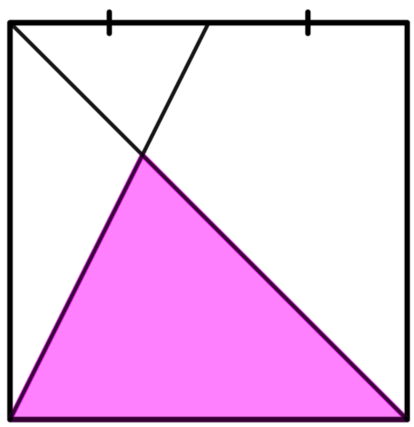
Approaches to the Pink Triangle Problem

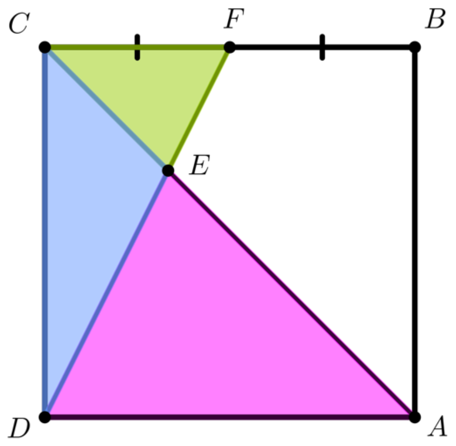
About a year ago I posted a fairly innocuous puzzle on Twitter. It was one of many thematic puzzles I regularly wrote at the time where a simple shape has a section shaded pink, and the puzzle solver needs to determine what fraction it represents. The difference on this occasion was that a newspaper decided to declare it 'viral', which in turn made it go viral across the globe. Whilst the specific ordering of those two events may feel a little dystopian, a nice side effect to 'going viral' is that I was sent an unusually large variety of approaches to solving it. A year later and the puzzle has risen its head again, this time in Spain. My notifications have yet again been inundated with numerous techniques to solve the "Pink Triangle Problem" as it is now more commonly known. I'm not sure what it is about this particular puzzle that people find so appealing, however I am sure that the experience highlights something wonderful – discovering new approaches is far more satisfying than solving it. Too often I cast aside a puzzle or mathematical conundrum once I've completed it, when in fact they're sometimes the gift that keeps on giving.



The puzzle is a square divided up as shown, and, spoiler alert, the answer is a third – but we're not interested in the answer, just the ways to get there.

Below I've selected ten different approaches to demonstrate how even after a puzzle is solved, you can still have many "a-ha" moments to enjoy. Notation below is as written by submission.

Approach 1



$$\measuredangle FEC = \measuredangle DEA$$

$$\measuredangle ECF = \measuredangle EAD$$

$$\Delta EFC = \frac{1}{4}(\Delta AED)$$

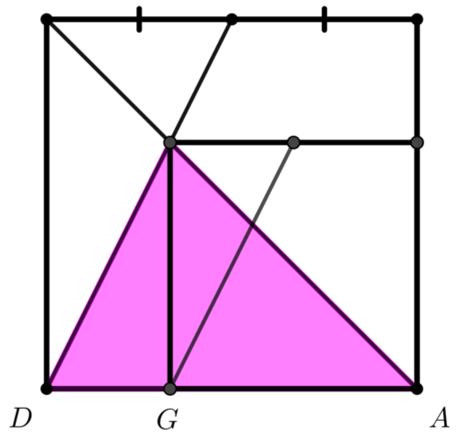
$$\Delta AED = \Delta ACD - (\Delta DFC - \Delta EFC)$$

$$\Delta AED = \frac{DA^2}{2} - \frac{DA^2}{4} + \frac{1}{4}(\Delta AED) = \frac{DA^2}{4} + \frac{1}{4}(\Delta AED)$$

$$\Delta AED = \frac{DA^2}{3}$$

$$\therefore \; fraction\ shaded = \frac{1}{3}$$

Approach 2

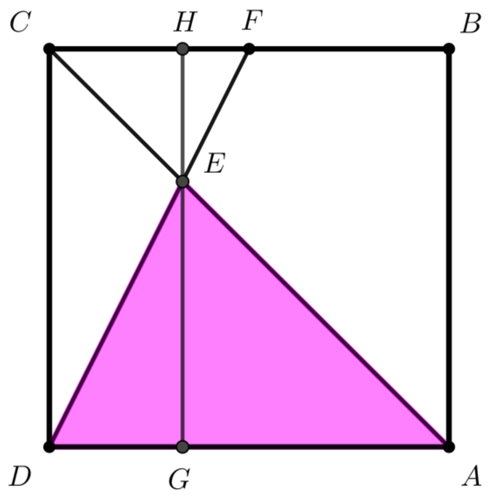


$$DG = \frac{GA}{2}$$

$$DA = \frac{3GA}{2}$$

$$\frac{\left(\frac{1}{2} \cdot \frac{3GA}{2} \cdot GA\right)}{\left(\frac{3GA}{2}\right)^2} = \frac{\frac{3GA^2}{4}}{\frac{9GA^2}{4}} = \frac{3}{9}$$

Approach 3



$$Let\ DA = 1$$

$$\Delta DEC + \frac{EG}{2} = \frac{1}{2}$$

$$\Delta DEC + \frac{1 - EG}{4} = \frac{1}{4}$$

$$2 \cdot \Delta DEC + EG = 1$$

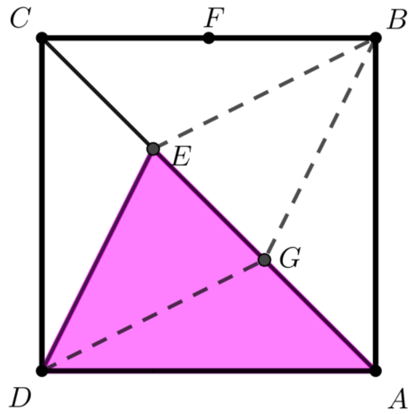
$$4 \cdot \Delta DEC - EG = 0$$

$$\therefore \Delta DEC = \frac{1}{6}$$

$$\therefore EG = \frac{2}{3}$$

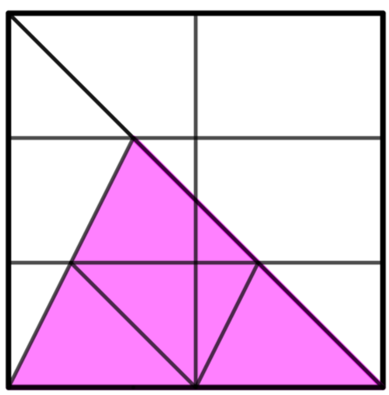
$$\therefore Fraction\ Shaded = \frac{1}{3}$$

Approach 4



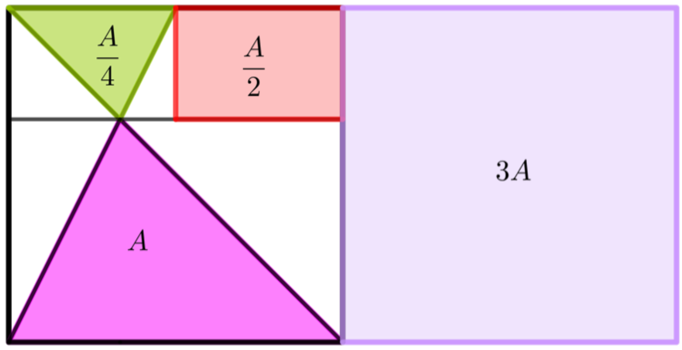
Six triangles sharing equal base and height.

Approach 5



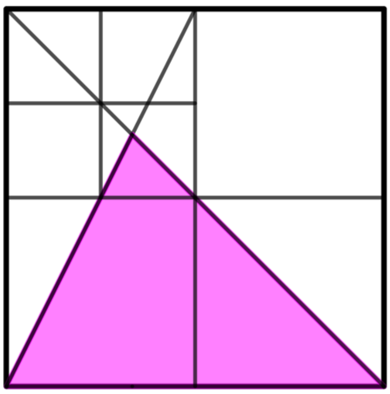
Observe that two of the four congruent pink triangles have a combined base equal to the side of the square, and thus take up half of the area of the bottom third of the square. Hence all four triangles take up a third of the square.

Approach 6



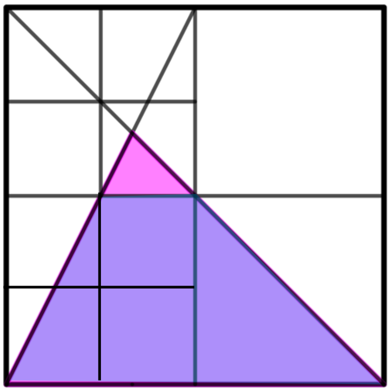
As the green triangle is ¼ the area of the pink triangle (they're similar, and the base is half as big), then we can deduce that the red rectangle is half the area of the pink triangle, therefore the full square is three times the size of the pink triangle.

Approach 7a



$$\frac{5}{16}\sum_{n=0}^{+\infty}\left(\frac{1}{16}\right)^n = \frac{5}{16}\cdot\frac{16}{15} = \frac{1}{3}$$

To prove this result, notice that the same figure is found in the little square representing a sixteenth of the initial square.



The small square shows a similar figure. The remaining area shaded in purple is $\frac{1}{8}+\frac{3}{16}=\frac{5}{16}$

of the initial square. Repeating the process to infinity, the total area is :

$$\frac{5}{16}+\frac{5}{16}\times\frac{1}{16}+\frac{5}{16}\times\left(\frac{1}{16}\right)^2+\frac{5}{16}\times\left(\frac{1}{16}\right)^3+\cdots$$

Which can be written as :

$$\frac{5}{16}\times\sum_{n=0}^{+\infty}\left(\frac{1}{16}\right)^n$$

Recall that for any real number q with $q\neq 1$,

$$1+q+q^2+\cdots+q^N=\sum_{n=0}^{N}q^n=\frac{1-q^{N+1}}{1-q}$$

When $q\in]-1,1[$, $\lim_{N\to+\infty} q^{N+1}=0$, so the sum converges to $\frac{1}{1-q}$.

Here, we have $q=\frac{1}{16}\in]-1,1[$, so the infinite sum is:

$$\sum_{n=0}^{+\infty}\left(\frac{1}{16}\right)^n=\frac{1}{1-\frac{1}{16}}=\frac{1}{15/16}=\frac{16}{15}$$

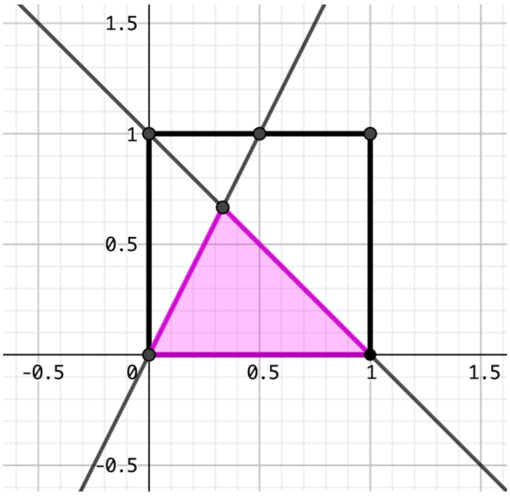
Or in context, as you like :

$$\sum_{n=0}^{N}\left(\frac{1}{16}\right)^n = \frac{1-\left(\frac{1}{16}\right)^{N+1}}{1-\frac{1}{16}} = \frac{16}{15}\times\left(1-\left(\frac{1}{16}\right)^{N+1}\right)$$

Since $\frac{1}{16} \in]-1,1[$, $\lim_{N\to+\infty}\left(\frac{1}{16}\right)^{N+1} = 0$ so:

$$\sum_{n=0}^{+\infty}\left(\frac{1}{16}\right)^n = \frac{16}{15}$$

Approach 8



$$Line\ 1:\ y = 2x$$

$$Line\ 2: y = 1 - x$$

$$Intersection:$$

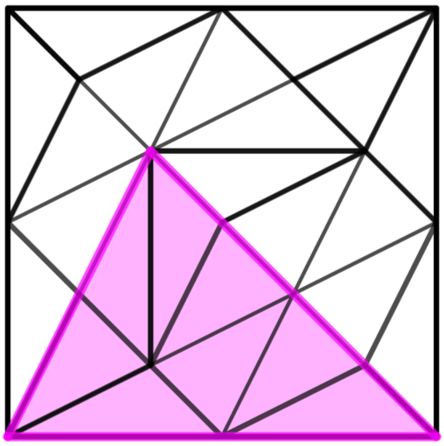
$$2x = 1 - x$$

$$3x = 1$$

$$intersection = \left(\frac{1}{3}, \frac{2}{3}\right)$$

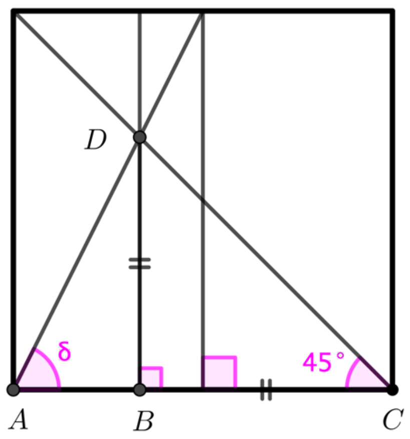
$$Shaded\ Area = \frac{1}{2} \cdot 1 \cdot \frac{2}{3} = \frac{1}{3}$$

Approach 9



Eight pink triangles out of twenty-four. All have equal area.

Approach 10



Assume the square has side length 2.

The area $\Delta ADC = (BC + AB)(\frac{BC}{2})$

$$Tan(\delta) = \frac{BC}{AB} = \frac{2}{1}$$

$$\therefore 2AB = BC$$

$$\therefore \Delta ADC = (2AB + AB)\left(\frac{2AB}{2}\right) = 3AB^2$$

Hence the shaded fraction is $\frac{3AB^2}{(3AB)^2} = \frac{1}{3}$

From the ten solutions above, it should be clear that the pursuit of a different approach can be a rich task that helps develop a deeper understanding of mathematics. A shift away from students tackling question after question, and towards looking deeper at alternative pathways, and how they work, can often be a more fulfilling task for both teacher and students alike.

This puzzle, and one of the solutions presented here are taken from More Geometry Snacks (Southall & Pantaloni, 2018). It is also featured in The Room in the Elephant (Pritchard, 2019).

References

Southall, E. & Pantaloni, V. (2018) *More Geometry Snacks,* Tarquin Publications

Pritchard, C. (2019) *The Room in the Elephant*, Mathematical Association