

Development of a Novel Methodology for Calculating the Thermal Efficiency of Clean Fuel Boilers based on Error Analysis Method

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ABSTRACT

Clean fuel boilers often use natural gas and wood pellets as their primary fuel, which result in reduced emissions from the boiler. Accurate determination of the thermal efficiency of these boilers plays a vital role in appropriately controlling the process parameters for enhanced thermal performance of the boilers. The traditional methods for calculating the thermal efficiency of clean fuel boilers require a number of input parameters, which is not suitable for fast calculation of the thermal efficiency. Moreover, the conventional methods are often significantly inaccurate in the determination of the thermal efficiency of boilers. Thus, a novel method for rapid and accurate determination of boiler's thermal efficiency is required. Therefore, using error analysis method, this study presents a novel mathematical model to calculate the thermal efficiency of an industrial boiler, fueled with natural gas and wood pellets. The main factors that affect the thermal efficiency of clean fuel industrial boilers are obtained based on the results of the thermal efficiency error analysis. A novel mathematical model to calculate the thermal efficiency of the boilers is developed as a function of these major factors. Finally, the calculated results, based on the model, are compared with the test values provided by Guangdong Special Equipment Inspection and Research Institute. The maximum deviation in comparative results has been observed to be within $\pm 3\%$, indicating the appropriateness and commercial viability of the novel methodology proposed in this study.

Keywords: Clean fuel boiler; Thermal efficiency; Mathematical model; Error analysis.

1. Introduction

Most of the industrial boilers used in China are fueled with clean fuels such as natural gas and wood pellets [1]. Use of natural gas significant improves local air quality and public health. When burned, natural gas emits 50% less CO₂ than coal. Moreover, there are negligible emissions of sulfur dioxide (SO₂), nitrogen oxides (NO_x), mercury (Hg) and particulates from natural gas as compared to other type of fuels [2]. As natural gas is expensive, a considerable number of industrial boilers use wood pellets as their primary fuel. Wood pellets are generally made from compacted sawdust [3] and industrial wastes from the milling of lumber, manufacturing of wood products, furniture and construction [4]. Because of the characteristics of low moisture content (below 10%) and low-ash content (below 5%) [5], wood pellets have been widely used as a clean fuel in existing industrial boilers [6]. Thus, natural gas and wood pellets are two major clean fuels being used by most of the industrial boilers in China.

Boiler engineers are always seeking innovative ways through which the fuel consumption of industrial boilers can be reduced. An important parameter that is widely used in the industry for gauging the performance of boilers is the thermal efficiency of the boiler. In order to make the industrial boilers more efficient, it

is important to determine the thermal efficiency of the boiler both rapidly and accurately. However, due to incomplete combustion and high carbon content in slugs from the use of natural gas and wood pellets as boiler fuel [7-8], calculating the thermal efficiency is a times taking process, which can be quite inaccurate as well. Conventionally, the thermal efficiency of a clean fuel boiler is calculated through the analysis of flue gas. There is a need to develop a novel method for calculating the thermal efficiency of clean fuel boilers that is inexpensive, quick, accurate and robust.

Many researchers have developed mathematical models to calculate energy efficiency of coal-fired industrial boilers, including ASME PTC4, GB10184 [9] and JIS B 8222 [10]. By using the concept of energy and exergy efficiency, Saidur et al [11] have found that the combustion chamber primarily contributes to the exergy loss in the boiler. Chao et al [12] developed a mathematical model for the analysis of thermal efficiency and optimal design of boiler systems. The results show that the thermal efficiency of an industrial boiler increases by 0.8% to 1% when the temperature of exhausted gases increases by 10°C to 15°C. Bujak [13] developed a mathematical model to calculate the thermal efficiency of a boiler room. The boiler model is treated as an open thermodynamic system, exchanging mass, energy and heat with the atmosphere. The author has

reported that the heat recovery is minimal when the heat load is low ($\leq 30\%$). Heat recovered from exhaust gases, in the system with an economizer at full power of the boiler, fluctuates between 4% and 7%. Satyavada and Baldi [14] have developed a mathematical model to monitor the energy efficiency of condensing boilers and have pointed out that the thermal efficiency degrades as inlet water temperature increases.

As mentioned earlier, there are many different mathematical models, reported in published literature, regarding the calculation of the thermal efficiency of coal-fired industrial boilers. However, the models for natural gas and wood pellets-fired industrial boilers are severely limited. Moreover, researchers around the world have been using Computational Fluid Dynamics (CFD) based techniques for flow diagnostics and calculation of thermal efficiency of a range of different flow handling systems [15-22]. To the best of authors' knowledge, there is no existing literature regarding the application of error analysis method to the development of a mathematical model for calculating the thermal efficiency of a coal-fired industrial boiler, let alone natural gas or wood pellets-fired industrial boilers. The advantage of error analysis of the thermal efficiency of industrial boilers is that it plays an important role in the development of mathematical models to calculate the thermal efficiency of a clean fuel boiler by considering the main factors affecting the thermal efficiency [23, 24]. Based on the results of error analysis, a simple and accurate mathematical model to calculate the thermal efficiency of a clean fuel industrial boiler can be developed. Thus, in the present study, an attempt has been made to develop an accurate and robust mathematical model to calculate the thermal efficiency of clean fuel boilers.

2. Methodology

Thermal efficiency of is a key performance indicator reflecting the operating conditions of a clean fuel industrial boiler. The thermal efficiency of a boiler can be expressed as [25]:

$$\eta_{gl} = q_1 = 100 - \sum_{i=2}^6 q_i \quad (\%) \quad (1)$$

where, q_i ($i = 2, 3, \dots, 6$) represents net heat q_1 , heat loss due to exhaust gas q_2 , heat loss due to unburnt gas q_3 , heat loss due to unburnt carbon q_4 (only for wood pellets boilers), heat loss due to radiation q_5 and heat loss due to sensible heat in slag q_6 (only for wood pellets boilers) respectively. The empirical equations that can be used to calculate the values of q_2 , q_3 , q_4 , q_5 and q_6 are presented here [26]. Heat loss due to exhaust gas can be calculated as:

$$q_2 = (m + n\alpha_{py}) \left(1 - \frac{q_4}{100}\right) \frac{\theta_{py} - t_{amb}}{100} \quad (\%) \quad (2)$$

where $m=0.5$, $n=3.45$, α_{py} is the excess air coefficient, θ_{py} is exhaust gas temperature in $^{\circ}\text{C}$ and t_{amb} is the ambient temperature in $^{\circ}\text{C}$. Heat loss due to unburnt gas can be calculated as:

$$q_3 = \lambda \alpha_{py} V_{CO} \quad (\%) \quad (3)$$

where $\alpha_{py} \approx \frac{0.21}{0.21 - V_{O_2}}$ and $\lambda = 3.2$. V_{O_2} and V_{CO} are the volumetric percentage of O_2 and CO in the exhaust flue gas [27]. Heat loss due to unburnt carbon can be calculated as:

$$q_4 = \frac{B A_{ar}}{Q_r} \left(\frac{a_{hz} C_{hz}}{100 - C_{hz}} + \frac{a_{fh} C_{fh}}{100 - C_{fh}} \right) \quad (\%) \quad (4)$$

where $B=33700$ kJ/kg, A_{ar} is the percentage of ash content in the fuel, Q_r is the low calorific value of in kJ/kg, C_{hz} and C_{fh} respectively represent the weight percentage of combustible materials in coal slag and fly ash. a_{hz} and a_{fh} respectively represent the percentage of coal slag and fly ash content in the fuel, and their values are $a_{hz}=0.1$ and $a_{fh}=0.9$ [28]. For natural gas boilers, as $a_{hz}=0$, $a_{fh}=0$, $C_{hz}=0$ and $C_{fh}=0$, thus, $q_4=0$.

Heat loss due to radiation can be calculated as:

$$q_5 = \frac{5.82 \times 0.62 \times X}{X_0} \quad (\%) \quad (5)$$

where X_0 is the rated load of the boiler and X is the current load of clean fuel boiler. The heat loss due to sensible heat in slag can be calculated as:

$$q_6 = a_{hz} \frac{100}{100 - C_{hz}} (c\vartheta)_{hz} \frac{A_{ar}}{Q_r} \quad (\%) \quad (6)$$

where $(c\vartheta)_{hz}$ is the enthalpy of coal slag. It can be calculated as:

$$(c\vartheta)_{hz} = 0.0002887 \theta_{py}^2 + 0.6851 \theta_{py} + 26.76 \quad (7)$$

For natural gas boilers, as $A_{ar}=0$, hence, $q_6=0$, in which case the thermal efficiency of the boiler can be expressed as:

$$\eta_{gl} = 100 - \left[(m + n\alpha_{py}) \left(1 - \frac{q_4}{100}\right) \frac{\theta_{py} - t_{amb}}{100} + \lambda \alpha_{py} V_{CO} + \frac{B A_{ar}}{Q_r} \left(\frac{a_{hz} C_{hz}}{100 - C_{hz}} + \frac{a_{fh} C_{fh}}{100 - C_{fh}} \right) + \left(-\frac{hX}{100X_0} + d \right) + a_{hz} \frac{100}{100 - C_{hz}} (c\vartheta)_{hz} \frac{A_{ar}}{Q_r} \right] \quad (\%) \quad (8)$$

Based on the above equations, the following general conclusion can be obtained:

$$\eta_{gl} = f(\theta_{py}, V_{CO}, C_{hz}, C_{fh}, A_{ar}, Q_r, V_{O_2}) = f(X_1, X_2, X_3, X_4, X_5, X_6, X_7) \quad (9)$$

Thus, based on the principle of calculus, the following equation can be achieved:

$$\Delta\eta_{gl} = \frac{\partial\eta_{gl}}{\partial\theta_{py}} \times \Delta\theta_{py} + \frac{\partial\eta_{gl}}{\partial V_{CO}} \times \Delta V_{CO} + \dots + \frac{\partial\eta_{gl}}{\partial Q_r} \times \Delta V_{O_2} = \sum_{i=1}^7 C_i \times \Delta X_i \quad (10)$$

where $\Delta\eta_{gl}$ is the test error of the thermal efficient of a clean fuel boiler (a natural gas or wood pellets-fired boiler), ΔX_i is the measurement error of X_i and $|C_i| = \left| \frac{\partial\eta_{gl}}{\partial X_i} \right|$ is the absolute value of partial derivative. The most important factor here is $|C_i|$ which represents the influence of X_i on η_{gl} [29].

3. Results and Discussions

The calculation results of $|C_i|$ for a 6t/hr wood pellets-fired boiler and a 6t/hr natural gas boiler are listed in table 1 and table 2 respectively. From the results summarized in table 1, it can be clearly seen that the minimum absolute value of partial derivative is $|C_6|$ which represents the low calorific value of the fuel. The value of $(|C_6|)$ is $2.73395e-05$. Similarly, the maximum value of partial derivative is $|C_5|$ which represents the ash content of the received base. The value of $(|C_5|)$ is 0.26175 . The corresponding minimum and maximum partial derivatives in case of natural gas boiler are $|C_6|$ and $|C_2|$ respectively, having values of 0 and 0.1231. The comparison of $|C_i|$ between the two clean fuel boilers discussed here is presented in figure 1.

Table 1. Calculation results of $|C_i|$ for a 6t/hr wood pellets-fired boiler

Items	Symbols	Units	Value	Absolute values of partial derivative ($ C_i $)
Exhaust flue gas temperature	θ_{py}	°C	173.9	0.00435 ($ C_1 $)
CO content in exhaust flue gas	V_{CO}	%	0.0171	0.06005 ($ C_2 $)
Unburnt carbon content in slag	C_{hz}	%	10.48	0.00842 ($ C_3 $)
Unburnt carbon content in fly ash	C_{fh}	%	22.37	0.03594 ($ C_4 $)
Ash percentage of the received base	A_{ar}	%	1.41	0.26175 ($ C_5 $)
Low calorific value	Q_r	kJ/kg	16090	2.75e-05 ($ C_6 $)
Oxygen content in exhaust flue gas	V_{O_2}	%	12.3	0.00926 ($ C_7 $)

Table 2. Calculation results of $|C_i|$ for a 6t/hr natural gas boiler

Items	Symbols	Units	Value	Absolute values of partial derivative ($ C_i $)
Exhaust flue gas temperature	θ_{py}	°C	118.2	0.0037 ($ C_1 $)
CO content in exhaust flue gas	V_{CO}	%	0.0123	0.1231 ($ C_2 $)
Unburnt carbon content in slag	C_{hz}	%	0	0 ($ C_3 $)
Unburnt carbon content in fly ash	C_{fh}	%	0	0 ($ C_4 $)
Ash percentage of the received base	A_{ar}	%	0	0 ($ C_5 $)
Low calorific value	Q_r	kJ/kg	36150	0 ($ C_6 $)
Oxygen content in exhaust flue gas	V_{O_2}	%	5.67	0.0224 ($ C_7 $)

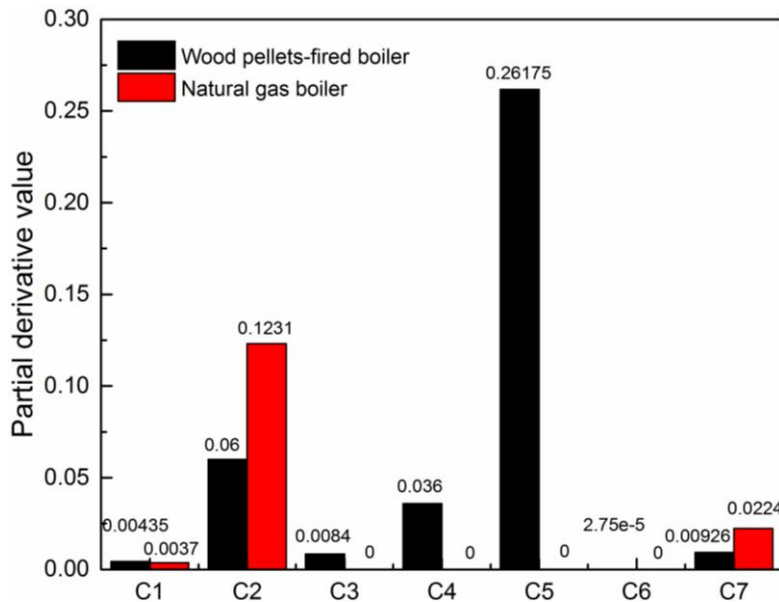


Figure 1. Values of partial derivative ($|C_i|$) for a 6t/hr wood pellets-fired boiler and a 6t/hr nature gas boiler

It can be clearly seen in figure 1 that the effect of $|C_6|$ is negligibly small, and hence the effect of calorific value (Q_r) on the thermal efficiency of the boiler (η_{gl}) can be ignored. Hence, the modified expression for the thermal efficiency of a clean fuel boiler is:

$$\eta_{gl} = f(\theta_{py}, V_{CO}, C_{hz}, C_{fh}, A_{ar}, V_{O_2}) = f(Y_1, Y_2, Y_3, Y_4, Y_5, Y_6) \quad (11)$$

which means that the thermal efficiency of a clean fuel boiler (η_{gl}) can be expressed as a function of $\theta_{py}, V_{CO}, C_{hz}, C_{fh}, A_{ar}$ and V_{O_2} . It is also noteworthy here that for natural gas boilers, as $C_{hz}=0, C_{fh}=0$ and $A_{ar}=0$; therefore, $Y_3=0, Y_4=0$ and $Y_5=0$. Hence, the thermal efficiency of a natural

gas boiler depends only on the temperature and CO content of exhaust gas.

In order to develop a mathematical model for the calculation of thermal efficiency of a clean fuel industrial boiler, multiple regression analysis has been carried out on the factors affecting the boiler's efficiency. Based on a number of test values of the thermal efficiency, the following equation to calculate the thermal efficiency (η_{gl}) is obtained:

$$\eta_{gl} = -0.05769 Y_1 - 6.1185 Y_2 + 0.05906 Y_3 + 0.01887 Y_4 - 1.6030 Y_5 - 0.3657 Y_6 + 99.735 \quad (12)$$

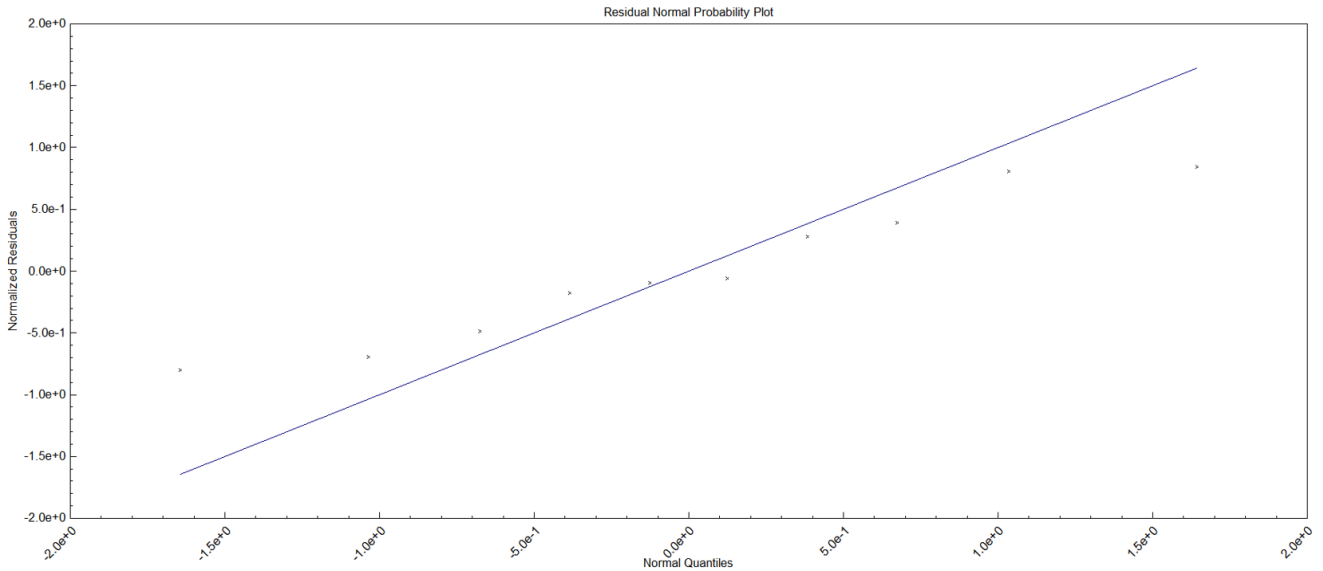


Figure 2. Relationship between the actual and predicted values of the thermal efficiency

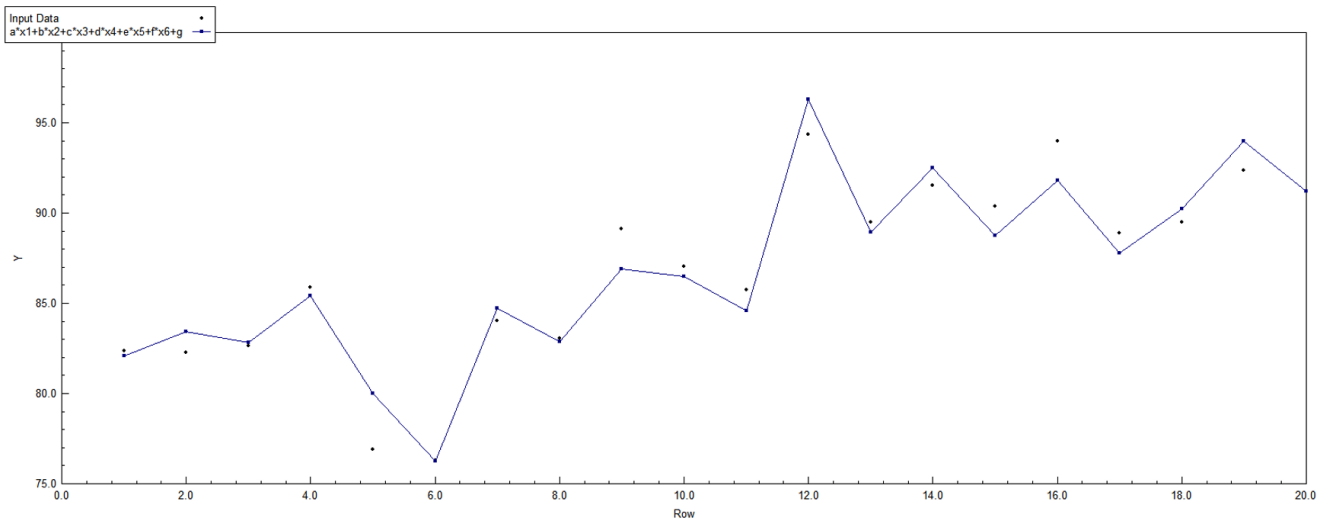


Figure 3. Predicted versus actual values of the thermal efficiency of wood pellets-fired industrial boilers

Figure 2 depicts the relationship between the actual and predicted values of the thermal efficiency, which indicates that the thermal efficiency model developed here is reasonably accurate as the residuals lie close to the diagonal line in the figure. Moreover, figure 3 depicts the predicted versus actual values of the thermal efficiency of a wood pellets-fired industrial boiler. It can be clearly seen that the relative deviation in the obtained results is small. In order to further quantitatively analyze the accuracy of the mathematical model, the values of the thermal efficiency of ten wood pellets-fired industrial boilers obtained from test reports are compared with the values calculated by using the model. The relative deviations between test values and predicted values have been summarised in table 2. It should be noted that the relative deviation is equal to $|\text{test value} - \text{predicted value}| / \text{test value}$.

It can be clearly seen from the table 2 that the test values and the calculated values match closely, where the maximum relative deviation between them is 2.04%. This indicates that the mathematical model developed in the present study, to calculate the thermal efficiency of a clean fuel industrial boiler, is accurate and robust. As this model is relatively simple than conventional models, the number of inputs to the model are less. This further

implies that the developed model is simple, quick and user-friendly.

4. Conclusions

A novel mathematical model has been developed to calculate the thermal efficiency of clean fuel industrial boilers, fired with natural gas and wood pellets. The model has been developed based on the error analysis method. The results of the error analysis of the thermal efficiency have indicated that the value of Q_f (low calorific value of the fuel) has negligibly small impact on the thermal efficiency of clean fuel industrial boilers. Moreover, the ash percentage of the received base and CO content in the exhaust gas have the most impact on the thermal efficiency of wood pellet-fired and natural gas boilers respectively.

Using multiple regression analysis on a number of test runs, a novel mathematical model has been developed to predict the thermal efficiency of clean fuel industrial boilers. The results obtained from this model have been verified against test data, which has shown a maximum variation of 2.04%, hence, validating the accuracy and appropriateness of the developed model. Thus, the developed model is more commercially viable than the conventional models.

Table 2. Relative deviations between test values and predicted values for the thermal efficiency of wood pellets-fired industrial boilers

Y ₁ (°C)	Y ₂ (%)	Y ₃ (%)	Y ₄ (%)	Y ₅ (%)	Y ₆ (%)	Test values	Predicted values	Relative deviations
183.8	0.0205	8.62	11.75	1.69	11.40	82.37	82.86	0.59%
217.3	0.0243	13.80	19.31	1.27	8.70	82.27	83.01	0.90%
220.4	0.0127	10.48	21.82	1.62	8.70	82.63	82.19	0.53%
139.7	0.0242	28.00	18.67	1.88	11.39	85.88	86.35	0.55%
146.0	0.0618	5.57	18.13	1.71	10.30	85.85	85.10	0.88%
180.3	0.0109	3.16	3.39	4.87	14.30	76.28	76.48	0.26%
156.8	0.0135	22.89	26.86	1.77	11.53	84.04	85.41	1.63%
173.9	0.0122	10.48	22.37	1.41	12.30	83.05	83.91	1.04%
150.0	0.0078	28.68	19.74	2.03	9.12	88.31	86.51	2.04%
136.0	0.0045	10.44	33.62	2.03	9.90	87.04	86.24	0.92%
185.3	0.0060	0	0	0	8.3	85.72	85.97	0.30%
50.50	0.0573	0	0	0	2.2	94.36	95.67	1.38%
111.5	0.1513	0	0	0	9.7	89.48	88.83	0.73%
128.9	0.0005	0	0	0	1	91.52	91.93	0.45%
128.0	0.0007	0	0	0	6.4	90.39	90.01	0.42%
103.6	0.003	0	0	0	3.9	93.97	92.31	1.76%
160.0	0.0005	0	0	0	5.5	88.91	88.49	0.47%
173.3	0.0018	0	0	0	1.1	89.48	89.32	0.17%
96.2	0.0261	0	0	0	1.7	92.38	93.40	1.11%
132.7	0.0012	0	0	0	2.6	91.19	91.12	0.08%

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