th>
<th>Crude Oil (CL)</th>
<th>Gold (GC)</th>
<th>Cooper (HG)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1 min.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>$-0.58 \cdot 10^{-4}$</td>
<td>$0.39 \cdot 10^{-5}$</td>
<td>$0.58 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>Minimum</td>
<td>-20.6631</td>
<td>-1.3742</td>
<td>-1.2316</td>
</tr>
<tr>
<td>Maximum</td>
<td>18.5318</td>
<td>2.0244</td>
<td>1.6887</td>
</tr>
<tr>
<td>Sd. Dev.</td>
<td>0.0966</td>
<td>0.0350</td>
<td>0.0448</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.1880</td>
<td>0.0955</td>
<td>0.0372</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3503.9019</td>
<td>55.9593</td>
<td>18.6008</td>
</tr>
<tr>
<td><strong>5 min.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>$-0.29 \cdot 10^{-3}$</td>
<td>$0.19 \cdot 10^{-4}$</td>
<td>$0.29 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>Minimum</td>
<td>-34.4720</td>
<td>-1.7700</td>
<td>-2.7153</td>
</tr>
<tr>
<td>Maximum</td>
<td>30.9188</td>
<td>1.9725</td>
<td>1.6887</td>
</tr>
<tr>
<td>Sd. Dev.</td>
<td>0.2219</td>
<td>0.0757</td>
<td>0.0963</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.8492</td>
<td>-0.0546</td>
<td>-0.1171</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3601.5295</td>
<td>25.0472</td>
<td>15.3554</td>
</tr>
</tbody>
</table>

Notes: This table reports the descriptive statistics for log-returns of crude oil (CL), gold (GC) and copper (HG) at one-minute and five-minute frequencies.

Table 2: Total Number of Trading Days with Bootstrap Significant Jumps and Co-jumps.

<table>
<thead>
<tr>
<th></th>
<th>CL</th>
<th>GC</th>
<th>HG</th>
<th>HG-GC</th>
<th>CL-GC</th>
<th>CL-HG</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>384</strong></td>
<td>595</td>
<td>376</td>
<td>220</td>
<td>157</td>
<td>161</td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table shows the highest number of bootstrap validated trading days with idiosyncratic jumps for crude oil (CL), gold (GC) and HG (copper); and three co-jumps.
Table 3: Impact of Co-jumps and Idiosyncratic Jumps on HG-GC, CL-GC and CL-HG Correlations.

<table>
<thead>
<tr>
<th></th>
<th>HG - GC</th>
<th>CL - GC</th>
<th>CL - HG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>-0.1595</td>
<td>-0.1073</td>
<td>-0.0558</td>
</tr>
<tr>
<td>$CJ_{T}^{{HG,GC}}$</td>
<td>$\beta_1$</td>
<td>5.2079</td>
<td>***</td>
</tr>
<tr>
<td>$CJ_{T}^{{CL,GC}}$</td>
<td>$\beta_2$</td>
<td>-</td>
<td>2.0313</td>
</tr>
<tr>
<td>$CJ_{T}^{{CL,HG}}$</td>
<td>$\beta_3$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1057</td>
<td>-0.0101</td>
<td>0.0982</td>
</tr>
<tr>
<td>$JJ_{T}^{{GC}}$</td>
<td>$\alpha_1$</td>
<td>-2.9732</td>
<td>***</td>
</tr>
<tr>
<td>$JJ_{T}^{{HG}}$</td>
<td>$\alpha_2$</td>
<td>-2.3921</td>
<td>***</td>
</tr>
<tr>
<td>$JJ_{T}^{{CL}}$</td>
<td>$\alpha_3$</td>
<td>-</td>
<td>-0.0251</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.0939</td>
<td>-0.0140</td>
<td>0.1003</td>
</tr>
<tr>
<td>$CJ_{T}^{{HG,CL}}$</td>
<td>$\beta_1$</td>
<td>5.1273</td>
<td>***</td>
</tr>
<tr>
<td>$CJ_{T}^{{CL,GC}}$</td>
<td>$\beta_2$</td>
<td>-</td>
<td>1.9979</td>
</tr>
<tr>
<td>$CJ_{T}^{{CL,HG}}$</td>
<td>$\beta_3$</td>
<td>-</td>
<td>3.1907</td>
</tr>
<tr>
<td>$JJ_{T}^{{GC}}$</td>
<td>$\alpha_1$</td>
<td>-2.9407</td>
<td>***</td>
</tr>
<tr>
<td>$JJ_{T}^{{HG}}$</td>
<td>$\alpha_2$</td>
<td>-2.3668</td>
<td>***</td>
</tr>
<tr>
<td>$JJ_{T}^{{CL}}$</td>
<td>$\alpha_3$</td>
<td>-</td>
<td>-0.0248</td>
</tr>
</tbody>
</table>

Notes: ***$p < 0.01$; **$p < 0.05$; *$p < 0.1$ for White’s heteroscedasticity consistent standard errors.
Table 4: Impact of Co-jumps and Idiosyncratic Jumps on Minimum Variance Realised Portfolio Allocation Decisions.

<table>
<thead>
<tr>
<th></th>
<th>GC</th>
<th>HG</th>
<th>CL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>-0.2597</td>
<td>0.3842</td>
<td>0.5317</td>
</tr>
<tr>
<td>$\tilde{CJ}_{T}^{{HG,GC}} \beta_1$</td>
<td>-0.1011</td>
<td>-1.5961</td>
<td>4.4286</td>
</tr>
<tr>
<td>$\tilde{CJ}_{T}^{{CL,GC}} \beta_2$</td>
<td>-0.4992</td>
<td>1.7214</td>
<td>-4.0373</td>
</tr>
<tr>
<td>$\tilde{CJ}_{T}^{{CL,HG}} \beta_3$</td>
<td>0.7936</td>
<td>-0.0499</td>
<td>-0.0052</td>
</tr>
<tr>
<td>$\theta$</td>
<td>-0.1154</td>
<td>0.3201</td>
<td>0.4176</td>
</tr>
<tr>
<td>$\tilde{JJ}_{T}^{{GC}} \alpha_1$</td>
<td>-6.4490</td>
<td>4.2264</td>
<td>1.2065</td>
</tr>
<tr>
<td>$\tilde{JJ}_{T}^{{HG}} \alpha_2$</td>
<td>2.0922</td>
<td>-2.6151</td>
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<td>$\tilde{JJ}_{T}^{{CL}} \alpha_3$</td>
<td>0.0405</td>
<td>-0.0244</td>
<td>0.0095</td>
</tr>
</tbody>
</table>

Notes: *** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$ for White’s heteroscedasticity consistent standard errors.
Table 5: Impact of Co-jumps and Idiosyncratic Jumps on Realised Conditional Diversification Benefits.

<table>
<thead>
<tr>
<th></th>
<th>CDB(^1)</th>
<th>CDB(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\hat{C}_{JT}^{{HG,GC}})</td>
<td>(\theta)</td>
<td>-0.2961 ***</td>
</tr>
<tr>
<td>(\hat{C}_{JT}^{{CL,GC}})</td>
<td>(\beta_1)</td>
<td>1.8659 ***</td>
</tr>
<tr>
<td>(\hat{C}_{JT}^{{CL,HG}})</td>
<td>(\beta_2)</td>
<td>-0.5906</td>
</tr>
<tr>
<td>(\hat{C}_{JT}^{{HG,GC}})</td>
<td>(\beta_3)</td>
<td>0.2242</td>
</tr>
</tbody>
</table>

\[2 \cdot w_T^{\{HG\}} w_T^{\{GC\}} \cdot \hat{C}_{JT}^{\{HG,GC\}}\]

<table>
<thead>
<tr>
<th></th>
<th>CDB(^1)</th>
<th>CDB(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_3)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[\sum 2 \cdot w_T^{(i)} w_T^{(j)} \cdot \hat{C}_{JT}^{(i,j)}\]

<table>
<thead>
<tr>
<th></th>
<th>CDB(^1)</th>
<th>CDB(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha_3)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[\sum w_T^{(i)} \cdot \hat{JJ}_T^{(i,i)}\]

Notes: **\(p < 0.01\); *\(p < 0.05\); *\(p < 0.1\) for White’s heteroskedasticity consistent standard errors. CDB\(^1\) denotes test setting in Eq. (13) and CDB\(^2\) denotes test setting in Eq. (14).
Figure 1: Jumps and Co-Jumps Distribution.

Notes: The top panel reports GC, HG and CL Jumps distribution, second panel shows Jumps frequency; third panel reports HG-GC, CL-GC and CL-HG Co-Jumps distribution; and the bottom panel shows Co-jumps frequency.
Figure 2: The impact of price discontinuities on realised correlations and diversification.

Notes: The top panel reports HG-GC, CL-GC and CL-HG Co-jump Variation; second panel shows continuous covariation; third panel reports realised correlation; and the bottom panel shows realised correlation difference due to jumps and Co-jumps. The rolling 21 trading day average for realised correlations and correlation differences due to jumps and co-jumps are in blue and red, respectively.
Figure 3: CG, HG and CL Idiosyncratic Jump Variation.

Notes: These figures report CG, HG and CL idiosyncratic jump variation.
Figure 4: Optimal Portfolio Weights and weights differences.

Notes: The top panel reports GC, HG and CL Minimum Variance Optimal Portfolio Weights, while the bottom panel shows the Optimal Weights Differences due to Jumps and Co-jumps. The rolling 21 trading day average for realised correlations and correlation differences due to jumps and co-jumps are in blue and red, respectively.
Figure 5: Realised portfolios.

Notes: CG, HG and CL Minimum Variance Portfolio Realised CDB (top left), Sum of Weighted Co-jump Variation (top right), and Differences for Portfolio Realised CDB due to Jumps and Co-jumps for True-Only VCOV Optimal Weights (bottom left) and True-Continuous VCOV Optimal Weights (bottom right). Rolling 21 trading day average for realised portfolio CDB and CDB difference due to jumps and co-jumps are in blue and red, respectively.
Appendix

Realised VCOV Estimation and Decomposition

For the fixed time period $t \in [0, T]$ with $0 \leq t \leq T$, consider a bivariate log-price process as:

$$Y_t = (Y_{t,l_1}, Y_{t,l_2})'$$

for simplicity of the methodology description. $Y_t$ can be decomposed into an underlying (latent) log-price process $X_t$ and i.i.d. zero mean finite variance microstructure noise process $\epsilon_t$, such that $Y_t = X_t + \epsilon_t$. Components of the latent log-price process $X_t$ are time-varying and follow the dynamics described by:

$$dX_{t,l_1} = \mu_{t,l_1} dt + \sigma_{t,l_1} dB_{t,l_1} + dJ_{t,l_1}$$
$$dX_{t,l_2} = \mu_{t,l_2} dt + \sigma_{t,l_2} dB_{t,l_2} + dJ_{t,l_2},$$

where $\mu_{t,l_j}$ and $\sigma_{t,l_j}$ are càdlàg stochastic processes, $B_{t,l_j}$ is a Brownian motion correlated with $\rho^{l_1,l_2}(B_{t,l_1}, B_{t,l_2})$, and $J_{t,l_j}$ is a right continuous pure jump process with finite and correlated jumps for $j = 1, 2$. Further, fixed period covariation of $X_{t,l_1}$ and $X_{t,l_2}$ returns can be represented by the quadratic returns covariation ($QV_{l_1,l_2}$) composed of two parts - integrated (continuous) covariance part $IC_{l_1,l_2}$ and co-jump (discontinuous) variation part $CJ_{l_1,l_2}$. That is,

$$QV_{l_1,l_2} = \int_0^T \sigma_{t,l_1} \sigma_{t,l_2} d\langle B_{l_1}, B_{l_2} \rangle_t + \sum_{0 \leq t \leq T} \Delta J_{t,l_1} \Delta J_{t,l_2}.$$  

The full VCOV matrix of the observed data pair is then given by:

$$\Sigma = IC + CJ = \begin{pmatrix}
IC_{l_1,l_1} + CJ_{l_1,l_1} & IC_{l_1,l_2} + CJ_{l_1,l_2} \\
IC_{l_2,l_1} + CJ_{l_2,l_1} & IC_{l_2,l_2} + CJ_{l_2,l_2}
\end{pmatrix},$$

(15)
where \( l_j = l_j \) elements represent quadratic variation and \( l_j \neq l_j \) elements represent quadratic covariation. In the investigation we focus on the impact of idiosyncratic jumps and co-jumps on correlations, portfolio allocation decisions and diversification benefits in crude oil, gold and copper futures portfolio; therefore, following Barunik & Vacha (2018) and Barunik & Fiser (2019), we estimate all continuous and discontinuous components of the 3x3 VCOV matrix similar to (15). If the observed price process is free from the microstructure noise, realised covariance estimator is a consistent estimator of the quadratic covariation (e.g. Andersen et al., 2003; Barndorff-Nielsen & Shephard, 2004). Realised covariance \( QV^{RC}_{t_1,t_2} \) for covariance associated with \((Y_{t_1,l_1}, Y_{t_2,l_2})\) can be computed as:

\[
\hat{QV}^{RC}_{t_1,t_2} = \sum_{i=1}^{N} \Delta_i Y_{t_1,l_1} \cdot \Delta_i Y_{t_2,l_2},
\]

where \( \Delta_i Y_{t_1,l_1} = Y_{t+i/N,t} - Y_{t+i-1/N,t} \). However, following Barunik & Vacha (2018) and Barunik & Fiser (2019), we are interested in covariation of the noise-free latent process \((X_{t_1,l_1}, X_{t_2,l_2})\). To achieve this, we employ wavelets to recover co-jumps and obtain covariation of the latent process with the two-scale microstructure noise robust estimators of the subsampled intraday returns as in Zhang (2011) and similar to Barunik & Vacha (2015) for univariate intraday series. For the jump size:

\[
\Delta_i J_{t,l} = \Delta_i Y_{t,l} \cdot 1_{|W_{l_1,k}^d| > \xi},
\]

where \( W_{l_1,k}^d \) is a wavelet coefficient at the first scale and:

\[
\xi = \sqrt{2} \text{median} (|W_{l_1,k}^d|) \sqrt{2 \log N} \cdot 1.48258,
\]

the jump and co-jump variation components of (15) can be estimated with:

\[
\hat{CJ}_{l_j,l_j} = \sum_{i=1}^{N} \Delta_i J_{t,l_j} \cdot \Delta_i J_{t,l_j}
\]

for \( j = 1, 2 \). Employing estimator in (16), the time-synchronised and jump-adjusted log-price process is then given by \( Y_{t,l_j}^d = Y_{t,l_j} - \hat{CJ}_{l_j,l_j} \), while noise robust wavelet covariance estimator
(JWC) of the integrated covariance (IC) for the jump-adjusted process is outlined by:

$$\hat{I}C_{t_1,t_2}^{JWC} = \sum_{j=1}^{m+1} c_N \left( \hat{I}C_{t_1,t_2}^{(G,J)}(j) - \frac{\bar{n}_G}{n_S} \hat{I}C_{t_1,t_2}^{(WRC,J)}(j) \right), \quad (17)$$

where $C_N$ is a small sample precision constant, $\bar{n}_G = (N-G+1)/G$ and $n_S = (N-S+1)/S$. Barunik & Vacha (2018) set $c_N = 1$, $S_N = 1$ and choose value of $G$ based on the number of intraday observations following the guidelines developed by Zhang (2011). Remaining components of the IC estimator in (17) can be obtained with wavelet based estimators following Barunik & Vacha (2015). For a wavelet scale $j$ and grid size $\bar{n} = N/G$, Barunik & Vacha (2018) obtain the first component with

$$\hat{I}C_{t_1,t_2}^{(G,J)}(j) = \frac{1}{G} \sum_{g=1}^{G} \sum_{k=1}^{N} W_{j,k}^{l_1} \cdot W_{j,k}^{l_2},$$

while the second term in (17) for the wavelet scale $j$ is then provided by:

$$\hat{I}C_{t_1,t_2}^{(WRC,J)}(j) = \sum_{k=1}^{N} W_{j,k}^{l_1} \cdot W_{j,k}^{l_2}.$$

Barunik et al. (2016) improves finite sample properties of jump tests based on the realised measures with bootstrap procedure in the univariate volatility setting, while Barunik & Vacha (2018) extends bootstrap of Barunik et al. (2016) to the multivariate framework and recommends computing bootstrap robust intraday VCOV components. Therefore, for each element of the realised VCOV, we perform 500 bootstrap realisations per trading day and obtain bootstrap and microstructure noise robust continuous and discontinuous components of the VCOV matrix for our estimations. To detect significant jump and co-jump variation, bootstrap of Barunik & Vacha (2018) contains several steps. First, generate $k$ intraday returns as:

$$\Delta_k y_{t_1}^* = \sqrt{\frac{1}{N} \hat{I}C_{t_1,t_1}^{JWC} \eta_{k,t_1}}$$

$$\Delta_k y_{t_2}^* = \sqrt{\frac{1}{N} \hat{I}C_{t_2,t_2}^{JWC} \left( \hat{\rho}_{t_1,t_2} \cdot \eta_{k,t_1} + \sqrt{1 - \hat{\rho}_{t_1,t_2}^2} \cdot \eta_{k,t_2} \right)},$$

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$$\Delta_k y_{t_2}^* = \sqrt{\frac{1}{N} \hat{I}C_{t_2,t_2}^{JWC} \left( \hat{\rho}_{t_1,t_2} \cdot \eta_{k,t_1} + \sqrt{1 - \hat{\rho}_{t_1,t_2}^2} \cdot \eta_{k,t_2} \right)},$$

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$$\Delta_k y_{t_1}^* = \sqrt{\frac{1}{N} \hat{I}C_{t_1,t_1}^{JWC} \eta_{k,t_1}}$$

$$\Delta_k y_{t_2}^* = \sqrt{\frac{1}{N} \hat{I}C_{t_2,t_2}^{JWC} \left( \hat{\rho}_{t_1,t_2} \cdot \eta_{k,t_1} + \sqrt{1 - \hat{\rho}_{t_1,t_2}^2} \cdot \eta_{k,t_2} \right)},$$

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$$\Delta_k y_{t_1}^* = \sqrt{\frac{1}{N} \hat{I}C_{t_1,t_1}^{JWC} \eta_{k,t_1}}$$

$$\Delta_k y_{t_2}^* = \sqrt{\frac{1}{N} \hat{I}C_{t_2,t_2}^{JWC} \left( \hat{\rho}_{t_1,t_2} \cdot \eta_{k,t_1} + \sqrt{1 - \hat{\rho}_{t_1,t_2}^2} \cdot \eta_{k,t_2} \right)},$$

Barunik et al. (2016) improves finite sample properties of jump tests based on the realised measures with bootstrap procedure in the univariate volatility setting, while Barunik & Vacha (2018) extends bootstrap of Barunik et al. (2016) to the multivariate framework and recommends computing bootstrap robust intraday VCOV components. Therefore, for each element of the realised VCOV, we perform 500 bootstrap realisations per trading day and obtain bootstrap and microstructure noise robust continuous and discontinuous components of the VCOV matrix for our estimations. To detect significant jump and co-jump variation, bootstrap of Barunik & Vacha (2018) contains several steps. First, generate $k$ intraday returns as:

$$\Delta_k y_{t_1}^* = \sqrt{\frac{1}{N} \hat{I}C_{t_1,t_1}^{JWC} \eta_{k,t_1}}$$

$$\Delta_k y_{t_2}^* = \sqrt{\frac{1}{N} \hat{I}C_{t_2,t_2}^{JWC} \left( \hat{\rho}_{t_1,t_2} \cdot \eta_{k,t_1} + \sqrt{1 - \hat{\rho}_{t_1,t_2}^2} \cdot \eta_{k,t_2} \right)},$$
with \( \hat{\rho}_{l_1,l_2} \) being the computed correlation from the \( \widehat{IC}^{JWC} \) matrix and \( \eta_{k,l_j} \sim \mathcal{N}(0,1) \) for \( j = 1, 2 \). Second, estimate \( \widehat{QV}^{RC}_{l_j,l_j} \) and \( \widehat{IC}^{JWC*}_{l_j,l_j} \) with simulated intraday data. Next, repeat first two steps for \( b = 1, \cdots, B \) bootstrap iterations and obtain:

\[
Z^*(b) = \frac{\widehat{QV}^{RC*}_{l_j,l_j} - \widehat{IC}^{JWC*}_{l_j,l_j}}{\widehat{QV}^{RC*}_{l_j,l_j}}
\]

for every realisation. Finally, compute a bootstrap statistic with

\[
Z = \frac{\widehat{QV}^{RC}_{l_j,l_j} - \widehat{IC}^{JWC}_{l_j,l_j}}{\sqrt{\text{Var}(Z^*)}} \sim \mathcal{N}(0,1)
\]

and construct all elements of the final VCOV matrix \( \widehat{IC}^{JWC*} \) as:

\[
\widehat{IC}^{JWC*}_{l_j,l_j} = \mathbb{1}_{\{|Z| \leq \phi_{1-\alpha/2}\}} \widehat{QV}^{RC}_{l_j,l_j} + \mathbb{1}_{\{|Z| > \phi_{1-\alpha/2}\}} \widehat{IC}^{JWC}_{l_j,l_j}
\]

where \( \phi_{1-\alpha/2} \) is a critical value for the two-sided test with a significance level \( \alpha \). Note that diagonal elements of the VCOV matrix are subject to the one-sided test in the final step of the bootstrap procedure (see Barunik et al., 2016, for details).

**Additional Estimation Results with Silver, Copper and Oil Realised VCOV Matrix**

In this section of the appendix, we provide key estimation results for the 3x3 VCOV matrix where GC has been substituted with silver (SI) continuous contract. SI is a common alternative to GC, and output obtained here confirms implications from the main text with the exception that SI demonstrates weaker “safe heaven” properties. These outputs are consistent with the literature on the relationship of oil with precious metals and supports our analysis of risks contained in the high-frequency data.
<table>
<thead>
<tr>
<th></th>
<th>HG - SI</th>
<th>CL - SI</th>
<th>CL - HG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>-0.0823 *</td>
<td>-0.0843 *</td>
<td>-0.0558</td>
</tr>
<tr>
<td>$\tilde{CJ}_{T}^{{HG,SI}}$</td>
<td>$\beta_1$ 3.5309 ***</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\tilde{CJ}_{T}^{{CL,SI}}$</td>
<td>$\beta_2$ -</td>
<td>2.1386 ***</td>
<td>-</td>
</tr>
<tr>
<td>$\tilde{CJ}_{T}^{{CL,HG}}$</td>
<td>$\beta_3$ -</td>
<td>-</td>
<td>3.1388 ***</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.2211 ***</td>
<td>0.0475 **</td>
<td>0.0982 *</td>
</tr>
<tr>
<td>$\tilde{JJ}_{T}^{{SI}}$</td>
<td>$\alpha_1$ -1.3487 ***</td>
<td>-0.6701 ***</td>
<td>-</td>
</tr>
<tr>
<td>$\tilde{JJ}_{T}^{{HG}}$</td>
<td>$\alpha_2$ -3.2332 ***</td>
<td>-</td>
<td>-3.3212 ***</td>
</tr>
<tr>
<td>$\tilde{JJ}_{T}^{{CL}}$</td>
<td>$\alpha_3$ -</td>
<td>-2.2294 *</td>
<td>-0.1029</td>
</tr>
</tbody>
</table>

Notes: ***$p < 0.01$; **$p < 0.05$; *$p < 0.1$ for White’s heteroscedasticity consistent standard errors.
Table A.2: Impact of Co-jumps and Idiosyncratic Jumps on Minimum Variance Realised Portfolio Allocation Decisions (Silver).

<table>
<thead>
<tr>
<th></th>
<th>SI</th>
<th>HG</th>
<th>CL</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>0.3049</td>
<td>**</td>
<td>-0.0099</td>
</tr>
<tr>
<td>( \hat{CJ}_{T} {HG,SI} ) ( \beta_1 )</td>
<td>-0.5703</td>
<td>0.4836</td>
<td>**</td>
</tr>
<tr>
<td>( \hat{CJ}_{T} {CL,SI} ) ( \beta_2 )</td>
<td>-0.2241</td>
<td>0.0819</td>
<td>-0.3835</td>
</tr>
<tr>
<td>( \hat{CJ}_{T} {CL,HG} ) ( \beta_3 )</td>
<td>0.5671</td>
<td>**</td>
<td>0.0480</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.2174</td>
<td>***</td>
<td>0.1573</td>
</tr>
<tr>
<td>( \hat{JJ}_{T} {SI} ) ( \alpha_1 )</td>
<td>-0.1984</td>
<td>*</td>
<td>0.2459</td>
</tr>
<tr>
<td>( \hat{JJ}_{T} {HG} ) ( \alpha_2 )</td>
<td>3.4634</td>
<td>***</td>
<td>-7.4162</td>
</tr>
<tr>
<td>( \hat{JJ}_{T} {CL} ) ( \alpha_3 )</td>
<td>0.0158</td>
<td>**</td>
<td>-0.0017</td>
</tr>
</tbody>
</table>

Notes: ***p < 0.01; **p < 0.05; *p < 0.1 for White’s heteroschedasticity consistent standard errors.
Table A.3: Impact of Co-jumps and Idiosyncratic Jumps on Realised Conditional Diversification Benefits (Silver).

<table>
<thead>
<tr>
<th></th>
<th>CDB(^1)</th>
<th>CDB(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(CDB^{†})</td>
<td>θ</td>
<td>-0.4010</td>
</tr>
<tr>
<td>(CDB^{‡})</td>
<td>β₁</td>
<td>0.0374</td>
</tr>
<tr>
<td>(CDB^{\beta})</td>
<td>β₂</td>
<td>-0.0183</td>
</tr>
<tr>
<td>(CDB^{γ})</td>
<td>β₃</td>
<td>0.1755</td>
</tr>
<tr>
<td>(2 \cdot w_{T}^{HG} \cdot \hat{C}J_{T}^{HG,SI})</td>
<td>θ</td>
<td>-0.4053</td>
</tr>
<tr>
<td>(2 \cdot w_{T}^{CL} \cdot \hat{C}J_{T}^{CL,SI})</td>
<td>β₁</td>
<td>3.4107</td>
</tr>
<tr>
<td>(2 \cdot w_{T}^{CL} \cdot \hat{C}J_{T}^{CL,HG})</td>
<td>β₂</td>
<td>-0.4266</td>
</tr>
<tr>
<td>(2 \cdot w_{T}^{CL} \cdot \hat{C}J_{T}^{CL,HG})</td>
<td>β₃</td>
<td>4.0308</td>
</tr>
<tr>
<td>(\sum 2 \cdot w_{T}^{(i)} \cdot \hat{C}J_{T}^{(i,j)})</td>
<td>θ</td>
<td>-0.4053</td>
</tr>
<tr>
<td>(\hat{C}J_{T}^{SI})</td>
<td>α₁</td>
<td>-0.7382</td>
</tr>
<tr>
<td>(\hat{C}J_{T}^{HG})</td>
<td>α₂</td>
<td>-1.7113</td>
</tr>
<tr>
<td>(\hat{C}J_{T}^{CL})</td>
<td>α₃</td>
<td>0.0026</td>
</tr>
<tr>
<td>(w_{T}^{2(GC)} \cdot \hat{C}J_{T}^{SI})</td>
<td>θ</td>
<td>-0.2890</td>
</tr>
<tr>
<td>(w_{T}^{2(HG)} \cdot \hat{C}J_{T}^{HG})</td>
<td>α₁</td>
<td>-7.1426</td>
</tr>
<tr>
<td>(w_{T}^{2(CL)} \cdot \hat{C}J_{T}^{CL})</td>
<td>α₂</td>
<td>-1.7722</td>
</tr>
<tr>
<td>(w_{T}^{2(CL)} \cdot \hat{C}J_{T}^{CL})</td>
<td>α₃</td>
<td>-31.9551</td>
</tr>
<tr>
<td>(\sum w_{T}^{(i)} \cdot \hat{C}J_{T}^{(i,i)})</td>
<td>θ</td>
<td>-0.2349</td>
</tr>
<tr>
<td>(\hat{C}J_{T}^{HG,SI})</td>
<td>β₁</td>
<td>-0.0189</td>
</tr>
<tr>
<td>(\hat{C}J_{T}^{CL,SI})</td>
<td>β₂</td>
<td>-0.0095</td>
</tr>
<tr>
<td>(\hat{C}J_{T}^{CL,HG})</td>
<td>β₃</td>
<td>0.2450</td>
</tr>
<tr>
<td>(\hat{C}J_{T}^{SI})</td>
<td>α₁</td>
<td>-0.7451</td>
</tr>
<tr>
<td>(\hat{C}J_{T}^{HG})</td>
<td>α₂</td>
<td>-1.7102</td>
</tr>
<tr>
<td>(\hat{C}J_{T}^{CL})</td>
<td>α₃</td>
<td>0.0026</td>
</tr>
<tr>
<td>(w_{T}^{2(GC)} \cdot \hat{C}J_{T}^{SI})</td>
<td>θ</td>
<td>-0.2438</td>
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<tr>
<td>(w_{T}^{2(HG)} \cdot \hat{C}J_{T}^{HG})</td>
<td>β₁</td>
<td>3.4591</td>
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<tr>
<td>(w_{T}^{2(CL)} \cdot \hat{C}J_{T}^{CL})</td>
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<td>(w_{T}^{2(CL)} \cdot \hat{C}J_{T}^{CL})</td>
<td>β₃</td>
<td>4.5559</td>
</tr>
<tr>
<td>(\sum w_{T}^{(i)} \cdot \hat{C}J_{T}^{(i,i)})</td>
<td>θ</td>
<td>-0.2933</td>
</tr>
</tbody>
</table>

Notes: ***\(p < 0.01; \) **\(p < 0.05; \) *\(p < 0.1\) for White’s heteroscedasticity consistent standard errors. CDB\(^1\) denotes test setting in Eq. (13) and CDB\(^2\) denotes test setting in Eq. (14).