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An Improved Optimisation Model for Horizontal Pipelines Transporting Solid Liquid mixtures

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ABSTRACT

The optimisation of solid-liquid two-phase flow pipelines is essential, to ensure commercial viability of these pipelines. A complete optimisation model needs elements of both mechanical design and hydraulic design to be included. In this work, an optimisation methodology has been developed that integrates both mechanical and hydraulic design aspects. The model allows determination of the size of the pipeline for adequate operational conditions and requirements that ensures least cost. All these models are interconnected together to integrate both the operational and commercial aspects into consideration. Robustness and user-friendliness are the two main features of the proposed model.

Keywords: *Optimisation ; Least-cost principle ; Slurry Pipeline.*
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1. Introduction

Slurry pipelines are being extensively used for transporting solid materials in bulk quantities over large distances in various industries [1-3]. The use of fluid flow for transportation purposes has been practiced for more than a millennium and detailed information on the flow behaviour of such complex mixtures in pipelines is still the subject of active research today. The optimal design of a slurry pipeline includes the selection of the correct pipe sizes, and material's characteristics for optimum energy consumption and reliable operation of the pipeline networks. Although various aspects of slurry pipelines have been analysed in detail, an integrated optimization model for designing such systems is not available [1-3]. It is proposed to develop an optimisation model in the current study based on the least cost principle. This model is designed based on the two-layer model, which is known be very accurate in estimating energy requirements for transporting the mixture, to find the cost of energy for running any slurry system. In addition, the model has been used to find the optimal diameter of horizontal pipelines transporting slurries.

2. Developed Least Cost Principle Method

The least cost principle model intends to optimize the design in order to have minimum cost. The cost of a pipeline includes the manufacturing cost and operation cost of the system. This model can also be applied to a slurry pipeline system if these two costs can be represented as a function of size of the pipe and other relevant variables.

The total cost of a slurry pipeline C_T can be defined as the following:

$$\mathbf{C_T = C_{manufacturing} + C_{Operating}} \quad (1)$$

In order to have an accurate and meaningful total cost, we have to calculate both the manufacturing and operating costs in over a specified time period. The operating cost can easily be calculated for one-year of operation. However, the manufacturing cost can also be considered per year by using system depreciation model.

2.1. System depreciation

Depreciation is an accounting method in order to allocate the total cost of any system over its expected operation life and for many other purposes in financial analysis. In this work, depreciation can be used in order to calculate the manufacturing cost per one year of operation.

There are different depreciation methods such as declining balance method and straight-line method. In this work, the straight-line method was used due to simplicity of the system and its applicability to the optimization model.

$$\textbf{Depreciation value} = \left(\frac{\textbf{System Value} - \textbf{Salvage value}}{\textbf{Useful Life}}\right) \quad (2)$$

2.2. Manufacturing cost

The manufacturing cost of most slurry systems can be divided into the pumping station cost and pipeline cost. The design selection of any pumping station is based on many parameters such as the minimum required solid throughput, the mixture properties, pipeline type and length.

$$\mathbf{C_{Manufacturing} = C_{pump} + C_{pipe}} \quad (3)$$

Where C_{pump} is the pump cost per year of operation which is constant depends on the type, size and brand of the used pump. The system depreciation equation, illustrated in the previous section, is used to calculate the annual cost of the pump.

The net annual cost of pipe per unit pipe length C_{pipe} is given by Chermisinoff [3] as a function of net annual cost of pipe per unit weight of pipe material C2:

$$\mathbf{C_{pipe} = \pi\, D\, t\, \gamma_p\, C_2} \quad (4)$$

where:

t : Thickness of pipe(m)

$\gamma_p = \rho_p g$ = Specific weight of the pipe (N/m^3)

ρ : Density (kg/m^3).

The standard dimension ratio SDR is the ratio of pipe diameter to the wall thickness of the pipe. Hence, the cost of the pipe would be:

$$C_{pipe} = \frac{\pi D^2\, \gamma_p\, C_2}{SDR} \quad (5)$$

The above equation uniquely defines the pipe cost as a function of pipe diameter and hence it will allow us to investigate functional variation of cost with diameter within operational range of the slurry pipeline. The pump cost is constant and can be calculated as a function of pipe cost.

2.3. Cost of Operating

The cost power for operating the system for one hour can be expressed as:

$$\mathbf{C_{Power/hr} = C_1 * P} \quad (6)$$

Where C_1 is the unit cost of power per one KW-Hr and P is the power needed for operating the pump. The power needed for operating the system can be defined as:

$$\mathbf{P = \gamma_m\, Q\, \Delta P/\eta} \quad (7)$$

Where:

$\gamma_m = \rho_m g$: Specific weight of the mixture (N/m^3)

Q_m: Mixture flow rate (m^3/sec)

ΔP: Pressure drop (m/m)

η: Efficiency of the Pumping Unit (%)

The mixture density can be defined as:

$$\mathbf{\rho_m = (C_v\, \rho_s) + (1 - C_v\, \rho_l)} \quad (8)$$

Where:

ρ_s : Solid density (kg/m^3)

ρ_l : Carrier fluid density (kg/m^3)

The mixture flow rate can be expressed as a function of the pipe cross-sectional area and operational velocity as below:

$$\mathbf{Q_m = (\pi\, D^2/4)\, V} \quad (9)$$

V : Flow Velocity (m/sec)

The total cost of power for operating the pipeline for one year can be expressed as:

$$\mathbf{C_{Operating} = \left(C_1\, \gamma_m\, Q_m\, \Delta P\, L_P\, H_{day}\, D_{year}\right)/\eta} \quad (10)$$

where:

L_P : Length of the pipe (m)

H_{day} : Operating hours per day (Hr/Day)

D_{year} : Operating days per year ($Day/Year$)

The pressure drop has been calculated in this model based on the two-layer model. In addition to the mentioned major losses, there are minor losses such as the loses due to pipe fittings and bends [4] and the losses due to crushing of particles [5]. For a long pipeline, the effect of these minor losses will be negligible [5].

3. The SRC Two-Layer Model to Compute Energy Consumption

The Saskatchewan research council (SRC) two-layer model of Gillies et al [6] divides the solids into two groups corresponding to fine and coarse particles (Figure 1). The model has been verified successfully against experimental data with reasonable accuracy by Gillies and Shook [7]. Furthermore, the model has undergone a number of refinements such as extension to higher solid volume fraction by Gillies et al [8] and higher velocities by Gillies et al [9]. Basically, the model considers two types of friction which are termed as kinematic and sliding bed frictions. Kinematic friction, also called as velocity-dependent friction, depends on the carrier fluid velocity and another component resulting from the particle dispersive stress [10, 11]. The Coulombic or sliding bed friction is produced by the action of the solid particles that are not suspended by the fluid turbulence and sliding against the pipe wall. The normal stress resulting from the immersed weight of the contact load particles is related to the shear stress required to move the sliding bed according to the Coulomb's law of friction [10, 12].The mass balance is represented by the following equation in the model. Where A is the area of the pipeline and V_1 and V_2 are velocities of upper and lower layers as shown in the figure1.

$$A\, V = A_1\, V_1 + A_2\, V_2 \quad (11)$$

The momentum for the upper layer, lower layer and entire pipe

is expressed in the equations (12), (13) and (14) respectively.

$$-\frac{d(P+\rho_1\, g\, h)}{dz} = \frac{\tau_1\, S_{1+}\, \tau_{12}\, S_{12}}{A_1} \quad (12)$$

$$-\frac{d(P+\rho_2\, g\, h)}{dz} = \frac{-\tau_{12} S_{12} + \tau_2 S_2}{A_2} \quad (13)$$

$$-\frac{d(P+\rho_m\, g\, h)}{dz} = \frac{\tau_1 S_1 + \tau_2 S_2}{A} \quad (14)$$

Where:

τ_1 : shear stress in layer 1 (pascal)

S_1: pipe boundary length of upper layer (m).

τ_2: shear stress in layer 2 (pascal)

S_2: pipe boundary length of lower layer (m).

τ_{12} : shear stress along the boundary between both layers

S_{12}: boundary length between upper and lower Layers.

Equations (11), (12) and (13) are solved to determine layer velocities and then equations (14) and (11) are solved to determine the pressure gradient. The details about SRC two-layer model have been explained in [13-16].

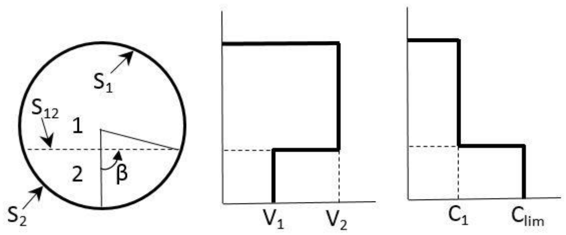

Figure 1. Idealised schematic

4. The Optimisation Model

An optimisation model has been proposed in this work based on developed least cost principles and prediction models that were proposed in the earlier section. The following steps should be followed to run the optimisation model. The input to the model is the solid throughput. A flow chart showing the outline of optimisation process is shown in figure 2.

1. Assume a value of D from a minimum value (for example 0.05m) to a maximum value (for example 0.2m) using step 0.05. This value should be chosen so that the solid throughput condition can be satisfied (target).
2. The length of the pipeline is already known.
3. Assume the value of solid concentration.
4. Mixture flow rate Q_m can be calculated using solid throughput and solid concentration.
5. For each value of D, the mixture flow velocity can be calculated using mixture flow rate and pipe cross-section area.
6. Calculating the cost of pipes depends on the information regarding the pipe materials and the market price.
7. For each value of D, find the optimum value of velocity to reach the required value of solid throughput.
8. Assume the value of the efficiency of the pumping unit (0.6 – 0.75) and then keep it fixed.
9. Calculate the pressure drop using two-layer model.
10. Find out the power requirement for the system.
11. Assume the useful life of the pipeline, such as 20 years.
12. Calculate the power cost of the pipeline for the period of useful life of the pipeline (20 years operation).
13. Calculate the total cost of the pipeline (Pipeline cost + operation cost) for the useful life period (20 years).
14. Save the data for each value of D and find the case where the value of the total cost is minimal.

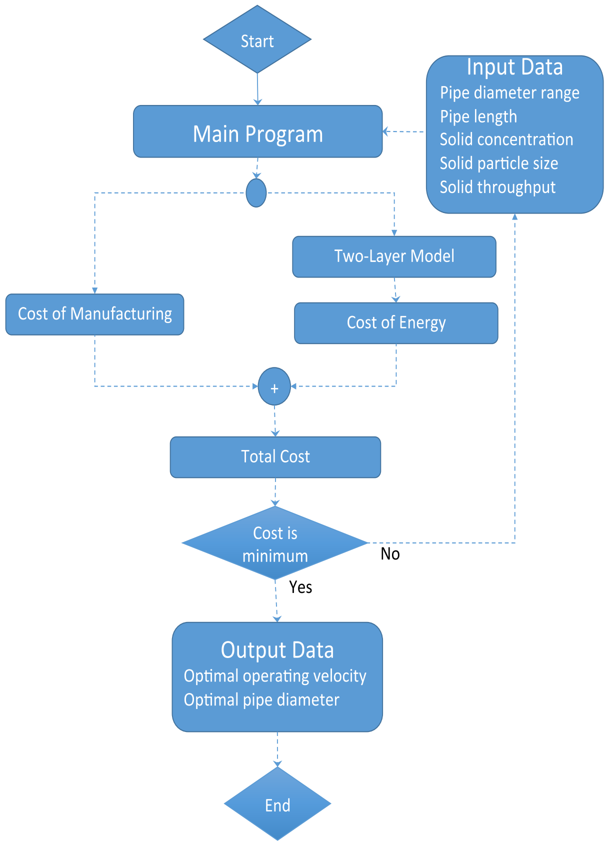

Figure 2. Flow chart of the model

5. Design Case Studies

In the following various case studies have been included to show the usefulness of the proposed model in designing pipelines for a variety of conditions.

5.1. Case study 1: Optimisation for Uni-size Slurry Pipeline

An optimal design is needed for a highly dense polyethylene pipeline transporting 0.7 mm diameter sand with 20% concentration. The solid density is 2650 kg/m^3 and the carrier fluid is water with 999 kg/m^3 density. The required solid throughput is 65 kg/hr. Assume that the cost of unit power is C1=£1.4 per KwH, the pipe density is 960 kg/m3 (HDPE) and C2= £1 per unit length of the pipe per year.

By applying the proposed optimisation model, the following results (Table 1) were obtained.

Table 1. Variations in total cost and pumping velocity w.r.t pipe diameter for Case study 1

$C_{Manufacturing}$ (£)	C_{Power} (£)	C_{Total} (£)	Diameter (m)	Operating velocity (m/s)
86.86	936.90	1023.76	0.08	4.88
109.93	519.49	629.42	0.09	3.86
135.71	307.14	442.86	0.1	3.13
164.21	191.24	355.45	0.11	2.58
195.43	124.26	319.68	0.12	2.17
229.36	83.66	313.02	0.13	1.85
266.00	58.06	324.06	0.14	1.59

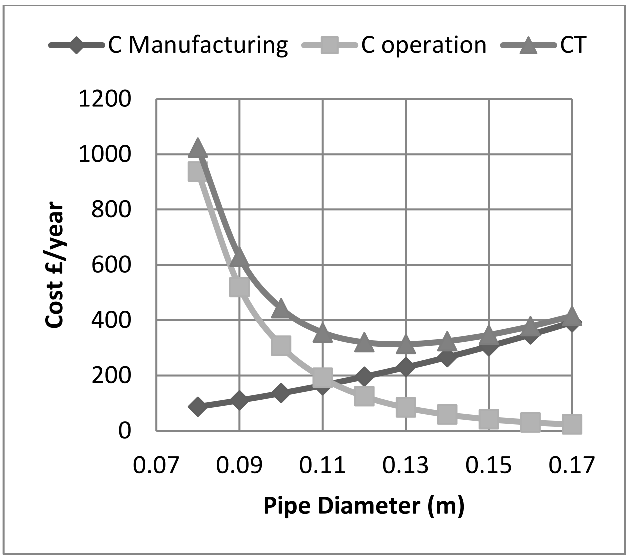

Figure 3. Variation of various slurry pipeline costs for case study 1.

The figure 3 above shows that as the pipe diameter increases the pumping cost of the pipe decreases whereas the capital cost of the pipe increases. The total cost initially decreases and then starts to increase. The total cost reaches a minimum value at the optimum diameter of the pipeline which in this case is 0.13 m. It must be mentioned that same process may be followed even when the constants may have different values depending on the market conditions.

5.2. Optimisation for Multi-Size Slurry Pipeline (Case study 2)

The developed methodology works for multisided slurry as well and to demonstrate this the following case study is presented. It must be mentioned that

An optimal design is needed for a high density polyethylene pipeline transporting multi-sized slurries with 15% concentration. The solid density is 2650 kg/m^3 and the carrier fluid is water with 999 kg/m^3 density. The required solid throughput is 60 kg/hr. Assume that the cost of unit power is C1=1.4 per KwH, the pipe density is 960 kg/m3 and C2=1. The solid size fractions (Pd in microns) and percentage finer (Sp) are as follows:

$P_{d(}$	2380	1190	841	595	420	297	210
S_p	1	0.962	0.881	0.674	0.237	0.039	0.008

By applying the proposed optimisation model, the following results (Table 2) were obtained.

The Table 2 and Figure 4 show that as the pipe diameter increases the pumping cost of the pipe decreases as in figure 3 and also the capital cost of the pipe increases. Furthermore, the total cost initially decreases and then starts to increase. The total cost reaches a minimum value at the optimum diameter of the pipeline which in this case is 0.11 m. It must be mentioned that same process may be followed even when the constants may have different values depending on the market conditions.

Table 2. Variations in total cost and pumping velocity w.r.t pipe diameter for Case study 2

$C_{Manufacturing}$ (£)	C_{Power} (£)	C_{Total} (£)	Diameter (m)	Operating velocity (m/s)
86.86	644.74	731.60	0.08	4.51
109.93	357.32	467.25	0.09	3.56
135.71	211.16	346.88	0.1	2.88
164.21	131.42	295.64	0.11	2.38
195.43	85.36	280.79	0.12	2.00
229.36	57.45	286.81	0.13	1.70
266.00	39.86	305.86	0.14	1.47
305.36	28.38	333.74	0.15	1.28
347.43	20.67	368.10	0.16	1.12
392.21	15.36	407.57	0.17	0.99
439.71	11.61	451.33	0.18	0.89

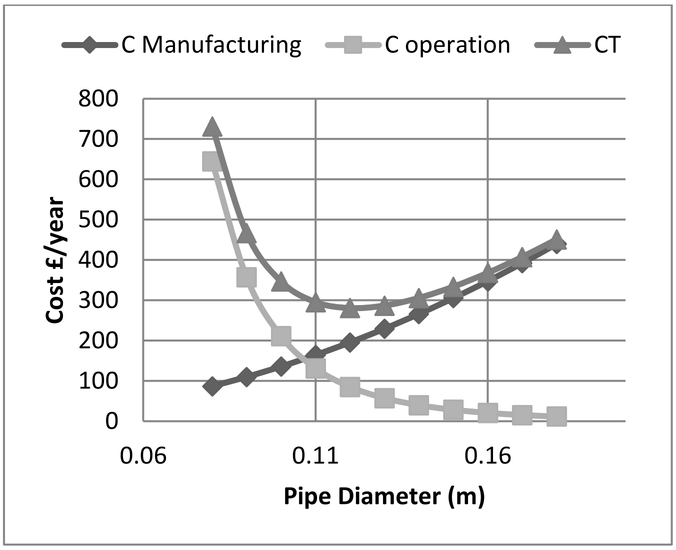

Figure 4. Variation of various slurry pipeline costs for case study 2.

5.3. Optimisation for Multi-Size Slurry Pipeline (Case study 3)

An optimal design is needed for a high density polyethylene pipeline transporting multi-sized slurries with 15% concentration. The solid density is 2650 kg/m3 and the carrier fluid is water with 999 kg/m3 density. The required solid throughput is 60 kg/hr. Assume that the cost of unit power is C1=1.4 per KwH, the pipe density is 960 kg/m3 and C2=1. The solid size fractions and percentage finer are as follows:

P_d	595	420	297	210	149	74	595
S_p	1	0.998	0.951	0.571	0.147	0.005	1

By applying the proposed optimisation model, the following results (Table 3) were obtained.

Table 3. Variations in total cost and pumping velocity w.r.t pipe diameter for Case study 3

$C_{Manufacturing}$ (£)	C_{Power} (£)	C_{Total} (£)	Diamete r (m)	Operating velocity(m/s)
109.93	558.88	668.81	0.09	4.15
135.71	329.85	465.56	0.1	3.36
164.21	205.04	369.25	0.11	2.78
195.43	133.02	328.45	0.12	2.34
209.36	109.44	318.80	0.13	1.99
266.00	61.99	327.99	0.14	1.71
305.36	44.10	349.46	0.15	1.49

347.43	32.10	379.52	0.16	1.31
392.21	23.83	416.04	0.17	1.16
109.93	558.88	668.81	0.18	1.04

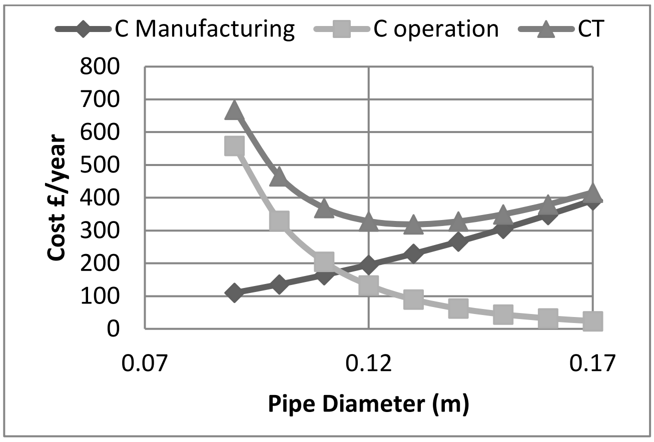

Figure 5. Variation of various slurry pipeline costs for case study 3.

In this case also the trends similar to previous cases have been obtained and the optimum diameter has been found to be 0.12 m. It can be seen that the particle size gradation has considerable impact on optimum pipe size for slurry transport.

Figure 6 depicts the variations in the pumping velocity and total head loss in the slurry pipeline i for different pipe diameters. It can be seen that as the mixture velocity drops for a given throughput as the pipe diameter increases. In addition, the figure clearly demonstrates that the total pressure drop also drops as the diameter of the pipeline increases.

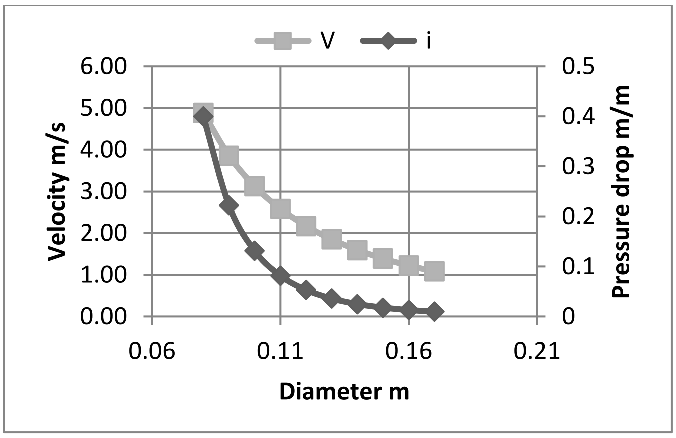

Figure 6. Variations of operating velocity and total head loss for various pipeline diameters for case study 3.

5.4. Summary about the case studies

In the previous sections, three different solid liquid mixtures have been used for optimal pipe size selection. In the first case study, the optimal operating velocity was 1.85 m/s for 700 micron uni- sized particulate slurries with 20% concentration. In case study 2, the concentration decreased to 15% and for multi-sized particulate slurries the optimal velocity increased to 2 m/s and the total cost decreased to £281. In third case study, finer slurries have been used with the same concentration 15%. The results show the optimal velocity remained unchanged with a significant increase in total cost due to the increase in operating cost. In the case study 3, higher head-loss will be produced and as a result, the cost of pumping will increase. The variations of total cost for optimal operating velocity and pipe diameter for all three case studies are illustrated in Table 4.

Table 4. Variations in total cost, optimal pumping velocity and optimal pipe diameter for the previous case studies.

Case studies	C_{Total} (£)	Optimal Diameter (m)	Optimal Operating velocity(m/s)
Case study 1	313.02	0.13	1.85
Case study 2	280.79	0.12	2.00
Case study 3	318.80	0.13	1.99

6. Conclusion

A detailed cost analysis of pipelines transporting slurries with fixed solid throughput gives the following results:

- The manufacturing cost is directly proportional to the pipe diameter (Figures 3, 4 and 5).
- The operating cost is inversely proportional to the pipe diameter (Figures 3, 4 and 5).
- The total cost is inversely proportional with diameter if the diameter is less than the diameter at the optimal point (Figures 3, 4 and 5).
- The total cost is directly proportional with diameter if the diameter is greater than the diameter in the optimal point (Figures 3, 4 and 5).
- The total head loss is inversely proportional to the pipe diameter and increases with mixture velocity increase (Figure 6).
- For the same concentration of case studies (2&3), the coarser particle size has a smaller optimal pipe diameter than the finer particles slurries. This is due to a higher pressure drop in the flow of the fine particle slurries.
- The useful life of any pipeline is an important parameter in the optimisation model. This value must be accurately assumed by the designer based on the special mathematical equations according to the quality of the used materials.

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