
Extension of an Analytical Method for Novel Volute Design

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Abstract: This paper extends and generalizes the analytical technique outlined by Eck (1973), for determining the outer boundary curve of a volute symmetric about the plane of rotation of the impeller of a centrifugal fan, under conditions of ideal flow, based on the principle of conservation of mass. The general technique is demonstrated using a circular cross-section and the method is then extended to spiral volutes of novel geometric cross-sections including the ellipse, semi-ellipses and conjoined semi-ellipses. The resulting volutes are presented in graphical form, together with corresponding equations.

Key words: volute, elliptic integral, centrifugal, compressor

INTRODUCTION

The volute is an essential component of a centrifugal pump or compressor, being the conduit between the environment and the impeller, controlling the balance between kinetic energy in the fluid and static pressure as the flow speed changes. Its design is a major determinant of performance in terms of operating range and efficiency (Xu et al., 2005). For this reason numerous attempts have been made (Mojaddam et al., 2012) to establish optimal volute shape and dimensions for various cross-sections. Most are computationally expensive.

The main principle adopted by Eck (1973) for modelling the air flow generated by the impeller of a centrifugal fan is that the flow is ideal in the aerodynamic sense of being inviscid, incompressible and devoid of turbulence. Further, the flow entering the volute is modelled by the streamlines generated by an isolated vortex located at the centre of the impeller. These cross the circular boundary representing the impeller limits, into the volute.

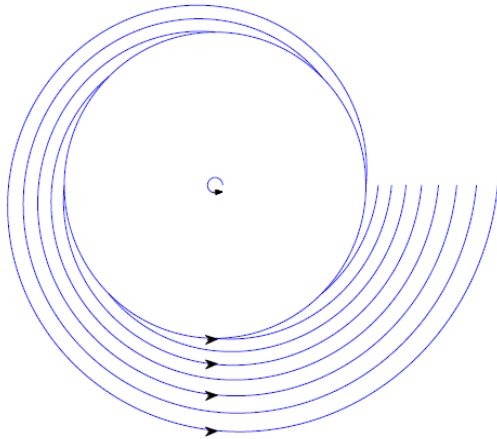


Figure 1: The spiral flow around a point vortex.

The circular inner edge of the volute, also represented by a streamline of the flow, models the circular orbit described by the outer edges of the impeller blades. The flow schematic illustrates the streamlines which, in this

case, follow a logarithmic spiral. The volute outline is shown in Figure 2.

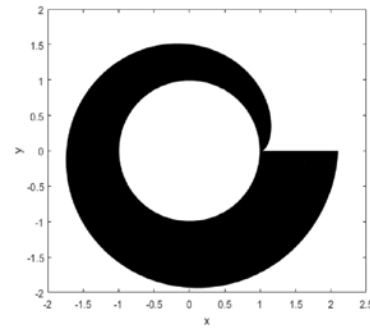


Figure 2: Typical volute outline based on streamlines

For the current work it was prudent to disregard the effects of friction in favour of the simplicity of the analysis. The reader is referred to Eck (1973) should details of the inclusion of friction be required. It is speculated here that the inclusion of friction would not add significantly to a model already burdened with the simplifying assumptions noted above.

The Model

The model is subject to two flow conditions: Firstly, $rc_u = const$ at distance r from the impeller centre, where c_u is the tangential velocity at that radius, this equation being associated with the isolated vortex. It is assumed that r is sufficiently large that the most local effects of the proximity of the impeller blades are negligible. Secondly, continuity of mass requires that

$$\frac{\phi V}{360} = \int_A c_m dA \quad (1)$$

where ϕ is the azimuth angle, V is the volumetric flow, c_m is the speed of flow through element dA of the cross-sectional area A at azimuth angle ϕ . In order to elucidate

the general method and verify its agreement with Eck (1973), the case of a volute with circular cross-section is first examined, the object being to determine the outer curve by calculating the diameter of the circle.

Circular cross-section

At a given azimuth angle, the equation of the circular cross-section is

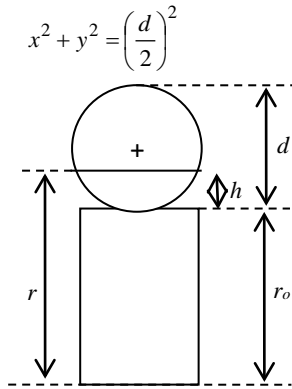


Figure 3: Volute geometry, circular cross-section

Conservation of mass (Equation (1)) then requires

$$\frac{\phi V}{360} = \int_A u_\phi \cdot dA = \int_0^d \frac{u_{\phi_0} r_0}{r_0 + h} \cdot 2\sqrt{h(d-h)} \cdot dh$$

where V is the volumetric flow, u_ϕ the flow velocity at (r, ϕ) and u_{ϕ_0} the flow velocity at (r_0, ϕ) .

Changing the variable of integration to $z = \frac{2h}{d} - 1$ and

setting $a = \frac{2r_0}{d} + 1$ then yields

$$\frac{\phi V}{360} = u_{\phi_0} r_0 d \int_{-1}^1 \frac{\sqrt{1-z^2}}{a+z} \cdot dz$$

Performing the integration leads¹ to the result:

$$\frac{\phi V}{360} = \pi u_{\phi_0} r_0 \left\{ (2r_0 + d) - \left[(2r_0 + d)^2 - d^2 \right]^{1/2} \right\}$$

Rearranging gives

$$\frac{d}{2} = \sqrt{\frac{\phi V (r_0 + \frac{d}{2})}{360 u_{\phi_0} r_0 \pi} - \frac{\phi^2 V^2}{(720 u_{\phi_0} r_0 \pi)^2}}$$

which is the formula given by Eck (1973).

Solving for d yields

¹ See Appendix for full details

$\frac{d}{2r_0} = \frac{1}{2}\Phi + \sqrt{\Phi}$ where $\Phi = \frac{\phi V}{360 u_{\phi_0} \pi r_0^2}$ is a dimensionless angular measure. The outer curve of the volute is then $r = r_0(1 + \sqrt{\Phi})^2$. Figure 4 shows the volute generated.

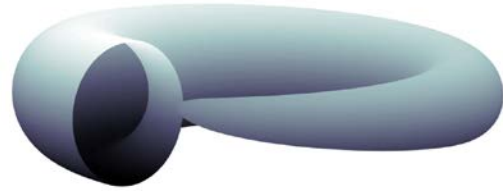


Figure 4: Volute of circular cross-section

Extension of the Method to Novel Cross-sections

Extending this technique to a variety of volute cross-sections leads to implicit or explicit equations for the outer curve. A few cases are presented here.

(i) Elliptical Cross-section

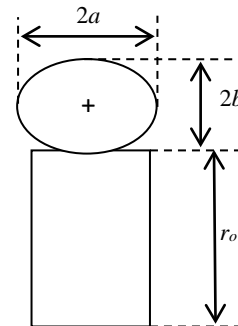


Figure 5: Volute geometry, elliptical cross-section

$$\text{Equation: } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Equation (1) then leads to Result (i):

$$\frac{\phi V}{360} = 2u_{\phi_0} r_0 a \pi \left\{ c - \sqrt{c^2 - 1} \right\} \text{ where } c = \frac{r_0}{b} + 1,$$

leading explicitly to

$$b = \sqrt{\frac{\phi V (r_0 + b) \left(\frac{b}{a}\right)}{360 u_{\phi_0} r_0 \pi} - \frac{\phi^2 V^2}{(720 u_{\phi_0} r_0 \pi)^2} \left(\frac{b}{a}\right)^2}$$

If the ratio $\frac{b}{a}$ is held constant through the volute, then

$$\frac{2a}{r_0} = \Phi \pm 2\sqrt{\frac{\Phi}{k}}$$

where $k = \frac{b}{a}$ and $\Phi = \frac{\phi V}{360u_{\phi_0}\pi r_0^2}$.

Figure 6 shows the volute obtained when $a:b = 4:1$.

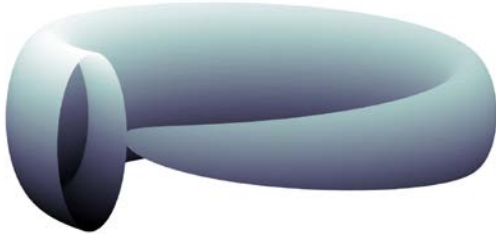


Figure 6: Volute of elliptical cross-section. Ratio of axes 4:1

(ii) Semi-ellipse cross-section, Type I

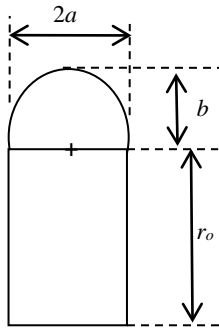


Figure 7: Volute geometry, semi-elliptical cross-section, Type I

$$\text{Equation: } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (y \geq 0)$$

Equation (1) leads to Result (ii):

$$\frac{\phi V}{360} = 2u_{\phi_0} r_0 a \left\{ \frac{\pi k}{2} - 1 - 2(k^2 - 1)^{\frac{1}{2}} \tan^{-1} \left(\frac{k-1}{k+1} \right)^{\frac{1}{2}} \right\},$$

where $k = \frac{r_0}{b}$, $k \geq 1$.

In this case there is no explicit formula for the outer curve of the volute. Variable k must be calculated iteratively at each azimuthal angle and b determined from it.

Figure 8 shows the volute obtained.

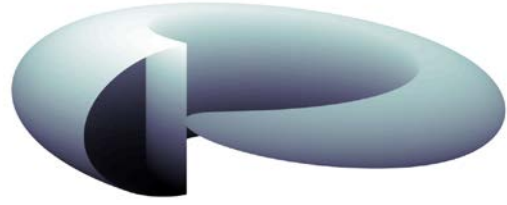


Figure 8: Volute of semi-elliptical cross-section (type 1)

(iii) Semi-ellipse cross-section, Type 2

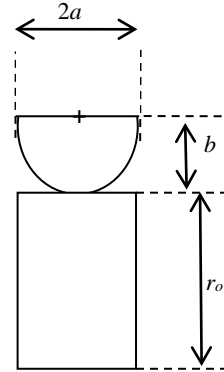


Figure 9: Volute geometry, semi-elliptical cross-section,

$$\text{Equation: } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (y \leq 0)$$

Equation (1) now leads to Result (iii):

$$\frac{\phi V}{360} = 2u_{\phi_0} r_0 a \left\{ \frac{\pi k}{2} + 1 - 2(k^2 - 1)^{\frac{1}{2}} \tan^{-1} \left(\frac{k+1}{k-1} \right)^{\frac{1}{2}} \right\}$$

where $k = \frac{r_0}{b} + 1$

As in case (ii) there is no explicit formula for the outer curve of the volute. Variable k must be calculated iteratively at each azimuthal angle and b determined from it.

Figure 10 shows the volute obtained.

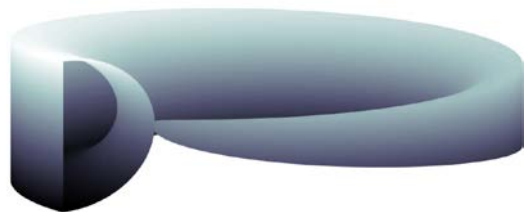


Figure 10: Volute of semi-elliptical cross-section (type 2)

(iv) Conjoined semi-elliptical cross-section

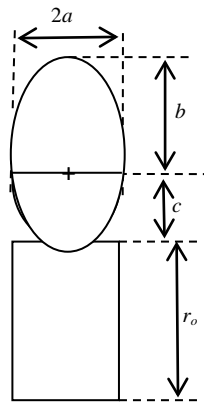


Figure 11: Volute geometry, conjoined semi-elliptical cross-section

$$\text{Upper semi-ellipse: } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (y \geq 0)$$

$$\text{Lower semi-ellipse: } \frac{x^2}{a^2} + \frac{y^2}{c^2} = 1 \quad (y \leq 0)$$

In this case Equation (1) leads to Result (iv):

$$\frac{\phi V}{360} = 2u_{\phi_0} r_0 a \left\{ \frac{\pi f}{2} - 2(f^2 - 1)^{\frac{1}{2}} \tan^{-1} \left(\frac{f+1}{f-1} \right)^{\frac{1}{2}} \right\} + 2u_{\phi_0} r_0 a \left\{ \frac{\pi g}{2} - 2(g^2 - 1)^{\frac{1}{2}} \tan^{-1} \left(\frac{g+1}{g-1} \right)^{\frac{1}{2}} \right\}$$

$$\text{where } f = \frac{r_0}{c} + 1 \text{ and } g = \frac{r_0}{c} + \frac{c}{b}$$

Putting $c=a$ or $b=a$ yields a semicircular cross-section. The case $a=b=c$ reduces to the circular case.

Again there is no explicit formula for the outer curve of the volute. In this case both $f(c)$ and $g(b,c)$ vary with azimuthal angle ϕ , as both b and c determine streamline in the flow. Fixing the ratios $a:b$ and $a:c$ enables both b and c to be expressed in terms of a . Variable a must then be calculated iteratively at each azimuthal angle and b and c determined from it.

Fixing the above ratios implicitly fixes the ratio $b:c$.

Figure 12 shows the volute obtained for $b:c = 1:3$

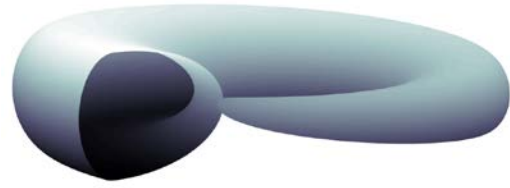


Figure 12: Volute with conjoined semi-elliptical cross-section. Ratio $b:c=1:3$

Conclusion

In conclusion, the elliptic integral used by Eck can be utilised to define a volute of almost any cross-section, provided it is symmetric with respect to the impeller plane. For some cross-sections with simple equations the integration may be performed analytically as in the above cases, giving Results (i) to (iv). For most cross-sections, including the above cases (ii) to (iv), the dimensions required are then implicit in the continuity equation obtained and must be determined iteratively at every azimuthal angle.

Future Work

The method may be applied to volutes whose cross-sections yield elliptic integrals which are not readily performed analytically, such as the hypercircle. It is presumed that such integrals may be evaluated numerically and that volutes may be generated from the resulting formulation. This is a more demanding aspect, but is to be pursued.

The method may be applied to volutes whose cross-sections are not simply connected, provided that the shapes enclosed by contours within the main cross-section are known and the integrations can be performed either analytically or numerically. Such volutes will be the subject of future papers.

Although this paper is intended first and foremost to demonstrate a method, the volutes obtained are at least a starting point for physical volute design, focussing particularly on the pressure change through the volute, viscosity, boundary layers, friction and turbulence. This is the domain of computational fluid dynamics and is the necessary developmental stage, using volutes derived by the method described here.

APPENDIX

To evaluate $\int_{-1}^1 \frac{(1-z^2)^{1/2}}{a+z} dz$, put

$\frac{(1-z^2)^{1/2}}{a+z} = \frac{(1-z^2)(a+z)}{(a+z)^2(1-z^2)^{1/2}}$ and resolve into partial fractions:

$$\frac{(1-z^2)^{1/2}}{a+z} \equiv \frac{a}{(1-z^2)^{1/2}} - \frac{z}{(1-z^2)^{1/2}} + \frac{(1-a^2)}{(1-z^2)^{1/2}(z+a)}$$

Then

$$\int_{-1}^1 \frac{(1-z^2)^{1/2}}{a+z} dz \equiv \int_{-1}^1 \frac{a}{(1-z^2)^{1/2}} dz - \int_{-1}^1 \frac{z}{(1-z^2)^{1/2}} dz + \int_{-1}^1 \frac{(1-a^2)}{(1-z^2)^{1/2}(z+a)} dz$$

$$= I_1 - I_2 + I_3, \text{ say.}$$

We have:

$$I_1 = \int_{-1}^1 \frac{a}{(1-z^2)^{1/2}} dz = [a \sin^{-1} z]_{-1}^1 = a\pi$$

$$I_2 = \int_{-1}^1 \frac{z}{(1-z^2)^{1/2}} dz = \left[-(1-z^2)^{1/2} \right]_{-1}^1 = 0$$

$$I_3 = \int_{-1}^1 \frac{(1-a^2)}{(1-z^2)^{1/2}(z+a)} dz = (1-a^2) \int_{-\pi/2}^{\pi/2} \frac{\cos t}{\cos t.(a + \sin t)} dt$$

where $z = \sin t$.

Then:

$$I_3 = (1-a^2) \int_{-\pi/2}^{\pi/2} \frac{dt}{a + \sin t} = (1-a^2) \cdot \frac{2}{(a^2-1)^{1/2}} \left[\tan^{-1} \left(\frac{a \tan(\frac{t}{2}) + 1}{(a^2-1)^{1/2}} \right) \right]_{-\pi/2}^{\pi/2}$$

$$= -2(a^2-1)^{1/2} \times \left\{ \tan^{-1} \left(\frac{a+1}{(a^2-1)^{1/2}} \right) - \tan^{-1} \left(\frac{-a+1}{(a^2-1)^{1/2}} \right) \right\}$$

leading eventually to:

$$I_3 = -\pi(a^2-1)^{1/2}$$

Ultimately we have

$$\int_{-1}^1 \frac{(1-z^2)^{1/2}}{a+z} dz = I_1 - I_2 + I_3 = \pi(a - (a^2-1)^{1/2})$$

and it is this result which leads to Eck's formula.

In pursuing the above integration the reader is referred to Abramowitz and Stegun, (1964), Articles: (3.3.44), (3.3.39), (4.3.131), and (4.4.34) for details.

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