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The Butterfly Schema in the Classical Instrumental Style: A Product of the Tendency for Congruence

Synthesising associative-statistical and generative theories of schemata through the notion of congruence

In the fields of music theory and music cognition, localised multiparametric schemata receive various explications and definitions. While these cannot be precisely pigeonholed, they do often favour classification in terms of a loosely associative-statistical or generative orientation. From an associative-statistical standpoint, schemata form in cognition through the mental association of statistically predominant features in time and place, such as the conglomerations of voice-leading patterns, figured progressions and metrical structures in Meyer (1973), Gjerdingen (1988, 1996, 2007) and Byros (2009a). For example, the 1–7…4–3 voice-leading schema appears to rise and fall in history in a life cycle of popularity and typicality resembling a bell curve, peaking in the early 1770s, ‘due to the way brains abstract stable categories from a continuum of historical change’ (Gjerdingen 1988, p. 99). Thus, schemata are a form of culturally situated cognition (Gjerdingen 2007, Byros 2012a). By contrast, in the generative paradigm, schemata emerge as stable descriptions of the tonal grammar, generated by a system of well-formedness and preference rules that are a product of universal cognitive capacities (Lerdahl and Jackendoff 1983, Lerdahl 2001, Temperley 2001), exemplified by Lerdahl and Jackendoff’s ‘normative structure’ (1983, p. 289).

Broadly, schemata are particular structures in associative-statistical theories, and general or universal structures in generative theories. In terms of underlying metaphysical foundations, the associative-statistical programme is loosely underpinned by empiricist theories of knowledge, following the British empiricist philosophers John Locke (1632–1704), George Berkeley (1685–1753) and David Hume (1711–1776), who were concerned with the influence of the environment or culture on behaviour. However, the generative paradigm embraces rationalist philosophy, stemming from the ‘continental rationalism’ of René Descartes (1596–1650), Baruch Spinoza (1632–1677) and Gottfried Leibniz (1646–1716), involving the representational, combinatorial and computational structure of internal cognitive capacities.

Notwithstanding these distinctions, both associative-statistical and generative models incorporate primary chords, regular harmonic rhythms, balanced phrases and regular textural and metrical structures, which might suggest that the two paradigms represent independent and opposing approaches for viewing the same phenomena. But the relationship between these schools of thought is more complex. Indeed, associative-statistical and generative conceptions do not engender a dichotomy because they can support mutually compatible theories. For example, Gjerdingen (1988) shows how schemata are perceived and cognised after being established in a style, which does not in principle contradict generative theory. Conversely, Lerdahl (2001) depicts Gjerdingen's voice-leading schemata (1996) in terms of global pitch-space and prolongational structure, identifying the higher-level tonal forces behind the statistically driven prototypes. Individual models can draw liberally from either conceptual pole. Associative-statistical models are, perhaps necessarily, absorbed into generative models and vice versa. For example, generative models (see especially Lerdahl 2001, pp. 231–42, and Temperley 2001, pp. 336–40), include Western-specific, top-down tonal structures as part of the knowledge of an 'experienced listener', and so must assume a type of inductive, abductive or associative mechanism that binds these learned phenomena to generative procedures. In Gjerdingen 1988, it is shown that some features and relations between features are preferred in cognition, drawing on notions of parallelism, symmetry and tonal stability that have foundations in generative theories.

Another complication is that modern music schema theory has taken a different direction from its earlier roots in associative (and cognitive) psychology. For example, while Gjerdingen (2007) is partly based on the associative foundations of earlier work (1988), there are decidedly different foci: historical musical practice and cultural interaction now play a central role. These amount to a type of 'archaeology' of style that uncovers the inherent compositional techniques and modes of practice, such as the *Règle de l*'*octave*, preserved in artefacts of the period like the *partimenti* and *solfeggi* exercises (Gjerdingen 2007, Byros 2012a, Sanguinetti 2012). These recent developments mean that perhaps only earlier schema theory might be legitimately placed within an associative-statistical programme.

The disjuncture between empiricist associative-statistical theories and rationalist generative theories is found in various avenues of cognitive science, influencing debate in philosophy, psychology, linguistics and computer science. It characterises the division between rationalist/nativist (Fodor 1975, 1998, 2008, Pinker and Prince 1988) and empiricist (Carnap 1950, Skinner 1953, Dennett 1991) theories of mind and behaviour, parallels the statistical/probabilistic (Shannon 1948, Rummelhart and McClelland 1986) versus symbolic/generative (Chomsky 1957, 1965) paradigmatic shift in linguistics, and is a defining point of contrast in the classical computationalism (Turing 1950) versus connectionism (McCulloch and Pitts 1943) schism in computer science. Notwithstanding, the most fruitful investigations are perhaps those that are receptive to both associative-statistical and generative perspectives (Price 1994, Wermter and Sun 2000). Generative approaches are useful because they account for the rule-based operations in cognition that are presumably innately endowed. However, statistical/probabilistic accounts might be profitably combined with generative methods: statistical methods distinguish between degrees of grammaticality, whereas generative methods require strict categorisation; statistical approaches are error-tolerant, accounting for messy data and fuzzy categories, but generative theories are relatively inflexible; and statistical models can learn 'on the fly', building structure according to the data itself, while generative theories rely on an innate symbolic architecture (Abney 1994).

For an examination of localised musical structures, a synthesis of associative-statistical and generative approaches is required because biases towards either theoretical orientation cannot determine the structure of schemata that are at the same time particularly and universally constrained. Statistical enumeration does not show how cognitive universals are partly responsible for the formation of local schemata, and generative theories do not explain how culturally specific schemata can be included in the cognitive architecture. An integration of the respective viewpoints would permit a more coherent model of schemata, as products of internal and external worlds, avoiding the respective strictures of both. Such an enterprise might be neither a bona fide theory of music cognition nor an empiricist explanation of the effect of culture on musical behaviour, but could determine the effects of culture and cognition on musical structure.

Local schemata of the Classical instrumental style[1] owe their existence to causes both learned and innate. Although the harmonic, textural and metrical characteristics of this style are ostensibly particular (i.e., learned through culture), the diatonic, rhythmic and metrical constraints that shape these features seem to be innate (i.e., universal). The main argument developed in this article is that external schemata (non-mental schemata) are constrained towards congruent relations by a universal 'tendency for congruence' (hereafter, TFC), thought to originate in cognition – although it must be assumed that such can be represented in mentality since they are also cognised before and after they form externally. Congruence is a resemblance relation between the structural elements or features in styles and schemata, characterised by stability, agreement, reinforcement, correspondence, simplicity, uniformity, parallelism or symmetry. Congruence is synonymic with resemblance because for phenomena to be congruent they must resemble each other at some abstract level. If the TFC is a constraint on styles and schemata, it means that congruent relations (viz., resemblances) should be commonly manifest in and between their particular features. The aim of this article is to show the predominance of a highly particular-and-congruent schema in a distinct style. The schema under examination is the 'butterfly' schema, thought to be a highly common and highly congruent schema in instrumental music of the Classical period and less common in some other styles of adjacent periods.

While the TFC presumably originates in cognition, why congruence should be generated from this source is explored in *Congruence in Styles and Schemata*. This involves an examination of the ontological structure of congruence and its corresponding reference and epistemology in mentality, with a critique on the induction of musical knowledge. A tentative conclusion is reached: congruence seems to be necessary to infer meaningful relations in music, but if this turns out not to be the case (in the fullness of time), it is nonetheless a useful notion for cognition because it increases the comprehensibility of musical structure and is a means for creating stylistic and expressive dialectal relations.

Butterfly Schema Features, uncovers the features of the Classical instrumental style and butterfly schema. We argue that these (and ipso facto butterfly schemata per se) are not supported in many other styles of the Baroque and Romantic periods. The Classical instrumental style exhibits a spectrum of congruent relations, but owing to the TFC (which equates commonality with congruence) congruent butterfly schemata are predominant therein. The butterfly schema comprises the following features: chords that are ‘close’ to the tonic in pitch space, starting and ending on the tonic with non-tonic chords in the middle; a textural structure that is uniform and forms a regular hierarchy on the main metrical level (and two levels directly above); and even harmonic rhythms produced at this level. The relatively abstract global tonic key and regular hierarchical metrical structure are implicit congruent features.

A Model and Survey of the Butterfly Schema, presents a full model of the butterfly schema and a survey of European instrumental music, *c*. 1750–1850. The survey compares the popularity and type of butterfly schemata in a Classical instrumental corpus, *c*. 1750–1800, with a Romantic instrumental corpus, *c*. 1800–1850 – the latter functioning as a control group. It is found that butterfly schemata, which are congruent structures, are more commonly produced in the Classical style, suggesting that they are caused by the TFC in a particular (culturally conditioned/selected) manifestation. The types of features that constitute butterfly schemata converge with this finding. Butterfly schemata can have both minimally and maximally congruent features, but they form more commonly with maximally congruent features, showing the influence of the TFC.[2]

Congruence in styles and schemata

This section explores the nature of congruence and its role in cognition and musical structure. It then outlines the types of congruence that become manifest in and between features.

Explaining congruence: *rules or resemblances*?

Longuet-Higgins and Steedman (1971, p. 223) introduce the ‘rule of congruence’[3] as a principle for parsing structure in automated tonic-finding and metrical analysis, where distinct congruent forms are necessary and sufficient for the computational induction of keys and metres, aiming to model the human cognition of these concepts. In one respect, this seems feasible since conventional musical notions (such as chords, keys and metres) are ontological categories that neatly correspond to mental concepts and so might be required by perception. For example, a major chord appears to be a sharply defined category, i.e., a fixed congruent form. By contrast, it is also possible to conceive harmony as comprising various levels of abstraction, where relatively congruent chords are convenient fictions with which to divide the harmonic landscape. Indeed, the ontological problem of whether musical phenomena exist as such (either as ontological categories or mental concepts) is a key issue in understanding the relation between congruence and cognition. Chords, keys and metres (etc.) are conceived either as ‘classical’ categories, with fixed boundaries (Lerdahl and Jackendoff 1983), or as ‘family resemblance’ categories with fuzzy boundaries (Gjerdingen 1988, Zbikowski 2002) (see Lakoff 1987, Pinker and Prince 1996, and Fodor 1998, 2008 for general discussion on categories, concepts and epistemology in cognitive science).

For Longuet-Higgins and Steedman (1971), key and metre are fixed categories that are inferred through congruence in the systems of pitch and rhythm, respectively. The tonic-finding algorithm captures only structures that are tonally congruent (in terms of the pitch criteria of the key), and the metrical analysis algorithm recognises only dactylic rhythms that are congruent with specific metres (of simple and compound types). This straightforward relation between congruence and cognition means that ambiguity is not tolerated; noncongruence is not parsed into a syntactic form. Accordingly, noncongruence is presumed not to be meaningful. The authors argue that ‘[m]usic would be incomprehensible if the key and metre were called into question before they were established’ (1971, p. 224), since from their perspective ontological categories have to be discrete and finite to individuate concepts and to be understandable. It is, nevertheless, questionable whether distinct congruent forms can be necessary and sufficient for the cognition of musical structure (see Fodor 1998, 2008 for broader discussion on the individuation of concepts). To take the first model, while certain pitch criteria seem to be important for the individuation of a key, these cannot be sufficient; there is an indefinite number of other factors (such as rhythmic criteria) involved in locking to a tonal centre. In this way, Longuet-Higgins and Steedman (1971), similar to many other inductive formal/computational models, commits the fallacy of affirming the consequent, conflating necessity with sufficiency. The same issues obtain, mutatis mutandis, to the individuation of metre in the rhythmic model of their paper.

In their models, a preconceived blueprint of musical structure determines the analytical output, rather than representations or cognitive operations being formed rationally, intuitively and flexibly from the input data. Thus they also come up sharply against the problem of induction (Hume 1739; Popper 1934), which concerns whether inductive reasoning can lead to knowledge, and so these systems are unlikely to amount to a feasible picture of musical understanding and meaning. While heuristics guide cognition, humans are able to change expectations and meaningful interpretations based on emerging data. Russell (1911, p. 35) and Popper (1934) point out in connection with arguments of inductive inference that we must be more critical about the principle of the uniformity of nature (from Hume 1739), because phenomena in the world can be indefinitely variable. We must be able to grasp emerging structures that depart from seemingly fixed percepts.

Although it has been established that inductive inference is a questionable means for music cognition, the underlying argument, that congruence is necessary for meaning, requires further consideration. It is probably not tendentious to say that congruence trivially increases comprehensibility, i.e., that congruence makes structures easier to comprehend, because it has the characteristic of simplicity. It is also likely that congruence is useful for expressive and stylistic dialectics. Prima facie, congruence does not however seem to be necessary for understanding and meaning because music that is generally and consistently noncongruent can also be meaningful (although the present authors think noncongruent music is rarer than that which is consistently congruent). For example, highly chromatic idioms of the nineteenth century (Wagner) and irregular contrapuntal structures of Renaissance music (Palestrina) are, superficially at least, largely noncongruent, but nonetheless understandable. That is, these styles seem to be consistently and predominantly noncongruent (and are ostensibly non-referential systems), yet human cognition is able to understand them. This suggests that congruence might not have a critical role in cognition after all. However, it could also be held that it is not clear how music can possibly be understandable if there are no congruent (viz., resemblance) relations to any extent between constituents (e.g., no stability, agreement or correspondence in and between features), because if structural interactions were completely noncongruent, each part would be de facto unrelated to other parts. The notion of entirely noncongruent music is thus illogical in a sense because it asserts there is no basis by which to infer meaning from musical structure – assuming music is fundamentally non-referential and the definition of music is not too wide to be intractable, of course. This contradiction concerning the apparent necessity of congruence for cognition and the seeming lack of it in a small number of styles might be termed the *musical congruence induction paradox,* and it is here considered an open problem; it is explored because it goes some way towards explaining why congruence is incorporated into musical structure.

One solution may be to assume that cognition has an a priori and pre-theoretical appreciation of the indefinitely many forms of congruent relations (viz., resemblances) in music, similar to how Russell considers resemblance relations to be universally understandable. He explains resemblance in terms of the concepts triangularity and whiteness:

> The beginner, in order to avoid error, often finds it useful to draw several triangles, as unlike each other as possible, in order to make sure that his reasoning is equally applicable to all of them. But a difficulty emerges as soon as we ask ourselves how we know that a thing is white or [is] a triangle. If we wish to avoid the universals *whiteness* and *triangularity*, we shall choose some particular patch of white or some particular triangle, and say that anything is white or a triangle if it has the right sort of resemblance to our chosen particular. But then the resemblance required will have to be a universal. Since there are many white things, the resemblance must hold between many pairs of particular white things; and this is the characteristic of a universal. It will be useless to say that there is a different resemblance for each pair, for then we shall have to say that these resemblances resemble each other, and thus at last we shall be forced to admit resemblance as a universal. The relation of resemblance, therefore, must be a true universal. (Russell 1911, p. 55)

As proposed at the outset, congruence in music is a relation of resemblance: in order for features to be congruent with each other, they must resemble each other at some abstract level. The tentative argument, put simply, is that cognition is able to glean meaning through an internal sense of congruent (viz., resemblance) relations at indefinite levels of abstraction. This means that congruence does not have to reside in a specific level of abstraction (which would presumably be the case if inductive inference was a feasible means of understanding). This rings true, because congruent relations must be used for the individuation of conventional concepts, such as chords, keys and metres, since these are understandable a priori, requiring little or no knowledge of facts of the external world, understanding of the level of abstraction, explanation or translation. Such concepts might be individuated (often unconsciously) in an indefinite number of ways, but there seem to be focal features for each concept also (although whether concepts have necessary and sufficient conditions is doubtful). To use a previous example, a chord can be understood as such because of congruent relations across an indefinite number of parameters used to individuate it, but there seem to be more critical features too, whether concrete or abstract (e.g., tonic, mediant and dominant pitches of the chord, according to Piston 1941 and Lerdahl 2001). Equating congruence with resemblance obliquely accords with schema theory, following Wittgenstein's (1953) notion of 'family resemblances', which are held to be strong factors in the conceptualisation and identification of schemata (Gjerdingen 2007, p. 10).

The argument that congruence is abstractly required for cognition is nonetheless challenged by ostensibly (and seemingly intentionally) completely noncongruent music, such as twentieth-century serial music, which radically departs from those relations of congruence that might be used to individuate conventional concepts (chords, keys, metres). However, it is possible that such highly noncongruent systems are meaningful in terms of radically different conceptual notions (see Lerdahl's related argument for 'compositional grammars', 1988a), although it is questionable whether music can be truly divorced from conventional concepts and still be describable as music (implied in Bernstein 1976).

In sum, it could be that for traditional styles that employ conventional concepts, congruence is required for cognition but at indefinite levels of abstraction. And while congruence does not have to be common to be a foundational principle upon which musical meaning is based (it only has to be present at some abstract level), this might explain why it is preferred and hence tends to be common in some styles (i.e. the TFC). However, even if the argument that congruence is understood through a priori and abstract relations of resemblance is incorrect, congruence is nonetheless useful for cognition because it trivially increases comprehensibility and is important for expressive and stylistic relations. From this discussion, three main suggestions have emerged about why the TFC in musical structure might have a cognitive origin:

1) Congruence trivially increases comprehensibility. (It weakly supports a cognitive origin of the TFC.)

2) Congruence, while perhaps not necessary for cognition, is useful for stylistic and expressive dialectal relations. (It gives moderate support for a cognitive origin of the TFC.)
3) Congruence is necessary for cognition (i.e., for meaningful interpretation), although not through inductive inference (for reasons given above), but because relations of congruence (viz., resemblance relations) are understood a priori at indefinite levels of abstraction. (It strongly supports a cognitive origin of the TFC.)

As argued, (1) is not tendentious because, all else equal, it is easy to accept that musicians and listeners might prefer congruent structures since they are simpler (as theorised in Bod 2002); although, as noted, it is also possible to conceive highly noncongruent structures. Furthermore, (2) can entail (1), and is likewise fairly uncontroversial, because it is evident that music gains much of its expressive and stylistic interest from the dialectic between congruence and noncongruence. (In support of this point, examples of expressive and stylistic use of congruent and noncongruent relations have been offered by a number of theorists, such as Cooper and Meyer 1960, Berry 1976, Mirka 2009, etc.) (3) can entail (1) and (2), and is the most compelling explanation of the TFC, while the most disputable. If congruence is necessary for music to be understandable, which is to say that it is a requirement for meaningful interpretation of structure (as suggested through this subsection), this must be with respect to conventional concepts only and at indefinite levels of abstraction. This means, broadly, that music might be cognised in terms of a priori congruent concepts at any level of abstraction, but also an indefinite number of a posteriori noncongruent concepts that are perhaps more concrete, to account for those intimated in non-conventional music, such as twentieth-century serial music.

It is unlikely that congruence is a product of culture (viz., memetic variation, selection and replication), since although schematised discretely for the sake of analysis, patterns of congruence are in actuality continuous, and occur in and between culturally conditioned features of styles and schemata. By contrast, units of culture (viz., memes) are selected and replicated singularly or in discrete collections (Jan 2007). If cultural conditioning were a driver of congruence, its structural manifestations would presumably be particulate. Also, congruent structures do not emerge in isolated places, as would be expected if they were selected and replicated by culture; congruence seems to be a widespread constraint in a multiplicity of cultures, largely irrespective of context, indicating a universal and flexible origin. Nevertheless, the extent and variety of congruence requires further study to examine its frequency and forms in various styles and schemata.

To consider a further contingency, congruence might be a separate notion to creativity. Creativity seems to be unbounded (Schlegel 1963) or even infinite, and therefore not easily characterised or open to straightforward quantification. It thus takes on an elusive presence in music, evading formal modelling (Wiggins 2007, Marsden 2012) and is often equated with novelty with respect to the probabilities and expectations of a musical system (Meyer 1956, Bernstein 1976). Since congruence is usually expected, it is unfeasible that creativity has a straightforward connection with it, that is, having a relation that necessarily involves resemblance, although congruence might have some role to play. It is not disputed whether models of musical structure that incorporate notions of congruence in order to shed (some) light on creativity therefore misunderstand the notion of congruence (such as Lerdahl and Jackendoff 1983, Bod 2002, Conklin 2003). These theories have a different goal to the present enquiry and so could be insightful, although the subject requires further investigation. In any case, the following account specifically examines the constraint for congruence on musical structure that is presumably an imposition of cognition. It is not an examination of how congruence is used in cognition; it is rather musical structure that is the focus.

A considerable amount of attention has now been given to unfolding arguments that illuminate the possible origins of congruence. These arguments are not conclusive, but taken together they provide some support for the idea that the TFC is a product of cognition. However, verification of the TFC does not actually hinge on cognition, culture, creative intuition or any other possible origins. Whether it produces butterfly schemata can be found valid or invalid independently of the above arguments; these are merely suggestive. Therefore, that congruence becomes manifest in musical structure via cognition is only a tentative proposal.

Categorising congruence

The analysis and statistical survey in the survey of the butterfly schema, below, require definitions of the basic types of congruent relations. Congruence in a single parameter can be termed ‘uniparametric congruence’ (hereafter, UC). UC is a resemblance relation between the elements of a single parametric feature, characterised by stability, agreement, reinforcement, correspondence, simplicity, uniformity, parallelism or symmetry. As examples, UC can describe tonal or harmonic stability, such as with the internal congruent relations in a progression of primary chords or parallelism or reinforcement between melodic groups in texture. Congruence can also be used to refer to the co-occurrence of uniparametrically congruent features in different parameters, and termed ‘multiparametric congruence’ (hereafter, MC). Examples of MC in the Classical instrumental style are where consonant harmony coincides with a strong beat in a regular and hierarchical metrical structure, or where an accent in harmonic rhythm occurs with relatively consonant harmony. The butterfly schema and Classical instrumental style comprise congruence that is manifest in (UC) and between (MC) their features.

Since UC and MC are both descriptions based on universal relations of resemblance, a distinction between the two forms may not always be pre-theoretical. UC and MC are nevertheless useful descriptors because they can show a statistical connection between features of different parameters. By contrast with Longuet-Higgins and Steedman’s rule of congruence, an equivalent ‘rule of MC’ is not appropriate because the present work does not use an inductive framework. It is here assumed that congruence might not correspond to necessary concepts in cognition but expresses congruent relations in and between the features of a style that seem to be a product of cognition. Indeed, particular congruent relations are not rules, because they are sometimes denied. While, broadly conceived, the butterfly schema comprises multiparametrically congruent patterns and emerges commonly in the Classical instrumental style, since congruence is graded – from the congruent, which is common, to the noncongruent, which is uncommon – schemata in musical structure cannot be natural kinds, nor do they have discrete boundaries (following Gjerdingen 1988 and Zbikowski 2002). Thus, the butterfly schema might be an arbitrary category in a sense, although it can be distinguished by the fact that it is congruent and so tends to be common. The intrinsic multiparametrically congruent interactions determine its form and commonality, securing its place as a cultural and cognitive category.

Multiparametric congruence and noncongruence are not really antithetical notions, since congruence is relative and graded, although these can be usefully, if perhaps artificially, schematised as being at opposite ends of a continuum. Structure is not usually completely lacking in either to warrant being categorised as such – although the description noncongruence might hold with styles that are devoid of conventional concepts (e.g., those that use serial techniques, as discussed above). Interaction can be better described as relatively congruent or noncongruent, which is usually implicit in the descriptions of this article. In the present formulation, multiparametric noncongruence occurs when unstable features coincide with other stable or unstable features. For example, when noncongruent features, including irregular textures (and irregular metrical structures), 'distant' chord progressions from the tonic or irregular harmonic rhythms coincide with each other or with congruent features. If the TFC is correct, multiparametrically noncongruent schemata should occur less frequently than multiparametrically congruent schemata. Although multiparametrically noncongruent schemata are not surveyed in this article, the overwhelming majority of schemata exhumed by theorists that focus on the Classical instrumental style are relatively multiparametrically congruent (with the possible exception of the *le–sol–fi–sol* schema, examined in Byros 2009a, which contains 'distant' pitch patterns and chords from the tonic), providing some indirect support for the present thesis that schemata are a product of the TFC.

Features of the butterfly schema

This section examines the congruent features of the Classical instrumental style and butterfly schema in the parameters of chord progression, textural grouping and harmonic-rhythm ratio. These contrast, by degree, with the common congruent features of many other styles. It is posed that some styles of the Baroque and Romantic periods do not have the necessary type of structural congruence to commonly support butterfly schemata.

Chord Progressions

Aspects of pitch-space theory, as theorised in Lerdahl (2001), are incorporated into the butterfly schema model to quantify congruent chord progressions. The conceptual distance between pitches, chords and keys are calculated using a set of 'basic spaces' available to cognition, such as diatonic, octatonic, hexatonic, pentatonic and whole-tone spaces, to name the most prominent. Basic spaces are loosely associated with particular periods of music history, as can be seen with the prominence of octatonic and hexatonic chromatic spaces in the Romantic period (see Lewin 1987, 1992, Cohn 1996, 1997, Lerdahl 2001, for discussions of tonal style-periodic associations). Diatonic space is here argued to be the predominant system in the Classical period. Diatonic space seems to best represent chordal relations of the Classical instrumental style – such as tonic–dominant chord progressions – as this yields smaller chord-distance values than when using octatonic, hexatonic, pentatonic or whole tone spaces (Lerdahl 2001, pp. 41–88); chromatic spaces more comfortably describe chromatic chord progressions commonly used in some Romantic styles. Generally, shorter distances are more congruent than larger distances in all spaces.

In Lerdahl, it is argued that listeners conceive pitch distance between chords by the shortest, most cognitively economical path in pitch space (described as the 'principle of the shortest path' (2001, pp. 73–7)). Thus, in Lerdahl, pitch-space calculations explain the cognition of music; as such, they are more than representations of pitch distance, but are necessary cognitive operations. This epistemological position differs subtly from the standpoint of the present article. For us, distances in pitch space are preferred but perhaps not necessary. That is, cognition may not always require specific short distances to understand passages, because there might be different forms of meaning involved, as we have noted above in the discussion surrounding the *musical congruence induction paradox*. It was determined that the question of whether congruence is necessary for cognition should remain open – although we think it is likely that congruence is requisite, but at indefinite levels of abstraction. This position avoids an inductive stance, where distinct congruent pitch percepts are necessary and sufficient to cognise pitch relations, which, for reasons given, is problematic. We propose that a propensity for short distances in pitch space (which is synonymic with the TFC) is a common product of cognition that becomes manifest in musical structure.

Lerdahl (2001, pp. 193–248) reinterprets Gjerdingen's (1996) voice-leading schemata through a system of harmonic functions which are determined through pitch-space distance. The functions usefully schematise abstract chordal movement (and therefore also congruent relations) rather than specifying concrete progressions. Fig. 1 shows a functional analysis of the 1–7…4–3 schema. Broadly, the departure ('Dep') and return ('Ret') functions denote non-tonic chords, and the tonic ('T') function represents tonic chords. The butterfly schema has a similar functional progression to this schema, observing a chiastic prolongational tension curve (low–high…high–low) which is the converse of its congruence curve (high–low…low–high). The functional progression of the butterfly schema can be a parent schema to a number of voice-leading (child) schemata of Gjerdingen (1988, 1996, 2007), specifically the 1–7…4–3 (Meyer), Jupiter, Aprile, Pastorella, Do–Re–Mi and Sol-Fa-Mi.

[INSERT Fig. 1 NEARBY]

In our formulation of pitch space, the butterfly schema is modelled using similar functions, which occupy its four stages. However, the functions in Lerdahl (Fig. 1) are applicable to tonal music of a variety of styles and periods, while our functions have a shallower tension/congruence path, being less distant in pitch space from the tonic (and thus more congruent). Our functions delineate closer distances between chords and the tonic particular to the butterfly schema and Classical instrumental style. In conjunction with the 'diatonic basic space', set to I/C (chord I in the key of C) in Fig. 2, we use the 'diatonic chord distance rule' as set out in Lerdahl (2001, p. 60) to derive the functions:

> **Diatonic chord distance rule (full version)** $\delta(x \rightarrow y) = i + j + k$, where $\delta(x \rightarrow y)$ = the distance between chord x and chord y; i = the number of applications of the regional circle-of-fifths rule needed to shift the diatonic collection that supports x into the diatonic collection that supports y; j = the number of applications of the chordal circle-of-fifths rule needed to shift x into y; and k = the number of distinctive pcs [pitch classes] in the basic space of y compared to those in the basic space of x.

[INSERT Fig. 2 NEARBY]

The diatonic chord distance rule and the diatonic basic space prioritise the root/octave, fifth and third levels for calculating distance. It calculates the number of steps between the respective keys of each chord, the number of steps separating chords in the circle-of-fifths and the number of pitch classes in common between chords at each level of their basic spaces. This rule, with information from the diatonic basic space, can be used to measure chord distance (δ). For example, in the progression I/C → V/C (I and V in the key of C) ($\delta(x \rightarrow y) = i + j + k$), the distance is 5 (0 + 1 + 4) (Fig. 3), which is a maximally congruent chord progression of the butterfly schema (explained shortly). The regional distance between the chords is zero (the V stays in the same collection) ($i = 0$); the V moves one step up the diatonic circle-of-fifths ($j = 1$); and there are four resultant non-common pitch classes in the octave, fifth and triadic levels in V compared with those of I ($k = 4$) (each non-common pitch class is underlined in Fig. 3) (Lerdahl 2007, p. 332).

[INSERT Fig. 3 NEARBY]

Contrary to the theorisation of Lerdahl, our Dep and Ret functions in the butterfly schema only sustain the chords of the diatonic collection in a single key, which are typically used in the Classical instrumental style. In Fig. 4, the relative distances between the tonic and the other chords of the diatonic collection are shown, termed the 'chordal core' (Lerdahl 2001).

[INSERT Fig. 4 NEARBY]

While the Dep and Ret functions of the butterfly schema are relatively close to the tonic, these chords are nevertheless distant enough to satisfy the minimal shallow prolongational tension/congruence curve. The following rules formally represent this trajectory:

(1) *Functional chords are in a single tonic key (or key area).*

(2) *The first and last (T) stages include tonic chords, where the distance (δ) = 0.*

(3) *The minimum hierarchical distance (δ) between functional chords in the second (Dep) and third (Ret/V) stages and the outer Ts is 5.*

(4) *The maximum hierarchical distance (δ) between functional chords in the second (Dep) and third (Ret/V) stages and the outer (Ts) is 8.*

(5) *T–Dep...Ret–T is the minimally uniparametrically congruent harmonic form of the butterfly schema.*

(6) *T–Dep…V–T is the maximally uniparametrically congruent harmonic form of the butterfly schema.*

As noted, a minimally congruent chord progression (rule 5) is required for a valid butterfly schema. In maximally congruent progressions (rule 6), V (as opposed to IV, for reasons given below) occupies the third (Ret/V) stage, which has a distance (δ) of 5 from the tonic (see Fig. 3 for the calculation). Maximal UC in the chord progression is necessary at this point of the schema to generate maximal MC, since this stage is accented in texture and harmonic rhythm.

Following the chord distance calculation of Lerdahl, distance is a product of the similarity of pitch content between chords and the number of steps separating chords on the circle of fifths. And as noted, owing to the TFC, there is a tendency for close distances between chords and the tonic. However, closeness (viz., congruence) might only be a loose metaphor for how pitch space is cognised, and only a first approximation of pitch-space representation. The present exposition is not primarily concerned with how pitch structure is cognised (or for that matter with how musical concepts are individuated), but with how the TFC influences compositional outputs. Thus, following rules 1–6, our pitch-space functions define close distances between chords in diatonic space that are common in the Classical instrumental style, and which are a manifestation of the TFC. Other styles are probably also governed by the TFC, presumably projected from cognition, but manifest congruence in other ways owing to the selective and replicative action of culture. The chord distance rule generally corresponds with traditional geometrical pitch-space representations (e.g., Weber’s 1821–1824 regional chord and key chart, and Riemann’s 1915 *Tonnetz*), and broadly follows the structure of the harmonic series, supporting its use as a model of the universal TFC in the Classical style. The chord distance rule is also apt because it attaches a numerical value to the representation that can be used formally in conjunction with other universal and particular constraints.

It is important now to distinguish between abstract/absolute representations of pitch distance between pitches, chords and keys formed irrespective of context, and concrete/relative representations cognised in specific situations. This distinction is significant in terms of conceiving the distance between pitches, chords and keys on the circle-of-fifths. In an abstract/absolute conception, which generally accords with our use of the diatonic chord distance rule of Lerdahl, distances on the circle-of-fifths are equal; we will explain why this is so shortly. However, in concrete/relative contexts, when pitches, chords and keys are individuated through their multiparametric connections, the perception of distance is marginally distorted owing to the direct effects of those multiparametric constellations. For example, in a concrete/relative context, when a tonic (pitch, chord and key) has been individuated (viz., tonicised), presumably through multiparametric interaction, the tonic chord is usually perceived as being closer to the dominant (chord and key) than to the subdominant (chord and key) (implicitly consistent with Schenker 1935; Krumhansl 1990, Lerdahl 2001, Lerdahl and Krumhansl 2007). The reason for this requires a small extension of our pitch-space conception for concrete contexts. When, in terms of the individuated tonic (pitch, chord and key) (which occurs in cognition) the progression I→V is sounded, the root of the dominant chord is a prolongation (of the fifth) of the tonic chord. But, in the case of the subdominant chord, i.e., I→IV, when likewise sounded (and likewise conceived in terms of an individuated tonic pitch, chord and key), only the fifth of the subdominant chord is a prolongation (of the root) of the tonic chord. Since the tonic chord prepares the dominant chord's root but only the subdominant chord's fifth, the dominant chord is perceived as being closer to the tonic chord than to the subdominant chord. To be clear, in concrete contexts where a tonic has been individuated, ceteris paribus, the progression I→V is marginally more congruent than I→IV. In the butterfly schema, this distinction applies only to the third stage in the maximally congruent version (see rule 6 above), owing to the concrete rhythmic and metrical conditions (elaborated below).

In the indefinitely many concrete/relative multiparametric contexts, various pitches, chords and keys are constantly (re-)individuated. Accordingly, any point on the circle-of-fifths can become the tonic (pitch, chord and key) – presumably through multiparametric interaction between features. It is therefore necessary in an abstract/absolute representation (which is how listeners represent pitch distance outside of a particular context) that each chord and key are equidistant on a circumplex circle-of-fifths, similar to the equidistant hours shown by points on a 12-hour clock. Indeed, biases between certain chords or keys on the circle-of-fifths must not exist in the abstract/absolute conception, because chords and keys are flexibly individuated in concrete/relative circumstances; just as any point on a clock is arbitrarily set to any time zone. This conception is necessary for the system to work holistically, in order for chords and keys to be individuated according to the specific multiparametric circumstances. We take this point to be uncontroversial, because if it were not the case, chords and keys would be biased in favour of a particular individuation (viz., tonicisation) a priori, regardless of the actual concrete and emerging structure. This would represent an incoherent picture of absolute pitch space, which would contradict the particular tonal forces underlying specific pieces. In sum, chords on the circle-of-fifths are equidistant in the abstract/absolute (such as where the distances in the progressions I→IV and I→V are identical), but might be subtly altered in various concrete/relative contexts.

When considering pitch space as a constraint, the pitch-space distance between chords in the four stages of the butterfly schema is usually calculated abstractly and hierarchically, because the tendency for MC that gives rise to the schema acts abstractly and hierarchically. Indeed, the abstract functional progression (or prolongational form) of the butterfly schema interacts with other multiparametric features at various abstract levels. Contrary to the theorisations in Lerdahl (2001), pitch-space perception is a more feasible notion in the abstract; in concrete circumstances, there are too many multiparametric contingencies that influence pitch space for it to be reliably modelled. However, while there is an infinite continuum between the concrete and the abstract (Schenker 1935, Swain 2002), in any single context there is always a concrete fact of the matter. Thus, for the maximally congruent butterfly schema (rule 6) it is necessary to include a relatively concrete chord V in the third stage to account for its highly specific and congruent conditions. It is not given a formal pitch-space distance value owing to the difficulty of defining concrete distances, as noted. This mixture of the concrete (chord V) and the abstract (functional progression) is permissible because these representations occur at different stages of the schema.

Lerdahl (2001) does not fully distinguish between abstract and concrete representations. The tonal tension rule mixes the abstract and concrete in a single calculation (viz., representation/cognitive operation) (Lerdahl 2001, pp. 142–192). It combines the chord distance rule, an abstract notion, with concrete ideas such as 'tonal attraction' and 'surface dissonance', which concern discrete note events and voice-leading relationships. The tonal attraction component uses the inverse square law to show the perceived 'gravitation' between concrete pitches close in frequency. As noted, such concrete representations of pitch space are of a different order to abstract conceptions and should logically be treated separately, since each makes the other redundant. They may be combined if they concern separate parts of a schema, as shown in our butterfly schema progression, but a mixture is incoherent in a single instance or calculation. A focus on pitch events in a Classical context is perhaps also incoherent because particular melodic structures, even leading-note and tritone relationships that seem to be significant for defining keys and chord relationships generally, are arguably not as important in the Classical instrumental style, since they mainly supervene on functional harmony (Berry 1976, Swain 2002). Indeed, it appears that key and chord features broadly dictate pitch attraction and surface dissonance contingencies in this style. This is a product of the gradual stylistic evolution from voice-leading dominance to functional harmony that occurred over a period of a hundred years in art music, commencing around the year 1600 (Swain 2002).

While the chord distance rule is useful for defining the butterfly schema, the process of interpretation and validation is problematic owing to the difficulty of modelling multiparametric interaction and fuzziness of schema categories in actual situations. A main concern is how, and to what extent, non-functional chords should occupy a valid functional chord progression. To deal with this, we incorporate two strategies from Lerdahl and Jackendoff (1983) to manage congruence relations: time-span significance and prolongational significance. The time-span reduction selects stable pitches with a preference for rhythmic criteria (its tendency is for points of tonal stability to be equidistantly spaced through a piece), whereas the prolongational reduction selects rhythmically significant events with a preference for pitch stability (its tendency is for the most harmonically stable events to be prolonged). These also tend towards congruence with each other, resulting in compromise.

In the butterfly schema, functional tonic chords can occupy any part of the first and last (T) stages, even weak beats, because the congruent prolonged tonic must be prioritised therein, although time-span significance is also active in these stages, since the initial points ('heads') of all stages are relatively time-span significant. When a tonic (pitch, chord, key) has been established, it tends to be projected into later structure (prolonged), even if subsequent non-tonic chords have rhythmic or metric significance. For this reason, tonic chords are invalid in the second (Dep) and third (Ret/V) stages, because these would create unwanted pitch stability regardless of rhythmic and metric placement, compromising the distinct tension curve of the schema.

There is, however, a notable difference between the second and third stages. As defined, the third stage when multiparametrically congruent has a dominant chord (V) (rule 6). This is because this stage takes place on a strong beat of the textural and metrical structure (and thus is time-span significant) and so a highly congruent chord is preferred (i.e., a chord that is prolongationally significant). The second (Dep) stage, by contrast, can sustain more distant chords, since it correlates with unaccented (non-salient) features where maximal MC could not feasibly occur (implicit in rules 5 and 6). Specifically, the Dep stage takes place at a relatively insignificant point in the textural grouping (see *Textural Grouping*). Thus, chords occupying this stage are relatively insignificant for constituting multiparametrically congruent relations in terms of the whole schema.

The following rules formalise these points for nuanced validation of butterfly schema progressions:

(7) *Chords that are not functional or specified* (*i.e.*, chord V) *in the second* (*Dep*) *and third* (*Ret*/*V*) *stages* (*i.e., diatonic or non-diatonic passing/interpolated/appoggiatura chords*[4]) *are valid even if significant in the time-span reduction of those stages* (*i.e., if they occur on the strongest beats therein*) *provided they resolve to the functional or specified chord on the next strongest beat in the stage*.
(8) *In the first and last* (*T*) *stages, non-functional chords* (*i.e., non-tonic chords*) *are valid on strong beats provided they resolve to a tonic chord in those stages on a weak or strong beat*.
(9) *Tonic chords* (δ = ~*0*) *are not valid in any part of the second* (*Dep*) *and third* (*Ret*/*V*) *stages*.

These fine-grained multiparametric rules explain the congruent and noncongruent interaction between pitch and rhythm in the four stages of the butterfly schema. The causal direction between multiparametrically congruent features is a complex consideration, however, and should be briefly explored. The causal inversion of a multiparametrically congruent relationship seems to be equally valid. For example, chords that are close to the tonic (being uniparametrically congruent) could be argued to cue strong beats in the metrical structure or, alternatively, strong beats might cue pitch stability, owing to the mutual and seemingly invertible congruent interaction between time-span significance and prolongational significance. In culture, causal direction between congruent features appears to depend on which features are dominant (established) as well as the depth of abstraction or generality. When an abstract congruent feature in a style has culturally selected a less abstract/more concrete congruent feature, these, in turn, select other, more concrete congruent features, creating a hierarchy of features constrained by the TFC (Rawbone 2017). In the Classical instrumental style, highly abstract tonal and metrical structure condition the more concrete features of low-level schemata (although this is generally the converse in real-time listening, in cognition). Rawbone (2017) provides further discussion on hierarchical cultural selection and causal interaction between multiparametric features in abstract and concrete contexts in culture and real-time listening, which is a topic that is beyond the scope of this article. While there are interactions between features in styles that nuance how congruence is directed, for present purposes and to a satisfactory level of approximation, it will suffice to establish the multiparametrically congruent correlation between features towards verifying the wider claim of the TFC.

Some issues of validation can now be explored in functional/chord progressions from Classical and Romantic instrumental styles. Ex. 1 shows a maximally congruent chord progression (T–Dep…V–T) (rule vi) in a butterfly schema in Beethoven's Piano Sonata Op. 2 No. 3, first movement. The functions/chords are shown beneath.

[INSERT Ex. 1 NEARBY]

The following more distant chord progression in Ex. 2, from Beethoven's Piano Sonata in C minor ('*Pathétique*') Op. 13, first movement, alludes to a butterfly schema progression but contains a number of noncongruent elements.

[INSERT Ex. 2 NEARBY]

In the second (Dep) stage, a <s>iv^{o} chord is used, which has a distance (δ) of 12 from the tonic ($\delta(x \rightarrow y) = i + j + k = 0 + 6 + 6 = 12$), exceeding the limit of 8 permitted by rule iv. It stays in the diatonic collection ($i = 0$), moves six steps away from the tonic chord on the circle-of-fifths ($j = 6$) and has six non-common pitches ($k = 6$) at each level of the basic spaces of the two chords. Despite the large distance of the <s>iv^{o} chord from the I, the V that follows it on the next strongest beat in the same stage provides a resolution which meets the requisite chord function distance for this stage ($\delta = 0 + 1 + 4 = 5$) (following rule 7). Thus, the stage is valid according to our definition.

The third (Ret) stage contains a diminished seventh chord (vii^{o7}) which also has a distance (δ) of 12 from the tonic chord ($\delta(x \rightarrow y) = i + j + k = 0 + 5 + 7 = 12$). This chord stays in the same diatonic collection ($i = 0$), moves five steps away from the tonic chord on the circle-of-fifths ($j = 5$), and has seven non-common pitches ($k = 7$) at each level of the basic spaces of the two chords. This stage is not valid because the vii^{o7} chord has a distance greater than $\delta = 8$ (contrary to rule 4) from the tonic chord and does not resolve to a functional chord on a relatively strong beat within the stage (as required by rule 7). The fourth (T) stage contains an appoggiatura chord on the first beat, but resolves to a functional tonic chord on the next beat in the same stage, so is valid (rule 8). Overall, this passage is not a butterfly schema progression as we define it (although it is very close to one), as it contains a non-resolved chord in the third (Ret) stage that is too distant from the tonic. However, all other stages (Ts and Dep) are permissible. As noted, the distant interpolated and non-functional chords that occur on the strong beats of the other stages are valid since they resolve to functional chords (following rules 7 and 8).

To take a more Wittgensteinian perspective, it is possible to see the opening progression of the *Pathétique* sonata as being associatively (or ‘psychologically’) related to the butterfly schema. The remainder of this section utilises conventions of the philosophy of mind and cognitive science to relate the various forms of the butterfly schema. The putative ontological structure is given in normal text (butterfly schema), the mental concept is capitalised (BUTTERFLY SCHEMA), and the meaning is italicised (*butterfly schema*). It is important to note that while categorisation and conceptualisation are required for the sake of explanation, it is doubtful whether listeners, excluding experts, do this explicitly; they need only understand congruent relations unconsciously. Also, for the following, it is probable that the relation between ontological categories and mental concepts is not isomorphic (Pinker and Prince 1996). Indeed, the process by which listeners individuate mental concepts from ontological categories is strongly disputed (Lakoff 1987, Fodor 1998). Furthermore, the relationship between a concept and its epistemic structure (its meaning or sense) is complex (Fodor 1998, 2008). For example, a concept might have various meanings.

From an associative-statistical perspective, and to a large extent a TFC perspective, this example is actually quite similar to the butterfly schema, as we define it. Indeed, it has similar features and relations between features that are relatively multiparametrically congruent. It might reliably invoke similar or coextensive concepts to the BUTTERFLY SCHEMA and be similar in meaning to the *butterfly schema* (although the concept individuated by the butterfly schema and any corresponding meanings are not explicitly examined in this article). We consider the chord progression not to fall within the defined category butterfly schema, because the third (Ret) stage is too distant from the tonic – according to our stipulated threshold of congruence. Styles and schemata have a variety of shades of congruence, from the highly congruent, which tends to be common, to the barely congruent and noncongruent, which tend to be uncommon. The purpose of this article is to define a highly congruent schema so that we might show it has a propensity towards commonality, vindicating the TFC as its cause (while cultural selection is another cause). Thus, an arbitrary threshold is better placed on the relatively congruent end of a theoretical congruence–noncongruence spectrum to demonstrate the TFC more forcefully. Although it would have been possible to present comparable analyses of marginally less congruent schemata, these would presumably be marginally less common, and the TFC would be less conclusively verified.

To be clear, the example is similar to the butterfly schema, containing almost the same features and in the same structural relations as the butterfly schema. Significantly, these are also relatively multiparametrically congruent, and thus probably generated by the TFC. We do not consider the example a valid butterfly schema because it falls short of the threshold of congruence required to belong to this category, as we define it. As noted, it is not unfeasible that a concept similar to, or coextensive with, the BUTTERFLY SCHEMA might be individuated through this example, and accordingly it may be understood as a *butterfly schema*, since it might be legitimately conceptualised and understood in various ways. Its borderline character makes it a particularly illustrative case, since it shows that congruence is a graded notion and that schemata can be categorised externally in terms of their degree of congruence without negating associative and meaningful interpretations. On pain of repetition, we wish to avoid the assumption that because we do not consider it a butterfly schema by our own highly congruent (and arbitrary) definition, then it may not be similar to the butterfly schema, BUTTERFLY SCHEMA and *butterfly schema*, or to similar or coextensive categories, concepts and meanings. The example is certainly similar to these and is presumably caused by the TFC, but it is merely outside the highly congruent boundaries that have been stipulated for the sake of categorisation.

Textural grouping

While the textures of schemata have been given attention in recent contributions to schema theory and generative theory (Temperley 2001, Mirka 2015), a more particular approach to the use of texture is required than the generalised presentations of many generative theories (such as Lerdahl and Jackendoff 1983) and a more encompassing portrayal is called for than the specialised considerations in schema theory and topic theory (as in Mirka 2015). This section examines the broad periodic, cultural and stylistic uses of texture in the Classical instrumental style and butterfly schemata, these being considered in terms of the universal TFC.

An important quality of texture is ‘textural accent’, which is the accentuation engendered through the coincidence of events at different levels of texture (Lester 1986, p. 29). Texture can be segmented horizontally and vertically into homogenous units by textural accent, here defined as ‘textural grouping’. Textural grouping that observes a regular nested hierarchy, where events at each level are fully contained within events at other levels, is proposed to be a uniparametrically congruent feature of the Classical style and butterfly schema, informing a regular nested metrical hierarchy at those levels. In one sense, textural grouping is similar to the notions of 'fusion' in Lerdahl and Jackendoff (1983, pp. 153–5) and contrapuntal grouping in Lerdahl (2001, pp. 32–4) and Temperley (2001, pp. 85–114), which likewise show that metrical structure is inferred from correspondences between accents in texture (viz., textural accentuation). Indeed, in our concept of textural grouping and the contrapuntal grouping of Lerdahl (2001) and Temperley (2001), texture is grouped vertically and horizontally by textural accentuation. However, textural grouping is a more specific notion than fusion and contrapuntal grouping, concerning the grouping of texture and metrical structure at particular levels and in particular styles; and, importantly, *textural grouping informs a regular nested metrical hierarchy if, inter alia, the former observes a regular nested hierarchy*.

This conception conflicts with Lerdahl (2001) and Temperley (2001), where distinct 'metres' can be inferred from variably congruent textures, even when those textures might not be regular nested hierarchies but involve only some degree of textural accentuation. This highlights a problem similar to that uncovered above, where formalisms were argued to affirm the consequent when there is a pre-conceived output that automatically parses the input. Likewise with these models, that such can reasonably infer a distinct and unequivocal 'metre' from variably congruent textures is contestable.

Although texture is often the principal cue for metrical structure (Rothstein 1995, McKee 2004), it is not always grouped regularly and hierarchically. Textural accentuation can be regular and irregular at distinct levels of styles and schemata, and so textural grouping and metrical structure can also be correspondingly regular, irregular, nested or un-nested at distinct levels of styles and schemata (Benjamin 1984). Formal models or parsers (such as Longuet-Higgins and Steedman 1971) do not permit this flexibility in metrical structure. The portrayal of metre in Komar (1971), Yeston (1976), Lerdahl and Jackendoff (1983), Lerdahl (2001) and London (2004) is that of a static nested structure. The Metrical Well-Formedness Rules (MWFR) 3 and 4 of Lerdahl and Jackendoff (1983, pp. 69–72) are cornerstones of this fixed, grid-like notion:

> **MWFR 3** At each metrical level, strong beats are spaced either two or three beats apart.
> **MWFR 4 (revised)** The tactus and immediately larger metrical levels must consist of beats equally spaced throughout the piece. At subtactus metrical levels, weak beats must be equally spaced between the surrounding strong beats.

These rules are simple and elegant, but essentialist in the assumption that metrical structure is always regular and uniform. London (2004, p. 84) presents similar metrical well-formedness rules, with modifications, positing that listeners perceptually entrain to well-formed metre automatically, with a degree of 'attentional invariance' to the input signal. From this viewpoint, which parallels Lerdahl (2001) and Temperley (2001), it might be assumed that textural accentuation of varying uniformity necessarily cues regular nested metrical hierarchies at all levels (where each level is neatly contained within others, without overlap), irrespective of the degree of congruence between other aspects of style and context. As just mentioned, regularity and hierarchy of texture/metrical structure exists only at particular levels in styles and cultures (Cone 1967, Benjamin 1984, Rothstein 1989, Hasty 1997). Indeed, the actual, as opposed to the notated metrical structures of both Western and non-Western music can be irregular, polymetrical or generally ill-formed at certain levels, while also being partly well-formed at other levels, depending on style and context (Benjamin 1984, Rothstein 1989, Clayton 1997).

Despite our scepticism of well-formedness rules, metrical structure is, inter alia and by degree, also implied by and inferred from the note values, contours and tonal implications of a single melodic group – that is, loosely, from its *Gestalt* – as fleshed out in Lerdahl and Jackendoff (1983). Broadly following 'Metrical Preference Rule 2 (MPR 2) Strong Beat Early' of Lerdahl and Jackendoff (1983, p. 76), the onset of a group can cue the onset of metrical structure. This means that melodic groupings do not only act on metrical structure through texture, but can act individually. Indeed, melodic groupings in texture have the potential to cue variably congruent or noncongruent metrical hierarchies (that is, those with irregular textural structure). A type of metrical conflict (/rhythmic dissonance/noncongruence) occurs when melodic groupings in texture contradict a previously established melodic/textural grouping or metrical structure (Benjamin 1984, pp. 371–2, Rothstein 1989, Mirka 2009, pp. 133–64). As noted, the notion of well-formed metre, in the sense conceived in Lerdahl and Jackendoff (1983) and London (2004), while useful when treated as a generalisation, is predicated on the idea that rhythmic groupings automatically and necessarily invoke distinct metres, which is strongly contested here. It is a problematic assumption because metrical structure is an abstract notion that cannot involve a direct relationship with variably congruent rhythmic/textural structure. (For this reason, we use the terms 'metric[al] structure', 'metric[al] hierarchy' or metrical 'profile' (Lester 1986) to describe abstract, variable and emergent multiparametric groupings, but confine 'metre' to essentialist notions of concrete metrical grids and schematic generalisations that seem to exist in cognition.)

It might now be apparent why bona fide contrapuntal textures (with independent voices) present a serious difficulty for generative theories of metre. In Lerdahl (2001) and Temperley (2001) melodic groups in contrapuntal textures are automatically grouped to cue regular and hierarchical metres. But as just outlined, these theories of contrapuntal metrical induction by texture are problematic in principle, because idiosyncratic melodic groupings should cue idiosyncratic (noncongruent) metrical structures. Since well-formed metre is conceived as a necessary product of contrapuntal grouping in generative models, it must shoehorn idiosyncratic melodic groups into artificially regular and homogeneous metrical units. In doing so, it makes a compromise between the multiplicities of independent melodic groupings that might individually have implicated noncongruent/ill-formed metrical structures. In cognition, it is possible to suspend judgment in the face of such ambiguity; cognition does not have to 'decide' on a definitive metre from a multiplicity of non-isochronous groupings. Lerdahl (2001) and Temperley (2001) offer no such nuances, since it is arguably the purpose of these models to reconcile groups in texture into a regular nested metrical analysis, precisely with the intention of showing how metre is cognised.

Notwithstanding these contentions, and as outlined, a direct relation between texture and metrical structure can be useful to draw when it is part of a programme that does not try to establish inductive inference (following our earlier explanation), and when the congruent interaction is unequivocal. In this case, there must be clear regularity of accent and homogeneity between melodic groups in texture at regular and nested levels of hierarchy so that a congruent metre is implied without question. Accordingly, for a butterfly schema, the textural grouping must be a regular nested hierarchy to cue a regular nested metrical structure.

As noted, many styles are metrically congruent at particular levels of hierarchy. Indeed, textural and metrical profiles observe various degrees of congruency, and often manifest regular nested hierarchies in and between distinct levels. The well-formedness rules of *GTTM* are presented as applicable to all tonal music (Lerdahl and Jackendoff 1983, p. 347), but do not account for the variable textural structure that differentiates between the metrical styles of tonal music. A regular nested hierarchy in textural grouping (that produces a regular nested metrical structure) is here formally defined by the expression H = 2L, where a higher-level note length (H) is equal to two lower-level note lengths (L). Fig. 5 compares textural/metrical grouping in the Baroque and Classical periods. The notational values are applied merely for explanation; only the mathematical relations between levels are significant. Also, the preferred levels can vary to a limited extent.

[INSERT Fig. 5 NEARBY]

Cone (1968, p. 72) argues that in the Classical style the bar level of metrical structure is the 'main metrical unit' (generally the fifth, sixth or seventh level of (hyper)metre, shown in Fig. 5). In the Classical style, this is the congruent level of accentuation in the textural and metrical structure that commonly corresponds with the level of regular functional harmonic change. While this is usually the 5th level of metrical structure in the Classical style it can occasionally be the 4th or 6th level, but rarely the 2nd, 3rd or 7th levels. The two (hyper)metrical levels directly above the level of regular functional harmonic change are generally texturally grouped with that level, resulting in a distinct regular nested metrical hierarchy, shown in Fig. 5 (similar to the portrayal of regular metre in Russell 1999 and Love 2015). The following rule summarises these points:

(10) *In the Classical instrumental style and butterfly schema, a regular nested hierarchy of textural grouping, defined as H = 2L, is uniparametrically congruent, forming at the level of regular functional harmonic change and at two immediately higher levels of metrical structure.*

Fig. 6 shows the relation between textural grouping and metrical structure in the butterfly schema and Classical instrumental style. Lower-level rhythmic groups are contained within higher-level rhythmic groups, satisfying the formula H = 2L and corresponding with regular nested metrical hierarchy.

[INSERT Fig. 6 NEARBY]

By contrast, in many Baroque metrical profiles the beat level is generally the primary metrical unit (Cone 1968, p. 66), corresponding with either the 1st, 2nd or 3rd levels of metrical structure, shown in Fig. 5. That is, textural grouping with H = 2L generally occurs at low levels. However, Baroque texture is relatively noncongruent at middle and high levels (Lester 1986, pp. 131–7); the primacy of the beat in Baroque music seems to facilitate freedom for higher multileveled pacings (Lester 1986, p. 128 and p. 153).

The Baroque chorale-with-instrumental-accompaniment style typically has noncongruence built-in to higher levels of texture (where H ≠ 2L), which might be characteristic of many contrapuntal styles of this period. Ex. 3 shows noncongruent metrical layering at the 4th and 5th levels of metrical structure in ‘O Mensch, bewein dein Sünde groß’ from Bach’s *St Mathew Passion*, BWV 244. Arabic numerals are used rather than dot structures – the preferred nomenclature in Lester (1986) and Rothstein (1989) – to show the superimposition of conflicting melodic groups and their implied metrical structures. In bars 17–19, the implied soprano metrical structure is out of phase (viz., noncongruent) with the implied orchestral metrical structure between the 4th and 5th levels of the metrical hierarchy. In bar 20, the alto, tenor and bass metrical structures are out of phase with the orchestral metrical structure between the same levels. Such noncongruence in texture and metrical structure at middle and high levels is uncharacteristic of the Classical instrumental style and butterfly schema, where, as theorised, a higher level of regular functional harmonic change (and two immediately higher levels) is congruent. If this depiction of Baroque textures obtains, broadly commensurate with the theorisations of Schenker (1935), Cone (1968) and Lester (1986), many Baroque styles do not have the appropriate infrastructure to foster butterfly schemata.

[INSERT Ex. 3 NEARBY]

In general, then, many fugal styles of the Baroque seem to sustain metrical noncongruence in and between middle and high levels of texture, while preserving texturally congruent grouping at low levels (i.e., the 1st, 2nd or 3rd level of metrical structure, shown in Fig. 5). This is supported by Schenker’s (1935/1975), Cone’s

(1968) and Lester's (1986, p. 252) portrayal of textural dissonance at middle and high levels in fugal writing of the Baroque period. We posit that noncongruence seems to be commonly engineered in two ways in Baroque fugues. The first form, which can be termed Type 1, and shown in Ex. 3, occurs through the conflict between two different melodic grouping structures in texture (and their implied metrical profiles), such as between themes and accompaniments (or between subjects and countersubjects). The second form, here termed Type 2, is the product of conflicting irregular repetitions/imitations of a theme, as exemplified by overlapping entries of fugal subjects. Both forms do not permit the inference of a regular constituent metrical hierarchy, because they do not have regular constituent textural grouping (i.e., they do not observe H = 2L). It should be noted that noncongruence can occur simply through irregularity of rhythm or pitch shape within a single melodic group, which can also result in implied metrical noncongruence, but this cannot be conveniently classified as a type so is put aside presently.

A small corpus study of Bach's *Well-Tempered Clavier*, book 1 (1722), was carried out to examine the commonality of the two types of textural/metrical noncongruence. It was found that 13 of the 24 fugues contained Type 1, Type 2 or both forms together, and when present were fairly consistently employed throughout. The results were as follows: Fugue in C major had Type 1 and Type 2; Fugue in C minor had Type 1; Fugue in C<s> major had Type 1; Fugue in C<s> minor had Type 1 and Type 2; Fugue in D<s> minor had Type 1 and Type 2; Fugue in E major had Type 2; Fugue in F<s> major had Type 1; Fugue in F<s> minor had Type 1; Fugue in G minor had Type 1 and Type 2; Fugue in A<f> major had Type 1; Fugue in G<s> minor had Type 1; Fugue in A major had Type 1; Fugue in B <f> major had Type 1. The remaining fugues of the corpus contained neither Type 1 nor Type 2 noncongruence. In addition, the fugues were checked for the butterfly schema: none was found.

The results are suggestive, although not conclusive owing to the corpus size, that these types of noncongruent textural and metrical hierarchies are predominant in Bach's fugues in general, and perhaps by extension, other Baroque fugues and contrapuntal forms. However, additional corpus studies are required to establish how noncongruence is incorporated therein. While no butterfly schemata were recorded in the survey, this of course cannot be taken as conclusive evidence that few exist in the contrapuntal idioms of Bach or Baroque genres overall, since the survey is too small-scaled. Nevertheless, it seems probable that the butterfly schema and its various voice-leading sub-schemata exist in smaller numbers in Bach (noted in Gjerdingen 1988, p. 130 for the 1–7…4–3 schema) and other Baroque instrumental styles.

While middle- and high-level textural and metrical noncongruence might characterise many contrapuntal textures of the Baroque period (and also many styles of the Renaissance period, as can be understood from Berry 1976 and DeFord 2015), other genres of this period more closely satisfy the generalised notions of metrical well-formedness in generative theories. Some metrical profiles of Baroque dance suites more closely conform to the generative archetype of well-formed grids – although others depart from them in a number of ways also. Comprehensive analysis on the character of Baroque dance suite metrical structures has been carried out in

Little and Jenne (1991), identifying a number of textural/metrical schemata. For example, the Baroque minuet strongly corresponds with metrical well-formedness, constructed from regular nested textural groups at multiple hierarchical levels. However, the French courante (not to be confused with the metrically grid-like Italianate corrente) has a metrical profile with built-in noncongruence at middle levels, using superimposed 3/4 and 6/8 grouping structures (Little and Jenne 1991, p. 18). A number of other Baroque dances are also relatively metrically noncongruent, but perhaps less acutely. Thus, this picture of the textural and metrical structure of dance suites corresponds with what we see as the wider trend in Baroque metrical profiles: that noncongruence is relatively common at middle and high levels of metrical structure but uncommon at low levels.

This overview of Baroque-period textures should be counterbalanced with a discussion of those typical of the Romantic period. Cone (1968, pp. 79–82) contends that metrical profiles of the Classical period are comparable with those of the Romantic period, but argues that the hypermetrical units are even more regular in the latter than in the former. While this might be an appropriate depiction of the salient symphonic textures of the late-Romantic period, exemplified in the symphonies of Bruckner (Cone 1968, p. 82), this is arguably but one profile used therein. The Romantic period comprises a number of styles that are ostensibly more complex and internally more idiosyncratic than the Classical period, including a greater variety of textural and metrical types. Although ‘light’ musical genres, such as those of Johann Strauss II and Offenbach (typically dance-style compositions), and some symphonic styles, like that found in the music of Bruckner, are often congruent at hypermetrical levels, ‘serious’ art music of the period, as in the work of Schumann, Wagner or Brahms, can be relatively noncongruent at middle and high levels (Berry 1976, pp. 266–80, Krebs 1999). Therefore, hypermetrical noncongruence, inter alia, might be a causal factor on the hypothesised paucity of butterfly schemata in the Romantic instrumental style, although this is an avenue for further research.

Textural grouping can now be examined in the butterfly schema and Classical instrumental style. The opening of Mozart’s Symphony in G Minor, K. 550, shown in Ex. 4, has challenged and divided theorists in terms of its textural and metrical structure (cf. Bernstein 1976, Lerdahl and Jackendoff 1983, Lester 1986, McKee 2004, Temperley 2009, Mirka 2015). In broad terms, there is disagreement about whether the hypermetrical structure begins on bar 2 or bar 3. From a TFC perspective, the texture forms a regular nested hierarchical group (H = 2L) in bars 3–9, which gives rise to the metrical structure (in real-time listening). The textural grouping occurs at the level of regular functional harmonic change (the 5th level in this case) and at two immediately higher levels (the 6th and 7th levels), as required by rule 10. The textural grouping, harmonic rhythm (the rhythm of harmonic change, explained in Piston 1941, pp. 189–203 and Swain 2002, and discussed below) and harmonic progression are multiparametrically congruent in these bars, supporting a butterfly schema. However, the melodic grouping of the first violins, which is segmented into two-bar units from bar 2, is out of phase with these multiparametrically congruent features, although this is relatively insignificant for a congruence analysis.

[INSERT Ex. 4 NEARBY]

McKee (2004) likewise argues for the primacy of texture over melodic phrase grouping for the inference of metrical structure in this passage. The ‘rule of texture’ (McKee 2004, p. 5) ascribes metrical strength to accompaniments when they begin before a melody, supporting the view that the metrical structure begins on bar 3. However, Lester (1986) places the main hypermetrical downbeat at the beginning of bar 2, which also has a relatively strong textural accent, but specifically marks the onset of melodic grouping. Lester’s reading is prima facie consistent with Lerdahl and Jackendoff’s (1983, p. 76) ‘Metrical Preference Rule 2 (MPR2) Strong Beat Early’ and Rothstein’s (1995) ‘rule of congruence’, which prefer melodic phrase grouping onsets to occur on strong beats of metrical structure, although the latter authors do not support this interpretation of the passage. Even in terms of ‘generative’ considerations (melodic accent and contrapuntal grouping) there are contrary metrical conceptions of this passage.

Mirka (2015) likewise argues for a hypermetrical structure beginning with the onset of the melody in bar 2, identifying a ‘gavotte schema’ at this point. One advantage of this reading is that the gavotte schema aligns with the following section (beginning on bar 10, not shown in Ex. 4), which enables a hypermetrical structure over a larger section of music. By contrast, Gjerdingen (1988, p. 221, 2007, p. 262) identifies a hypermetrical Sol-Fa-Mi (5–4…4–3) voice-leading schema beginning at bar 3. Gjerdingen shows that this is nested in the melodic phrase grouping. As noted, the melodic phrase grouping, in contrast to the textural grouping, is not a multiparametrically congruent feature of this passage. (The textural grouping has an onset on bar 3 that is congruent with the two-bar harmonic rhythm and chord progression also starting on bar 3, which are primary cues for (hyper)metrical structure in the Classical instrumental style and butterfly schema.) In this example, the Sol-Fa-Mi cannot be a child schema of the parent butterfly schema, because the melodic grouping conflicts with other features. However, this represents a relatively minor noncongruence, and the structure as a whole is still more broadly generated by the TFC. But in Gjerdingenian schemata, MC is not actually an essential criterion, since structures can have a more flexible arrangement of features, so the Sol-Fa-Mi might be valid from an associative-statistical perspective. More generally, it is possible to consider this passage as an opening ‘presentation phrase’ of what Caplin (1998, pp. 35–48) describes as a ‘sentence’, a relatively generic two-part opening schema. The second section of the sentence is characterised by a destabilisation of grouping, rhythm and harmony, shown in this piece in later bars (not in Ex. 4). In terms of present considerations, this Caplinesque reading is coherent with generative models, schema theory and a TFC analysis, since it is compatible with a (hyper)metrical structure that begins either on bar 2 or bar 3.

It is important to note that although a TFC analysis prefers a bar-3 onset for the validation of the butterfly schema here, it does not preclude various voice-leading, generative or formal-functional analyses that have different onsets; these are legitimate from knowledgeable listeners' and cultural insiders' perspectives. Thus, a bar-3 onset does not negate understanding this example in terms of a voice-leading structure that begins on bar 2. Temperley's (2009) suggestion that a bar-3 hypermetrical beginning is a more sophisticated reading (perhaps commensurate with an 'experienced listener'), might be misleading since the passage is designed to be ambiguous. The present congruence analysis, with a bar-3 onset (broadly in line with Bernstein 1976, Lerdahl and Jackendoff 1983, McKee 2004 and Temperley 2009), is preferable from a standpoint that defines the butterfly schema in terms of MC, but it does not refute the existence of schemata that are depicted through other forms of understanding. It is presumed that listeners are able to attend simultaneously to conflicting representations of musical structure, perceiving a multiplicity of schemata.

Harmonic-rhythm ratios

While harmonic progression is widely acknowledged as an important feature of schemata in both associative-statistical and generative schools of thought, less commonly considered concepts in these spheres are harmonic change and harmonic rhythm. However, theorists not directly occupied with schema theory or generative theory often view harmonic change and harmonic rhythm to be amongst the strongest influences on other multiparametric features (Piston 1941, Berry 1976, Lester 1986, p. 58, pp. 66–68, Rothstein 1995, Swain 2002, Mirka 2007). For example, Rothstein's 'rule of harmony' (1995, p. 173) prefers changes of harmony to occur on the inception of strong beats in the metrical structure, although it is likely that this partly depends on historical and stylistic context. Prima facie, harmonic change seems to be a more important stylistic cue for metrical structure in serious Classical music than in serious Romantic music, because in the latter harmonic change and metrical structure are less consistently aligned. This general inconsistency between harmonic change and metrical structure in serious instrumental Romantic music might be a factor in the projected paucity of butterfly schemata therein.

As pointed out earlier, the metrical level of regular functional harmonic change is commonly multiparametrically congruent with other uniparametrically congruent features of the butterfly schema and Classical instrumental style. Simple ratios of harmonic change, termed 'harmonic-rhythm ratios', form at this level, corresponding with each stage of the schema, excepting occasionally the midpoint (which is accented by textural grouping). Indeed, the ratios 1:1 and 3:1 are uniparametrically congruent and multiparametrically congruent with other parametric features. Ex. 5 illustrates a butterfly schema with a 1:1 ratio, and Ex. 6 depicts one with a 3:1 ratio. The following rule defines these ratios:

(11) *The harmonic-rhythm ratios 1*:*1 and 3*:*1 are uniparametrically congruent features in the Classical instrumental style and butterfly schema, forming at the level of regular functional harmonic change*.

[INSERT Ex. 5 and Ex. 6 NEARBY]

Duple structuring at multiple levels in harmonic rhythm permits more extended and regular (vertically and horizontally) nested (hyper)metrical hierarchies (loosely following Lerdahl 2001, p. 286). (Note that duple structuring was also shown above to be required for textural grouping to cue regular nested metrical hierarchies, i.e., H = 2L.) Harmonic-rhythm ratios in the butterfly schema, 1:1 and 3:1, imply duple structuring at high levels. The simplest ratio, 1:1, is more uniparametrically and multiparametrically congruent than the 3:1 ratio because it permits greater unbroken duple hierarchical depth. That is, the 1:1 ratio generates UC and MC at more (hyper)metrical levels than the 3:1 ratio. Nevertheless, these ratios are both more uniparametrically and thus more multiparametrically congruent (that is, they permit deeper duple structures) than complex ratios, such as 2:1 or 5:1. The ratios 2:1 and 5:1 inform triple structuring at relatively high levels of texture and metre, conflicting with the universal preference for duple structuring; they do not permit hierarchies as regular as 1:1 and 3:1. The following rules depict the minimally and maximally congruent harmonic-rhythm ratios of the butterfly schema and Classical instrumental style:

(12) *A 1:1 harmonic-rhythm ratio is maximally uniparametrically congruent*.
(13) *A 3:1 harmonic-rhythm ratio is minimally uniparametrically congruent*.

A model and survey of the butterfly schema, circa 1750–1850

Fig. 7 shows a multiparametrically congruent model of the butterfly schema, comprising the features presented above within chord progression, textural grouping and harmonic-rhythm ratio. While chord progression and harmonic-rhythm ratio are variably congruent in this presentation, textural grouping is non-variable. (However, it is likely that textural grouping is in actuality of graded congruence, but a model that accounts for this would require a level of detail beyond the scope of this article.) The harmonic-rhythm ratios 1:1 and 3:1 are expressed numerically and through the corresponding length of the rectangles; the harmonic functions are inscribed therein; the main textural groupings are illustrated using hemispheres; and the metrical hierarchy is shown with dot structures. (A congruent tonal structure is implicit.) We present minimally and maximally multiparametrically congruent versions of the butterfly schema. In the former, the degree of congruence is generalised; a minimally multiparametrically congruent version can have an admixture of minimally and maximally uniparametrically congruent features. But for a maximally multiparametrically congruent version, all features must be maximally uniparametrically congruent (see Ex. 5 for an instance of this type). This distinction was made to provide a clear boundary between minimal and maximal MC for the survey. The following rule sets show how the uniparametric rules combine to define minimally and maximally multiparametrically congruent forms:

(14) rule x + (rule v + rule xiii) ***or*** (rule v + rule xii) ***or*** (rule vi + rule xiii) = minimally congruent butterfly schema.

(15) rule x + rule vi + rule xii = maximally congruent butterfly schema.

[INSERT Fig. 7 NEARBY]

A survey of European instrumental music c. 1750–1850, summarised in Tables 1 and 2, was undertaken to examine the distribution of butterfly schemata in samples of the Classical and Romantic instrumental styles. We faced a number of challenges owing to the structure of the corpus analysed. The most pertinent methodological issues included the following: How are samples chosen and how are they delimited? Which composers should be included? How many presentations of a schema in a work or movement should count statistically, and what should they be counted against?

It was important that the samples were defined in a way that instances of the schema could not be 'cherry-picked' (as recognised in Meyer 1989, pp. 57–59 and Cavett-Dunsby 1990). Entire genres of composers were used to demarcate samples because they comprise predefined collections of works and movements. It was not feasible to randomly select works or movements from a database of each period since the process for choosing a database (from which to randomly select data) must itself be somehow determined – although this would have been preferable in one sense because it would have permitted an equal number of movements to have been surveyed. The election of pre-defined corpora of seminal composers of the periods (i.e., instrumental genres) was thought favourable because these are part of a distinct lineage in the Western instrumental music canon that offers a coherent connection between periods. However, instrumental works of the Classical period generally comprise multiple movements, whereas Romantic instrumental works often have single movements. Thus, if the survey examined an equal quantity of genres from each period, it could not compare an equal quantity of movements. To account for this inconsistency, schemata were counted against the number of movements (or single-movement works) in each genre, and the percentage of schemata found for each period was compared. If quantities against the number of whole works had been calculated, butterfly schemata would have had a much greater score in the Classical-period samples than the Romantic-period samples. This finding would have better suited the hypothesis, but of course would not have been a fair representation.

It would have also better fitted the hypothesis to permit recording an unlimited quantity of butterfly schemata in each movement/single-movement work. The TFC generates structure that often results in several butterfly schemata forming together, and repeated sections of a piece would mean counting the 'same' schema two or three times, whereas movements that have no butterfly schemata are only able to record a single negative score. Thus, we limited the count to one per movement and single-movement work to avoid a false comparison between works where butterfly schemata were present and those where they were absent. In sum, since the use of a selection of salient composers and complete genres was necessary to compare like for like between periods, it was necessary also to compare percentages of schemata found and to limit the tally to one per movement and single-movement work.

Table 1 indicates the quantity and types of butterfly schemata found in the two sets of samples, which total nine hundred and seventy-three movements (/single-movement works) from ten genres by five composers. The survey shows also whether butterfly schemata found had maximal or minimal MC and whether Gjerdingen's (1988) 1–7…4–3 (Meyer) schema was embedded within. (Other voice-leading schemata, such as the Jupiter, Aprile, Pastorella, Do–Re–Mi and Sol-Fa-Mi schemata, which can reside in the parent butterfly schema, were found in insufficiently large quantities to represent.)

[INSERT Table 1 NEARBY]

In the Classical-period set of samples, 22% of movements contain the butterfly schema compared with 7.6% of movements (and single-movement works) in the Romantic-period set of samples. That is to say, the butterfly schema is almost three times more common in the Classical-period set of samples than the Romantic-period set. Since the butterfly schema is a multiparametrically congruent structure, this suggests that the TFC in the particular manifestations presented is a constraint on its formation. An independent t-test compared the variability between sets against the variability within sets. It was found that the percentages of schemata found in the Classical-period samples were consistently different from the percentages in the Romantic-period set (see Table 2). This difference is unlikely to have occurred by chance ($t = 6.10$; $df = 10$; $p = 0.0001$, two-tailed distribution).

[INSERT Table 2 NEARBY]

We can now consider the second component of the survey, which concerns the type of features that form in butterfly schemata. Of the butterfly schemata found in the Classical samples, 77% have maximal MC, while the remaining 23% have minimal MC. The predominance of butterfly schemata with maximal MC converges with the above findings on the commonality of butterfly schemata per se, showing that the TFC in a particular manifestation is a constraint on their formation.

This article has considered whether the butterfly schema is a particular and localised manifestation of the TFC. That it is a predominant congruent structure in the Classical instrumental samples but less common in the Romantic instrumental samples, and more commonly has features that are maximally congruent than minimally congruent, is evidence that supports this hypothesis. Baroque styles were also explored, but were shown not to contain the type of multiparametrically congruent infrastructure required to consistently produce this schema.

We have questioned the generalisability of generative theories. Music of the common practice period is often schematised as a homogenous body in generative theories (Lerdahl and Jackendoff 1983, Lerdahl 2001, Temperley 2001). However, that textures necessarily cue well-formed metre contradicts the variety of interaction between voices in texture and other features that result in various metrical structures in tonal styles. For example, texture and metrical structure are often highly specialised in contrapuntal styles of the Baroque and Renaissance, where particular levels are articulated. The theories of contrapuntal grouping in Temperley (2001) and Lerdahl (2001) are too coarse-grained to account for the distinct type of textural grouping in the Classical instrumental style and butterfly schema. Harmonic rhythm is also not fully incorporated into generative models. We have shown that harmonic-rhythm ratios are multiparametrically congruent with other uniparametrically congruent features of the butterfly schema and Classical instrumental style. This is important for the inference of higher-level concepts in this style, such as tonal and metrical structure, as well as lower-level concepts, such as localised schemata.

Associative-statistical theories present situated models of style corresponding to culturally particular understanding, although are arguably less revelatory about cognitive universals. The survey shows that the 1–7…4–3 (Meyer) schema was a relatively common voice-leading schema in the Classical set of samples, embedded within 17% of butterfly schemata. This is a considerable presence, especially when considering it is here formulated as a child schema of the butterfly schema, where the imposed congruent strictures do not form part of a Gjerdingenian analysis. Indeed, that specific voice-leading schemata are less common than the butterfly schema does not directly challenge the claims of schema theory (i.e., that voice-leading schemata offer a more coherent representation of Galant/Classical styles than generative models, e.g., Gjerdingen 2007), because in the present model, voice-leading schemata are defined as sub-schemata of the butterfly schema. Thus, any particular voice-leading schema might be expected to be less common than the butterfly schema, since the availability of voice-leading paths in the butterfly schema is many times greater than that of a single voice-leading schema.

Voice-leading structures largely supervene on the functional harmonic structure in the Classical instrumental style; the latter more commonly forms multiparametrically congruent relationships with other abstract features. A number of theorists, including Berry (1976) and Swain (2002), have noted the increasing importance of functional harmony over voice-leading structure in the eighteenth century, although it is clear that in certain contexts in the Classical instrumental style, voice-leading structure dictates functional harmony (see Rohrmeier and Neuwirth 2015 for further discussion). The significance of voice-leading structure in this style is shown in our survey through the commonality of the 1–7…4–3 structure. It is clear that voice-leading structure involves a more fine-grained conception than that currently available through the theory of the TFC. It was established earlier that the notion of the TFC can be held simultaneously with theories of situated cognition, since these refer to distinct, although related categories and concepts. Indeed, in some of the butterfly schema examples presented, voice-leading schemata might be meaningfully picked out when considered from historically situated listening practices (following Gjerdingen 2007 and Sanguinetti 2012). In sum, voice-leading schemata are products of conventions that co-exist with multiparametrically congruent structural relationships in the Classical instrumental style.

That butterfly schemata emerge in small quantities in the Romantic-period samples seems to contradict the main argument of this article, that butterfly schemata are a particular and situated manifestation of the TFC. Also, maximally congruent butterfly schemata were the most common form in the Romantic samples (80% of butterfly schemata had maximal MC; 20% had minimal MC). These mercurial findings must be counterbalanced with the broader evidence that the butterfly schema is consistently more popular in the Classical set of samples than in the Romantic set of samples. Despite the blurred style boundaries between periods, there is a dramatic difference between the respective distributions of butterfly schemata. During the evolution from the Classical instrumental style to the Romantic instrumental style, it seems that relatively congruent vestigial features of the old style remain, although they become less frequent and are usurped by novel features and evolved interrelationships between new and extant features (Jan 2007). A style is never completely pure, changing seamlessly between periods. Since congruence seems to be generated from a universal source (i.e., cognition), there is a propensity for similar multiparametrically congruent structures to form in various periods and styles.

As a fraction of the total musical possibilities in the Classical instrumental style, the butterfly schema might be of relatively modest proportions. However, the individual features that make it up occur much more globally. And so this study has wider significance than the confines of the butterfly schema; it concerns the interaction of features over the Classical instrumental style as a whole – even though only the most congruent combinations become schemata. While one or two features are commonly congruent in the Classical instrumental style according with the TFC, the condition that many features coincide in particular formulations to satisfy a definition of a local schema significantly lowers commonality. By extension, it appears that the popularity of a schema is relative to the degree of congruence in and between its features, but increasing the quantity and particularity of features considerably reduces prevalence. The notion of the TFC might pertain to the interaction of features in other styles and periods. While it requires further investigation it is a tantalising notion, especially if it turns out that congruence is necessary for musical understanding at indefinite levels of abstraction, as has been the outlying suggestion in this article. While it looks more certain that the TFC has a broadly cognitive origin, further analysis on the relationships between features in different styles and schemata is required to establish a more detailed picture.

[1] The Classical instrumental style is considered a sub-style of the Classical style, and so the former shares a large part of the latter’s infrastructure.

[2] This paper extends the model presented in Rawbone & Jan (2015) and Rawbone (2017).
[3] The 'rule of congruence' of Longuet-Higgins and Steedman (1971) has a different and more general meaning than Rothstein's (1995) usage; the latter concerns the specific congruent interaction between grouping and metre.
[4] Appoggiatura chords are those that are dissonant with a bass note, after which a new chord is placed that is consonant with the same bass note (Piston 1941, p. 126).