Hierarchy, Symmetry and Scale in Mathematics and Bi-Logic in Psychoanalysis, with Consequences

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Abstract

Hierarchy has properties of symmetry and scale. All that is related to hierarchy provides important perspectives on many other domains. The primary focus here is on renowned psychoanalyst, Ignacio Matte Blanco's Bi-Logic. Bi-Logic relates to the two modes of being, respectively symmetry and asymmetry in thought and reasoning and brain processes, in unconscious and in conscious modes of being. Some further consequences and implications are noted. These include relevance and potential in social sciences; in qualitative as well as quantitative literary analysis, and content analytics of text, speech, visual recording, and so on; in security and forensics; and in the contemporary theme of research and development relating to Big Data analytics.

1 Introduction

At issue in this paper is Geometric Data Analysis [1] and related formal approaches for Matte Blanco's Bi-Logic, relating to symmetry and asymmetry in unconscious and conscious thought processes.

The Chilean psychoanalyst, Ignacio Matte Blanco (1908–1995) was the originator [2] of an innovative approach to both conscious and unconscious logic. For Matte Blanco, there are "two fundamental types of being which exist within the unity of every man: that of the 'structural' id (or unrepressed unconscious or system unconscious or symmetrical being) which becomes understandable with the help of the principle of symmetry; and that visible in conscious thinking, which can roughly be comprehended in Aristotelian logic." (Ref. 2, p. 13). Within a class, when symmetry logic applies, there is no contradiction, absence of negation, displacement, space and time vanish, no relations of contiguity, no order. Furthermore, "the unconscious does not know individuals but only classes or propositional functions which define the class" (Ref. 2, p. 139). "Consciousness ... when confronted by a whole class can only consider it in two ways: either it focuses on the limits (or definition) of the class, that is, on those precise features which characterize it and distinguish it from all other classes, or it concentrates on the individuals which form the class." (Ref. 2, p. 139).

A class comes about through condensation. In computer programming algorithms, a term such as assignment to a cluster is used, and cluster formation. The latter, cluster formation, allows the data to be summarized by the clusters that are determined. So, quite clearly, in this context, a class, also termed a cluster, or a group (of individuals, or of objects) or a set, can be expressed in the Freudian term, condensation. Condensation, in this context, is the assimilation, or pulling together, or ramification, of what we are dealing with. The latter, what we are dealing with, can be expressed in terms of data elements, individuals, objects, observations, concepts, and so on.

The principle of generalization relates different classes. "Symmetrical being alone is not observable in man." Even delineating it is "already an asymmetrical ... activity" (Ref. 2, p. 104). (So the symmetrical (and unconscious) is measurable but only in the context of the asymmetrical (and physical or empirical world). "We must ... keep in mind the possibility that if things are viewed in terms of multidimensional space, symmetrical being can actually unfold into an infinite number of asymmetrical relations." (Ref. 2, p. 110).

It has been observed [3,4,5] just how well Matte Blanco's bi-logic can be mapped out and understood in geometric data analysis terms, based on geometry and, especially, topology. It is our view that this becomes very important from the point of view of therapy and diagnostics, for allowing deeper analysis of the unconscious and its integral relation to conscious reasoning processes, and to new application domains also (such as, potentially and to be investigated further, education, health and well-being, security, trust and identity, and many other domains). Furthermore, this theoretical approach can also be connected to the underlying physiological level by considering mirroring systems and other neural network phenomena [6] .

2 Symmetry and Hierarchy

Scale and proportion are, of course, inherent in, and integral to, hierarchical structure. For visually observed and recorded information, providing us with images and video, hierarchy underpins much of the processing and analytics that will be carried out for all analytical or exploratory needs and requirements [7,8] .

Weyl [9] makes the case for the fundamental importance of symmetry in science, engineering, architecture, art and other areas, and especially in group theory in mathematics. The following are quotations in Weyl (Ref. 9, pp. 3, 5): "*Beauty* is bound up with symmetry."; "*Ebenmass* is a good German equivalent for the Greek symmetry"; "... a modern poet [Anna Wickham] addresses the Divine Being as 'Thou Great Symmetry' ".

As a "guiding principle" (Ref. 9, p. 144), "Whenever you have to do with a structure-endowed entity ... try to determine its group of automorphisms, the group of those element-wise transformations which leave all structural relations undisturbed. You can expect to gain a deep insight in the constitution of [the structure-endowed entity] in this way. After that you may start to investigate symmetric configurations of elements, i.e. configurations which are invariant under a certain subgroup of the group of all automorphisms; ..."

Herbert A. Simon, Nobel Laureate in Economics, originator of the concepts "bounded rationality" and of "satisficing", believed in hierarchy as the basis of the human and social sciences, as the following quotation [10] (p. 184) shows: "... my central theme is that complexity frequently takes the form of hierarchy and that hierarchic systems have some common properties independent of their specific content. Hierarchy, I shall argue, is one of the central structural schemes that the architect of complexity uses."

Another quotation from Simon: "My thesis has been that one path to the construction of a nontrivial theory of complex systems is by way of a theory of hierarchy." (Ref. 10, p. 216).

Symmetries are therefore ubiquitous in data, such that the data represent complex phenomena, and the symmetries provide a model for understanding these complex phenomena. Hierarchy gives rise to a rich expanse of symmetries. Thus, we will be concerned mostly with hierarchy.

Partitioning a set of observations leads to some very simple symmetries. This is one approach to clustering and data mining. But such approaches, often based on optimization, are of less direct interest here.

Instead, the theme pointed to by Simon will be pursued, namely that the notion of hierarchy is fundamental for interpreting data and the complex reality which the data expresses.

2.1 Hierarchy in Mathematical Number Theory, in Geometry, in Topology

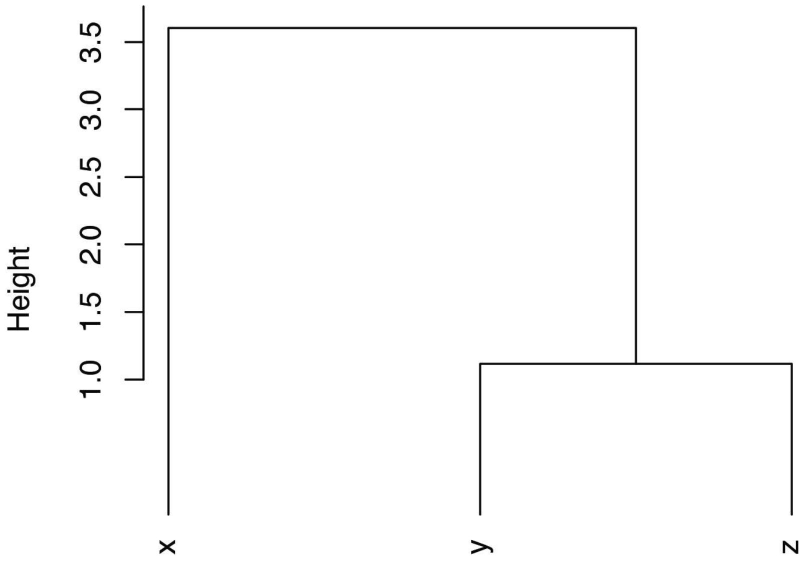
Practical implementing, exploiting and benefiting from hierarchical clustering can be very effective and efficient [11] . Number systems can be very worthwhile to consider. The real number system is commonly used and a natural tree or hierarchy representation of any decimal, also termed 10-ary (or 10-adic), number is a regular 10-way tree. Apart from having any other base system for number representation, there is often consideration of p-adic number systems, where p is a prime number. A p-adic number has a natural tree or hierarchy representation as a regular p-way tree. Consider just how computers are based on binary numbers, also termed bits, binary digits. In earlier times, there were proponents for computers to be based on ternary, i.e. 3-adic numbers [12] .

In topology in mathematical terms, a hierarchy, also termed a tree structure, is a representation of an ultrametric topology. Such a structuring of the data as a tree is defined by considering distance between the points defining the nodes of this tree. Given any two points, i,j, symmetry and positive definiteness properties are: $d(i,j) \geq 0$, $d(i,j)=0$ implies $i=j$; $d(i,j)=d(j,i)$. The further inequality that holds is the strong triangular inequality, also termed the ultrametric inequality: $d(i,k) \leq \max\{d(i,j), d(j,k)\}$ for distance d and for a triplet of points, i,j,k. This is informally expressed as "stronger" relative to the fundamental property of what constitutes a metric or distance: $d(i,k) \leq d(i,j)+d(j,k)$. Now, the natural geometric ordering of metric valuations is: a real line. But when we have partial order, alternatively expressed, hierarchical order, then this is the ultrametric case: at issue is a hierarchy or rooted tree.

Informally expressed, in terms of genealogy, a genealogical tree, the ultrametric distance is the least common ancestor distance.

In regard to terminology, a Euclidean metric endowed space can be termed an Archimedean space, while an ultrametric or tree topology, that is, an ultrametric endowed space, is termed a non-Archimedean space.

The latter, triangular inequality that defines a metric, can be most informally described as follows: consider travelling from location i to location k. If there is deviation via location j, then the travel distance will be longer, or at the very best, equal to what it is without this deviation. That is just a very informal description. Now an ultrametric is a metric but with a stronger requirement in the strong triangular, or ultrametric, inequality. For this to always hold, then all triplets of points comprise an isosceles triangle, i.e., with two equal length sides, with small base. Additionally, an equilateral triangle also satisfies the strong triangular or ultrametric inequality. An illustration is in Figure 1, right, of such an approximation. Informally, in the graph display here, we may claim that we are seeking the closest black points to the upper right hand side red point, and we are prepared to approximate, with quite a good approximation, the isosceles triangle with approximately equal longer sides. From the point of view of the two black points in the base now of this triangle, the upper right hand side red point is an exception or anomaly relative to their (i.e. these black points) proximity. In Figure 1, top, there is the explanation, based on the display, of the ultrametric inequality.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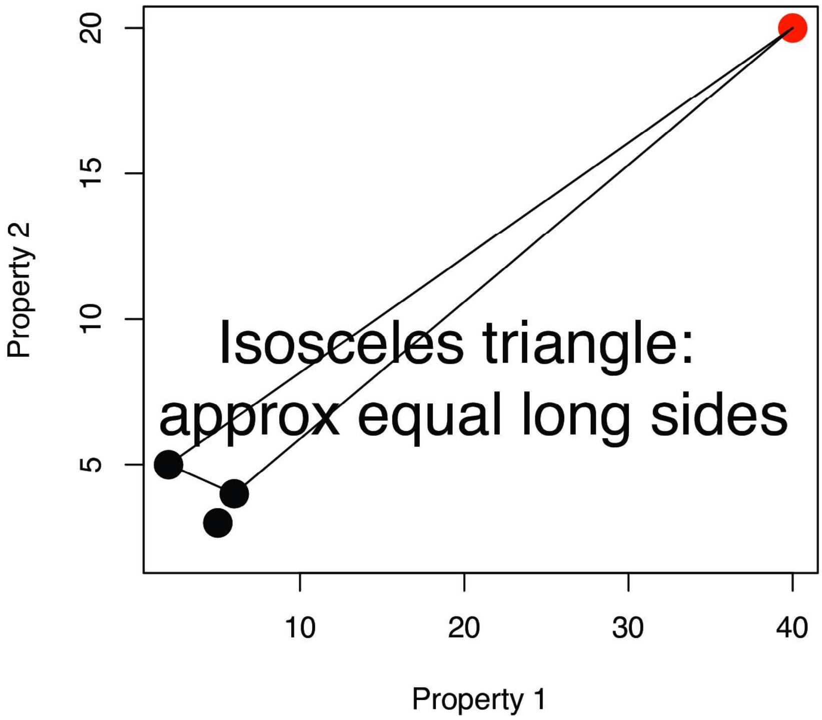

Figure 1: Illustration of how a hierarchical display can depict what is exceptional or anomalous. (Top) Hierarchy, or rooted tree, depiction. The strong triangular inequality defines an ultrametric: every triplet of points satisfies the relationship: $d(x,z) \leq \max\{d(x,y), d(y,z)\}$ for distance d. Cf. by reading off the hierarchy, how this is verified for all x,y,z: $d(x,z)=3.6; d(x,y)=3.6; d(y,z)=1.1$. In addition the symmetry and positive definiteness conditions hold for any pair of points. (Bottom) Illustration of how hierarchy can be a graphical representation of exceptionality. Illustration of how a hierarchical display can depict what is exceptional or anomalous. Once we either set up, or have as an approximation here, a triangle of points that form an isosceles triangle with small base, then there is a hierarchical depiction.

2.2 Metric Geometry and Utrametric Topology, Hierarchy and Symmetry

An ultrametric representation is quite appropriate for genealogical relations, and the ultrametric, i.e. hierarchy- or tree-derived distance, as can be seen in Figure 1, top, where the closest common ancestor distance is. So this refers to genealogical or genetic, and sometimes even creative processes starting with the root, and then growing the outputs and outcomes in a tree or hierarchy structure. Of course, a tree itself is relevant here. Phylogeny and evolution are very relevant for a tree, hierarchical, hence ultrametric representation or expression. This is relevant also for, e.g., a thesaurus, or all that is related to the brain cerebellum.

Our presentation in this paper has focused on the ultrametric or tree topology, and on how number systems can be associated too, with such topologies. For practical data analytics, the computational construction of a hierarchical clustering from data is usually carried out as follows. The analysis starts with the set of terminal nodes. This is followed by carrying out, stepwise, an agglomeration into a new cluster, that is associated with a non-terminal node in the rooted tree. Mathematically, a hierarchy is a rooted tree, i.e. it has the root node. In practical applications it may be a case of constructing the hierarchy, termed a dendrogram, and then cutting it horizontally in order to yield a partition of the object set at issue, i.e. the set of terminal nodes in the dendrogram.

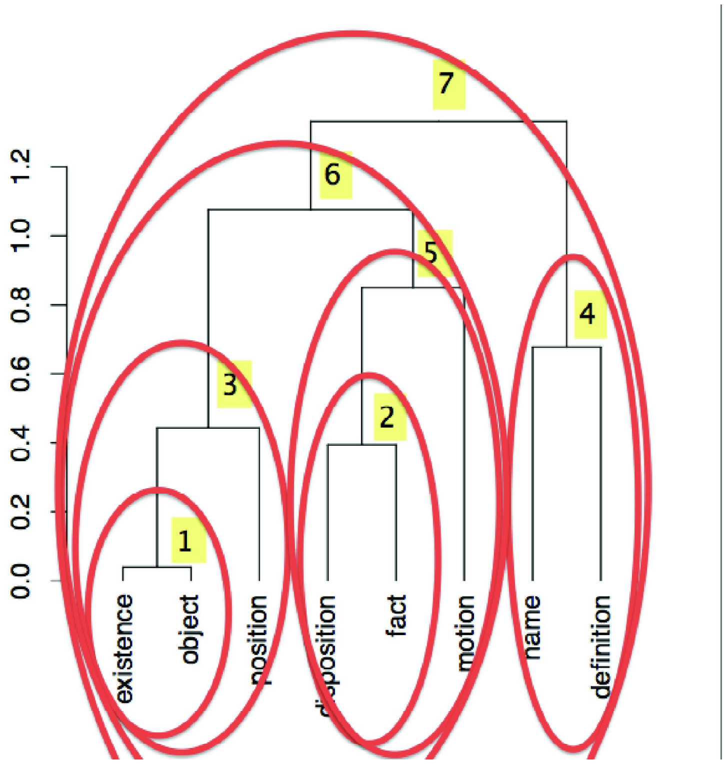

Figure 2: Hierarchical clustering of 8 terms. The data on which this was based were the frequencies of occurrence of the 8 nouns in 24 successive, non-overlapping segments of Aristotle's *Categories*. The 8 terms comprise the terminal (or leaf) nodes. In the hierarchy these are singleton clusters. The non-terminal nodes, i.e. clusters, are numbered in sequence (the numbers beside the relevant nodes). If one cuts horizontally through the dendrogram or tree or hierarchy, one gets a partition of the set of 8 terms. The clusters associated with the non-terminal nodes are shown with ellipses here.

Some main geometrical properties of an ultrametric space are as follows [11, 13] : (1) In an ultrametric space, all triangles are either isosceles with small base, or equilateral. (2) Every point of a circle in an ultrametric space is a centre of the circle. (3) In an ultrametric topology, every ball is both open and closed, termed *clopen*. (4) An ultrametric space is 0-dimensional. (5) Informally, in an ultrametric space everything "lives" in a hierarchy expressed by a tree. Later there will be a reference to such properties of ultrametric spaces when Matte Blanco's Bi-Logic is considered. Matte Blanco's symmetry, i.e. unconscious reasoning, is most appropriately described in terms of an ultrametric topology [3] .

In Murtagh [14] there is description of various symmetries that are integral to hierarchy: a hierarchy is an example of a wreath product group in mathematical group theory – this is just the fact that each and every node in the hierarchical tree structure can rotate or, in general, permute its child nodes. A major symmetry too, to be at issue in Matte Blanco's, is the taking as similar and at least practically, and application-wise, the cluster contents. This will be related, later in this paper, as a perspective on, Freudian condensation and how Matte Blanco has this as the fundamental property of one's unconscious reasoning.

2.3 Ultrametric Topology: in Physics, Time Series Analysis, Literature

Hierarchy, as a branching process, is a most appropriate means of expressing suboptimal and/or discrete energy states or levels. That includes quantum physics, and lots of forms of genealogy, evolutionary processes, etc. A hierarchy, as seen, incorporates a sequence of partitions. There is a partial order on the clusters/nodes, and a total order on the partitions. Related to this can be the important role of p-adic representation for physics on very small scales (i.e., for quantum physics). Volovich [15] poses the general principle that the fundamental physical laws should be invariant under the change of the number field. This leads to the following ambitious statement: "If these ideas are true then number theory and the corresponding branches of algebraic geometry are ... the ultimate and unified physical theory".

Physics has been considered now, and the following, related to physics, is from Khrennikov [16] (page xiii): "Human thinking (as well as many other information processes) is fundamentally a hierarchical process. ... In our information modeling the main distinguishing feature of p-adic numbers is the treelike hierarchical structure."

Ultrametric topology was introduced by Marc Krasner in 1944. The ultrametric inequality was formulated by Hausdorff in 1934.

Above it has been indicated that (i), according to Simon, ultrametric, i.e. hierarchical perspectives are crucial for the handling of complex systems; (ii) that ultrametric and closely linked p-adic perspectives are important for physics; (iii) later it will be shown how fundamental this all is for psychoanalysis and the human unconscious and conscious modes of being; (iv) and the following is really important, how fundamental this is for its implications, and also for such aspects as the remarkable simplicity of all that is associated with high dimensional spatial representations for data and information.

As ambient dimensionality increases, distances became more and more ultrametric [17,18,19,20] . That is to say, a hierarchical embedding becomes more and more immediate and direct as dimensionality increases. Hence there is inherent hierarchical structure in high dimensional data spaces. Points in high dimensional spaces become increasingly equidistant with increase in dimensionality. (Ref. 20, p. 9453) : "not only are the points [of a Gaussian cloud in very high dimensional space] on the convex hull, but all reasonable-sized subsets span faces of the convex hull. This is wildly different than the behavior that would be expected by traditional low-dimensional thinking".

There are remarkable symmetries in very high dimensional spaces: very simple structures in very high dimensions are not necessarily trivial. Even very simple structures (hence with many symmetries) can be used to support fast and perhaps even constant time worst case proximity search. In the machine learning framework, there are important implications ensuing from the simple high dimensional structures. Very high dimensional clustered data contain symmetries that in fact can be exploited to read off the clusters in a computationally efficient way. What we might want to look for in contexts of considerable symmetry are (what we might characterize as) the impurities or small irregularities that detract from the overall dominant picture.

In Murtagh [21] there is the ultrametric embedding of time-varying signals, including biomedical, meteorological, financial and other data. At issue are the inherent hierarchical properties in the data. One finding is that eyegaze trace data, including saccades which are the jumping around of human eye direction, can be very high in ultrametricity. Otherwise expressed, eyegaze trace data can be well expressed, or represented, as a hierarchy. A further outcome from EEG data is in regard to ultrametricity characterizng normal sleep behaviour and forms of epilepsy.

In (Ref. 5) there is degree of ultrametric, hence inherent hierarchical, content in literature and in other forms of written narratives and text. Included are tales from the Brothers Grimm, Jane Austen novels, dream reports, air accident reports, and James Joyce's Ulysses. Dream reports and Joyce's Ulysses are the most ultrametric, i.e. the most inherently hierarchical in their semantic content.

Finally, it is the case that ultrametric spaces offer excellent computational benefits. Nearest neighbour search in ultrametric space can be carried out in $O(1)$ or constant time [11,22] .

Quite clearly, one's conscious mental processes are in the context of 3-dimensional ambient space, and the time dimension, so that Euclidean-distance endowed space is most relevant. Then if the ambient space is very high dimensional, and perhaps even of infinite dimensionality, the ultrametric is fully appropriate. That is, such a very high dimensional space is inherently ultrametric, hierarchical. No wonder, therefore, that the human unconscious is to be considered as an infinite space.

3 The Unconscious: One's Symmetric Mode of Being

Ignacio Matte Blanco, born in Santiago, Chile, in 1908, trained in London, England, in psychiatry and in psychoanalysis. He worked in the US, Chile and Italy and died in Rome in 1995. His major work, *The Unconscious as Infinite Sets*, was published in 1975.

For convenience of exposition, this extends some of what was in the first section above. Within a class, when symmetry logic applies, there is: no contradiction, absence of negation, displacement, space and time vanish, no relations of contiguity, no order. Such symmetry logic applies to the unconscious: "the unconscious does not know individuals but only classes or propositional functions which define the class" (Ref. 2, p. 139). "The only unity for the (symmetrical) unconscious is the class or set, in which all individuals belonging to it are included. The unconsciousness cannot, therefore, deal with parts, except by treating them as classes or sets" (Ref. 2, p. 141).

This corresponds very well with the mathematical term, *clopen*. This term is an expression for being topologically both open and closed, and applies to a set (or class, or ball, or cluster) in an ultrametric space.

Counterposed to the unconscious is the conscious. "Consciousness ... when confronted by a whole class can only consider it in two ways: either it focuses on the limits (or definition) of the class, that is, on those precise features which characterize it and distinguish it from all other classes, or it concentrates on the individuals which form the class." (Ref. 2, p. 97).

In human reasoning, a class comes about through condensation. The principle of generalization relates different classes. "Symmetrical being alone is not observable in man." Even delineating it is "already an asymmetrical ... activity" (Ref. 2, p. 104). So the symmetrical (and unconscious) is measurable but only in the context of the asymmetrical (and physical or empirical world). "We must ... keep in mind the possibility that if things are viewed in terms of multidimensional space, symmetrical being can actually unfold into an infinite number of asymmetrical relations." (Ref. 2, p. 110).

So, from the hierarchical clustering vantage point, there is the following. Cluster members, as members of the cluster are conflated, they are "identical". Each cluster/ball is topologically open and closed. This is referred to as sets being *clopen*. Clusters are either non-overlapping or are embedded, thus forming the hierarchy that we have seen.

The foregoing are properties of an ultrametric topology, i.e. a tree topology. Informally we might say: in an ultrametric topology everything exists in a tree or hierarchical structure. As a visual, or representational, model, it does rather well – very well, in fact – in expressing and encapsulating Matte Blanco's unconscious reasoning. Also it – the hierarchy, and the ultrametric topology that underpins the hierarchy – can be induced from the data.

Words, and language, are tracers for what lies behind. "Consciousness cannot exist without asymmetrical relations, because the essence of consciousness is to distinguish and to differentiate and that cannot be done with symmetrical relations alone." (Ref. 2, p. 96). "Symmetrical being is translated into asymmetrical terms by means of words. Words (i.e. their meanings) are the asymmetrical tools of the translating-unfolding function." (Ref. 2, p. 115). We have that "words, abstract things, fulfill the function of differentiating between concepts and also between other things. They are bound to be, therefore, highly asymmetrical in their structure." (Ref. 2, p. 115).

Up to now, analytical methodology and its context have been the major focus. A most important aspect is what data should be at issue. In line with eminent social scientist, Pierre Bourdieu's work, for example, we need to find the data that supports the investigation that we want to pursue. Needed is induction (and transduction, and maybe even some deduction). The data used can be written, or verbal, or other activity expressed by the subject, encompassing conscious reasoning, and unconscious reasoning processes.

Pierre Bourdieu, 1930–2002, renowned French social scientist, had his major work, *La Distinction, Critique Sociale du Jugement*, *Distinction, A Social Critique of the Judgement of Taste*, published in French in 1979 and in English in 1984. In the preface to the German edition of "Le Métier du Sociologue" ("The Craft of Sociology"), Bourdieu wrote: "I use Correspondence Analysis very much, because I think that it is essentially a relational procedure whose philosophy fully expresses what in my view constitutes social reality. It is a procedure that 'thinks' in relations, as I try to do it with the concept of field."

4 Brief Review of Applications of Matte's Blanco's Bi-Logic in Many Domains

A comprehensive review of Matte Blanco's Bi-Logic, its relevance and use in analyzing narratives in textual data, and in literature, is available in Murtagh [23] . Particular dream report analyses are carried out in Murtagh [24] .

Developing this work further, especially from the mathematical perspective, has been achieved in specifying and describing "Formal foundations for the origins of human consciousness" [25] .

In Murtagh [23] there is some discussion of the vantage point of Dijksterhuis and Nordgren [26] who note the enormous superiority of human reason, in its efficiency and possibly also in its effectiveness, relating to expectancies and schemas, for unconscious mental processes relative to conscious mental processes.

Using principles that are very much related to all that is under discussion in this article, emotion, i.e. manifestation of the unconscious thought processes, are investigated using film script and literature (the Casablanca movie and Gustave Flaubert's 19th century novel, Madame Bovary) [27] .

Aylward [28] bases his insights on Matte Blanco Bi-Logic, with a focus on triangulation, based on dyadic and, more relevantly, triadic relationships. In creating one's affective or emotional being, as a newborn, respectively these relationships start with one's maternal and paternal relationships. There is a great deal at issue in Aylward [28] . There is analysis, relating terrible crimes to the emotional state and background of those commiting the crimes, including: 16 children and a teacher shot dead in a primary school in Dunblane, Scotland, in 1996; murder of 77 people in Norway in 2011, by Anders Breivak.

Based on all that is at issue here, aspects of family life, with implications for the good side and the bad side of human behaviour, are at issue in Murtagh and Iurato [29] . The positive side of human behaviour includes creativity in all its myriad forms, and intelligence. On the very negative side of human behaviour, there is crime and malevolent behaviours, also including ultimately, depression and mental collapse,

5 Conclusion

From the perspectives of Bi-Logic, the unconscious and the conscious modes of being, and also triadic and dyadic relationships [22, 28, 29] , there are numerous foundations here for important individual and social themes. We have here a means of understanding the basis, the foundations, for emotions, i.e. the profound manifestations of one's unconscious mental processes.

The greatness of all of Matte Blanco's work is nicely expressed by the first sentence of the Preface of his major work: "This book is written for psycho-analysts as well as for mathematical philosophers." (Ref. 2, p. xxv). The insight is so very impressive, as is also the practical nature of all of Matte Blanco's work. "As will be seen, some of the notions employed in my presentation – such for instance, as those of relation and correspondence or mapping – are at the very foundation of both logic and scientific knowledge. The use in psycho-analysis of precise mathematical concepts permits, I believe, the development of a new and wider view of the mind, of a greater degree of precision, and leads to a synthesis and a unity of apparently widely disparate subjects" (Ref. 2, p. xxv).

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