

Abstract

Since the financial crisis, risk management has been of growing interest to investors and the approach of Value-at-Risk has gained wide acceptance. Investing in Cryptocurrencies brings not only huge rewards but also huge risks. For this purpose, this paper investigates whether Cryptocurrencies investors' decisions can rely on the pragmatic and parsimonious approaches for Value-at-Risk forecasting. Specifically, we suggest a parsimonious reflected gamma specification under the GAS framework, consider other GAS special cases and the Exponential Weights driven nonparametric methods, which fall into the same modelling category as the well-known and widely recognised original RiskMetricsTM approach. We focus on the returns for BTC, LTC and ETH and find that progress upon RiskMetricsTM may provide valuable gains in exposure modelling of Cryptocurrencies under the rough and primary backtesting conditions, though not all of the considered approaches demonstrate consistency at the selected risk confidence levels. In our setting, Laplace GAS specification, which controls for time-variation both in scale (volatility) and skewness (asymmetric responses to positive and negative volatility) parameters, performs the best at the most of the levels. We also find that controlling for time-variation in the degrees of freedom (tails) of the Student's t may be a worthwhile consideration, though such approach may still yield more conservative investors' strategies than its Laplace asymmetric alternative. Reflected gamma and Extreme Value Theory linked Double Pareto specifications also demonstrate a modest performance, but likely suffer from the lack of asymmetry in their parameters, as our Reflected Gamma parametrisation accounts for time-variation in the tails, unlike Pareto specifications and does not outperform asymmetric Laplace specification. Data-driven nonparametric methods seem to struggle the most in approximating downside tail risks due to the sharp corrections in Cryptocurrencies' value.

Keywords: RiskMetrics, Exponential smoothing, Generalized autoregressive score models, Kernel density estimation, Time-varying quantiles, Value-at-Risk, Cryptocurrencies

1 Introduction and Motivation

Investing in Cryptocurrencies has not only a benefit for excess returns (e.g. Cheah & Fry, 2015; Gregoriou, 2019) but also huge risks (e.g. Corbet, Lucey, & Yarovaya, 2018) due to unique properties of these emerging financial assets (e.g. Phillip et al., 2018). Corbet et al. (2019) document great interest in these financial assets from academics, financial regulators and various investors groups, while we also note an emerging trend in the literature aiming to assess the performance of various approaches for Value-at-Risk (VaR) and/or Expected Shortfall forecasting in this volatile market environment (e.g. Gkillas & Katsiampa, 2018; Peng et al., 2018; Caporale & Zekokh, 2019; Trucíos, 2019). Indeed, assessing Cryptocurrencies exposure is vital. Cryptocurrencies can serve as a hedging tool against financial markets uncertainty (e.g. Demir et al., 2018; Fang et al., 2019) or global geopolitical risks (e.g. Aysan et al., 2019). Moreover, Klein et al. (2018) argue that Bitcoin and Gold are financial assets demonstrating unique relationships with international stock markets. Besides, Corbet, Meegan, et al. (2018), Beneki et al. (2019) and Ji et al. (2019) document that the connectedness among Cryptocurrencies is not stable and begins to critically fluctuate after the late 2016 price peaks. Therefore, we aim to empirically test if investors in Cryptocurrencies may rely on the parsimonious schemes for their risk exposure evaluations and contribute to the growing empirical literature on the properties of these assets.

J.P. Morgan's (1996) RiskMetricsTM (RM) is the most recognised and basic benchmark model in financial research for VaR estimation (e.g. Boucher et al., 2014; Nieto & Ruiz, 2016). Its original form of the exponentially weighted moving average (EWMA) conditional variance for Gaussian distributed returns has been critically reviewed (e.g. Guermat & Harris, 2002), but its intrinsic simplicity and pragmatism is still appealing to practitioners (e.g. Zumbach, 2007) as well as academics (e.g. Gerlach et al., 2013; Dupuis et al., 2014; Lucas & Zhang, 2016) to introduce necessary upgrades and keep it attractive for applied financial practices such as VaR estimations (e.g. Pafka & Kondor, 2001; Taylor, 2007; McMillan & Kambouroudis, 2009; Boucher et al., 2014). Therefore in this paper, we compile together some of the up-to-date EWMA VaR methods and question whether this widely familiar to the financial audiences weighting scheme can be still valid for risk measurement of emerging and high volatile financial assets under the basic and easy to interpret VaR backtesting framework.

To achieve our research objective, we mostly focus on the methods under or related to the Generalized Autoregressive Score (GAS) time series framework. This includes Student’s t (t -GAS) based EWMA VaR of Lucas & Zhang (2016), GAS parametrizations of the “robust” Laplace scheme (L-GAS) of Guermat & Harris (2002), its skewed GAS (L-GAS(p)) extension of Gerlach et al. (2013) and the special case GAS EWMA VaR “bias robust” double Generalized Pareto (D-GAS) model of Dupuis et al. (2014). To complement our GAS EWMA VaR analysis, we also suggest a reflected gamma distribution (G-GAS) EWMA specification. Unlike other Laplace distribution based special cases in Dupuis et al.’s (2014), it allows for time-varying scale and tail parameters as well as is parsimonious at the implementation stage. We also consider nonparametric EWMA kernel (kCDF) based alternative to RM of A. Harvey & Oryshchenko (2012) and its EWMA VaR empirical distribution function (eCDF) version similar to Taylor (2007). These approaches form a comprehensive set of EWMA models and ensure a fruitful investigation with valuable insights for various groups of finance academics and practitioners.

For backtesting our EWMA VaR forecasts we use quantile tests of Kupiec (1995), Christoffersen (1998) and Engle & Manganelli (2004) and Model Confidence Set (MCS) procedure of Hansen et al. (2011). There are several interesting findings from the results and analysis we conducted. First, we find that L-GAS(p) specification of Gerlach et al. (2013), if controlling for time-variation both in scale (volatility) and skewness (asymmetric responses to positive and negative volatility), consistently performs the best at most of the VaR levels. Second, while we expect parametric specifications to outperform their nonparametric alternatives at more extreme levels, we find that considered nonparametric EWMA specifications struggle approximating VaR in lower tail domains of Cryptocurrencies. On the other hand, we find that RM approximates LTC downside risk relatively well. Gkillas & Katsiampa (2018) also conclude that LTC is not the riskiest Cryptocurrency; however, our results demonstrate that it can be modelled with the standard RM approach rather than with more elaborate Extreme Value Theory. In the rapidly evolving Cryptocurrencies market, where new empirical findings reported monthly (Corbet et al., 2019), we conclude that our LTC RM results should be considered as an exception, similar to the well-known GARCH (1,1) case of Hansen & Lunde (2005). Overall, our results shall be insightful to perform comparisons to other currencies, commodities and other financial securities, while practitioners may successfully employ L-GAS(p) specification of Gerlach et al. (2013) in their applied daily analysis

of Cryptocurrencies.

Our work is organised as follows: Section 2 briefly introduces GAS EWMA framework and t -GAS, L-GAS, L-GAS(p), D-GAS, G-GAS, kCDF and eCDF EWMA specifications. Section 3 describes selected Cryptocurrencies data and provides general estimation details of our VaR forecasts. Section 4 formally specifies VaR and illustrates how VaR forecasts can be obtained with our G-GAS specifications. Section 5 describes first stage of our backtesting exercise with tests of Kupiec (1995), Christoffersen (1998) and Engle & Manganelli (2004), while Section 6 analyses these tests results. Section 7 introduces the MCS procedure and provides a final stage of our VaR analysis. Section 8 concludes our EWMA VaR forecasts of Cryptocurrencies investigation.

2 RiskMetrics, GAS and Some Nonparametric Approaches

For the Probability Density Function (PDF)

$$f(x_t | \mathbf{F}_{t-1}; \mathbf{f}_t, \theta) = \frac{1}{\sqrt{2\pi\sigma_t^2}} e^{-\frac{x_t^2}{2\sigma_t^2}}, \quad (1)$$

where x_t denotes daily Cryptocurrency logarithmic returns, \mathbf{F}_{t-1} is the information set available at time $t - 1$, \mathbf{f}_t and θ are vectors of time-varying and static parameters respectively; setting $\mathbf{f}_t = \sigma_t^2$ produces J.P. Morgan's (1996) RiskMetricsTM which parametrises volatility as the weighted sum of the past squared observations given by the following recursive form

$$\sigma_{t+1}^2 = \omega \cdot \sigma_t^2 + (1 - \omega) \cdot x_t^2, \quad 0 < \omega < 1; \quad (2)$$

equivalently expressed as

$$\sigma_{t+1}^2 = (1 - \omega) \sum_{i=1}^t \omega^i x_{t-i}^2 \quad (3)$$

or by

$$\sigma_{t+1}^2 = \frac{1 - \omega}{1 - \omega^t} \sum_{i=1}^t \omega^i x_{t-i}^2 \quad (4)$$

which ensures weights always sum to 1 over $i = 1, \dots, t$ and is a zero intercept particular case of Bollerslev's (1986) Integrated GARCH (1,1) (IGARCH) model. The more general form of the

IGARCH model is

$$\sigma_{t+1}^2 = c + A \cdot x_t^2 + B \cdot \sigma_t^2 = c + A \cdot (x_t^2 - \sigma_t^2) + (A + B) \cdot \sigma_t^2 \quad (5)$$

and the special case occurs when $c = 0$, $B = \omega$ and $A = 1 - B$ (see Bollerslev et al., 1994, for details).

For the Gaussian PDF in (1) and under the GAS framework

$$\mathbf{f}_{t+1} = c + A \cdot s_t + B \cdot \mathbf{f}_t \quad (6)$$

where $s_t = \mathbf{S}_t \cdot \frac{\partial \mathcal{L}_t}{\partial \mathbf{f}_t}$ for $\mathbf{S}_t = \mathbf{S}(\mathbf{f}_t, \mathbf{F}_{t-1}; \theta)$ and $\mathcal{L}_t = \log f(x_t | \mathbf{F}_{t-1}; \mathbf{f}_t, \theta)$ with $\mathcal{L}_t(\cdot)$ denoting the logarithm of the conditional PDF and $\mathbf{S}_t(\cdot)$ a scaling function, which as in Lucas & Zhang (2016) is the inverse diagonal of the Fisher information matrix (see Creal et al., 2013; A. Harvey, 2013, for more details or other scaling options), setting $c = 0$ and $B = 1$ Creal et al. (2013) show that the Integrated GAS (IGAS) reduces to

$$\mathbf{f}_{t+1} = A \cdot s_t + \mathbf{f}_t \quad (7)$$

and is identical to the IGARCH in (5) if $A = 1 - \omega$.

For the Student's t PDF

$$f(x_t | \mathbf{F}_{t-1}; \mathbf{f}_t, \theta) = \frac{\Gamma\left(\frac{\nu_t + 1}{2}\right)}{\Gamma\left(\frac{\nu_t}{2}\right) \sqrt{(\nu_t - 2)\pi\sigma_t^2}} \left(1 + \frac{x_t^2}{(\nu_t - 2)\sigma_t^2}\right)^{-\frac{\nu_t + 1}{2}} \quad (8)$$

and with $\sigma_t^2 = \mathbf{f}_{1,t}$ and $\nu_t = 2 + \exp(\mathbf{f}_{2,t})$ Lucas & Zhang (2016) provide closed form recursions for the t -GAS form of RiskMetrics. The recursions are outlined by

$$\mathbf{f}_{1,t+1} = \mathbf{f}_{1,t} + A_{\sigma_t^2} \cdot (1 + 3\nu_t^{-1}) \cdot \left(\frac{\nu_t + 1}{\nu_t - 2 + x_t^2/\mathbf{f}_{1,t}} \cdot x_t^2 - \mathbf{f}_{1,t}\right) \quad (9)$$

for σ_{t+1}^2 and ¹

$$\begin{aligned} \mathbf{f}_{2,t+1} = & \mathbf{f}_{2,t} - A_{\nu_t} \cdot \frac{2}{\nu_t - 2} \cdot \left(\gamma'' \left(\frac{\nu_t + 1}{2} \right) - \gamma'' \left(\frac{\nu_t}{2} \right) + \right. \\ & \left. \frac{2(\nu_t + 4)(\nu_t - 3)}{(\nu_t + 1)(\nu_t + 3)(\nu_t - 2)^2} \right)^{-1} \cdot \left(\gamma' \left(\frac{\nu_t + 1}{2} \right) - \right. \\ & \left. \gamma' \left(\frac{\nu_t}{2} \right) - \frac{1}{\nu_t - 2} - \log \left(1 + \frac{x_t^2}{(\nu_t - 2)\mathbf{f}_{1,t}} \right) + \right. \\ & \left. \frac{\nu_t + 1}{\nu_t - 2} \cdot \frac{x_t^2}{(\nu_t - 2)\mathbf{f}_{1,t} + x_t^2} \right), \end{aligned} \quad (10)$$

where $\Gamma(y) = \int_0^\infty z^{y-1} \exp(-z) dz$, $\gamma'(\cdot)$ and $\gamma''(\cdot)$ are the first and second order derivatives of $\gamma(\cdot) = \log \Gamma(\cdot)$, for ν_{t+1} under $A_t > 0$ restriction for both (9) and (10).

On the other hand, a less involved alternative to recursions in (9) and (10) is presented by Guermat & Harris (2002) under the functional form of Laplace distribution. Laplace PDF for estimations is given by

$$f(x_t | \mathbf{F}_{t-1}; \mathbf{f}_t, \theta) = \frac{1}{\sqrt{2}\sigma_t} e^{-\frac{\sqrt{2}|x_t|}{\sigma_t}}, \quad (11)$$

while its IGAS dynamics are specified as

$$\mathbf{f}_{3,t+1} = c + 2A \cdot \sqrt{2}|x_t|\sigma_t + (B - 2A) \cdot \mathbf{f}_{3,t}, \quad (12)$$

which under $c = 0$, $A = \frac{1 - \omega}{2}$ and $B = 1$ takes the “robust” form of Guermat & Harris (2002), given by

$$\sigma_{t+1}^2 = \omega \cdot \sigma_t^2 + (1 - \omega) \cdot \sqrt{2}|x_t|\sigma_t, \quad (13)$$

as shown by Lucas & Zhang (2016) for L-GAS EWMA parametrisation.

Further, to introduce the functionality of asymmetric responses to “negative” and “positive” volatility in (11), Gerlach et al. (2013) consider a skewed Laplace PDF given by

$$f(x_t | \mathbf{F}_{t-1}; \mathbf{f}_t, \theta) = \frac{k_t}{\sigma_t} \exp \left(- \left[\frac{1}{1 - p_t} \mathbb{I}_{\{x_t > 0\}} + \frac{1}{p_t} \mathbb{I}_{\{x_t < 0\}} \right] \frac{k_t |x_t|}{\sigma_t} \right), \quad (14)$$

¹Note that setting $\omega = A \cdot (1 + 3\nu_t^{-1})$ in (9) provides a recursive form similar to (2), see Lucas & Zhang (2016) for details.

where $k_t = \sqrt{p_t^2 + (1 - p_t)^2}$ and with $\mathbf{f}_{4,t} = \sigma_t^2$ yielding the following GAS recursions:

$$\begin{aligned}\sigma_{t+1}^2 &= \omega_1 \cdot \sigma_t^2 + (1 - \omega_1) \cdot \sigma_t \cdot |x_t| \cdot \left(\frac{k_t}{1 - p_t} \mathbb{I}_{\{x_t > 0\}} + \frac{k_t}{p_t} \mathbb{I}_{\{x_t < 0\}} \right), \\ u_{t+1} &= \omega_2 \cdot u_t + (1 - \omega_2) \cdot |x_t| \cdot \mathbb{I}_{\{x_t > 0\}}, \\ v_{t+1} &= \omega_3 \cdot v_t + (1 - \omega_3) \cdot |x_t| \cdot \mathbb{I}_{\{x_t < 0\}}, \\ p_{t+1} &= \frac{1}{1 + \sqrt{\frac{u_{t+1}}{v_{t+1}}}}.\end{aligned}\tag{15}$$

On the other hand, Dupuis et al. (2014) concerned with limited tails functionality of the Laplace distribution, suggest employing a more flexible, but still parsimonious, double Generalised Pareto distribution model. Its PDF for estimations is given by

$$f(x_t | \mathbf{F}_{t-1}; \mathbf{f}_t, \theta) = \frac{1}{2\sigma_t} \left(1 + \frac{\xi |x_t|}{\sigma_t} \right)^{-1/\xi-1},\tag{16}$$

while $\mathbf{f}_{5,t+1} = \sigma_{t+1}$ and is obtained iteratively by solving

$$\sum_{i=1}^t \frac{(1 - \omega)\omega^{t-i}}{1 - \omega^t} \left(\frac{1}{1 + \xi |x_i|/\sigma_t} - \frac{1}{1 + \xi} \right) = 0,\tag{17}$$

where ξ can be set at

$$\xi = \xi_{\mathbf{Hill}}^{\lfloor t_{0.05} \rfloor} = \frac{1}{\lfloor t_{0.05} \rfloor} \sum_{j=1}^{\lfloor t_{0.05} \rfloor} \log \left(\frac{x_{j,t}}{x_{\lfloor t_{0.05} \rfloor, t}} \right)$$

for $x_{1,t} \leq \dots \leq x_{j,t}$ of $|x_i|$, $i = 1, \dots, t$ and $\lfloor t_{0.05} \rfloor$ denoting integer of x for 5% of the largest absolute returns on the estimation time t , as per rule-of-thumb suggestion of Dupuis et al. (2014). Asymmetric volatility responses for D-GAS can also be allowed. To implement this, Dupuis et al. (2014) consider a double Pareto PDF of the following form:

$$f(x_t | \mathbf{F}_{t-1}; \mathbf{f}_t, \theta) = \begin{cases} \frac{1-p}{\sigma_t^-} \left(1 - \frac{\xi x_t}{\sigma_t^-} \right)^{-1/\xi-1} & \text{for } x_t < 0 \\ \frac{p}{\sigma_t^+} \left(1 + \frac{\xi x_t}{\sigma_t^+} \right)^{-1/\xi-1} & \text{for } x_t > 0 \end{cases},\tag{18}$$

where $\mathbf{f}_{6,t+1} = \sigma_{t+1}^+$ and $\mathbf{f}_{7,t+1} = \sigma_{t+1}^-$ are obtained by solving:

$$\sum_{i=1}^t \frac{(1 - \omega_1)\omega_1^{t-i}}{1 - \omega_1^t} \cdot \mathbb{I}_{\{x_i > 0\}} \cdot \left(\frac{1 + \xi}{\xi + \frac{\sigma_i^+}{x_i}} - 1 \right) = 0 \quad (19)$$

and

$$\sum_{i=1}^t \frac{(1 - \omega_2)\omega_2^{t-i}}{1 - \omega_2^t} \cdot \mathbb{I}_{\{x_i < 0\}} \cdot \left(\frac{1 + \xi}{\xi - \frac{\sigma_i^-}{x_i}} - 1 \right) = 0 \quad (20)$$

respectively.

As an alternative to the RiskMetrics approach outlined by Dupuis et al. (2014), we suggest approximating data generating process of returns by employing a reflected gamma PDF as discussed in Nadarajah (2004) and given by

$$f(x_t | \mathbf{F}_{t-1}; \mathbf{f}_t, \theta) = \frac{1}{2\sigma_t \Gamma(\nu_t)} \cdot \left| \frac{x_t}{\sigma_t} \right|^{\nu_t - 1} \exp \left\{ - \left| \frac{x_t}{\sigma_t} \right| \right\}, \quad (21)$$

where $\sigma_t > 0$ and $\nu_t > 0$. It is straightforward to see that if $\nu_t = 1$, PDF in (21) takes a form of the Laplace distribution similar to the robust symmetric EWMA in Gerlach et al. (2013) and to what is shown by Nadarajah et al. (2013) for the PDF in (16) if $\xi = 0$. Now, if $\log \sigma_t = \mathbf{f}_{8,t}$ and $\log \nu_t = \mathbf{f}_{9,t}$ for PDF in (21) under dynamics in (6) it can be shown that

$$\mathbf{f}_{8,t+1} = \mathbf{f}_{8,t} + A_{\sigma_t} \cdot \left(\frac{|x_t|}{\sigma_t \cdot \nu_t} - 1 \right) \quad (22)$$

and

$$\mathbf{f}_{9,t+1} = \mathbf{f}_{9,t} + A_{\nu_t} \cdot \mathbb{I}_{\{x_t \neq 0\}} \cdot \left(\frac{\log|x_t| - \mathbf{f}_{8,t} - \gamma'(\nu_t)}{\nu_t \cdot \gamma''(\nu_t)} \right) \quad (23)$$

respectively. Recursions in (22) and (23) are notably more straightforward at the implementation stage than those outlined by Lucas & Zhang (2016) for t -GAS RiskMetrics, but most important, they allow time-variation in the shape of the tails, unlike D-GAS specifications.²

Another pragmatic strategy under RiskMetrics type weightings may result from removing any particular form of the distributional parametrisations. From A. Harvey & Oryshchenko (2012), a time-varying CDF of financial returns for dynamic quantiles and VaR mining can be estimated

²We provide empirical illustrations on the reflected gamma, its EWMA GAS parameters and VaR in Section 4.

using a kernel in the form of CDF and is given by

$$\mathbf{F}_{1,t+1}(x) = \frac{1 - \omega}{1 - \omega^t} \sum_{i=1}^t W\left(\frac{x - x_i}{\beta}\right) \omega^i, \quad (24)$$

where $W(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-z^2/2} dz$, $x \in \mathbb{R}$ and β is the CDF optimal bandwidth parameter.³ Moreover, the latter form may be further simplified to

$$\mathbf{F}_{2,t+1}(x) = \frac{1 - \omega}{1 - \omega^t} \sum_{i=1}^t \mathbb{I}_{\{x_i \leq x\}} \omega^i, \quad (25)$$

following the guidelines of Taylor (2007) for nonparametric quantile regressions with exponentially declining weights.

3 Data

For our VaR estimations, we employ daily USD exchange rates for Bitcoin (BTC), Litecoin (LTC) and Ethereum (ETH) from Kraken Cryptocurrency exchange.⁴ BTC, LTC and ETH are on the list of the top five most highly capitalised Cryptocurrencies (as of coinmarketcap.com data on the 25th of February, 2019) and are often subjects for investigation in the relevant literature (e.g. Katsiampa (2019); Caporale & Zekokh (2019)). Therefore, they formulate a small set of representative Cryptocurrencies for our investigation.

We obtain returns for computations as follows:

$$x_t = \log\left(\frac{P_t}{P_{t-1}}\right) \cdot 100$$

and describe some of their properties in Table 1. Obtained returns are illustrated in Figure 1.

[Table 1 and Figure 1 around here.]

³Semeyutin & O’Neill (2019) empirically show that kernel functional form is not the most important component of forecasting with these estimators. Therefore, we employ most commonly used Gaussian kernel in our estimations.

⁴BTC data prior to 06.10.2013 was obtained from coindesk.com and was no longer publicly available at the moment of producing this work. It is available upon request from the corresponding author.

We split each series in “training” and “testing” subsamples as reported in Table 1. For BTC and LTC, we then perform 1000 observations rolling window parameters updates for our one-day-ahead VaR forecasts similar to Laporta et al. (2018) among others. For ETH, to insure that VaR testing period covers the “boom and bust” of Cryptocurrencies, parameters for one-day-ahead forecasts are first obtained by recursively adding daily data from the testing subsample, and when ETH estimation sample reaches 1000 observations, parameters are updated using a rolling window approach similar to BTC and LTC. Please see Ardia & Hoogerheide (2014) for relevant and thorough discussion of the parameters estimation strategies and their impact on VaR forecasts.

Parameters for (1), (8), (11), (14), (16) and (18) are obtained employing maximum likelihood as prescribed by Creal et al. (2013) and for (24) and (25) with accordingly modified least-squares routine of Bowman et al. (1998) as discussed in Semeyutin & O’Neill (2019) for nonparametric estimators of A. Harvey & Oryshchenko (2012).⁵ Note that for stable evaluations with (17), (19), (20), (24) and (25) more observations may be necessary at the initial recursive iterations. Therefore, recursions for these parameters are initialised at the 250th observation, however still employing all preceding observations.

4 Value-at-Risk and Reflected Gamma Quantile Function

Our daily out-of-sample VaR forecasts are performed assuming “long” position in the selected Cryptocurrencies for 95%, 97.5%, 99% and 99.5% risk confidence levels and are backtested employing unconditional coverage test of Kupiec (1995), conditional coverage test of Christoffersen (1998), dynamic quantiles test of Engle & Manganelli (2004) and MCS procedure of Hansen et al. (2011). We describe tests of Kupiec (1995), Christoffersen (1998) and Engle & Manganelli (2004) in Section 5, while the procedure of Hansen et al. (2011) in Section 7. Overall, our VaR backtesting framework can be described as the most standard (e.g. see Nieto & Ruiz, 2016, for comprehensive VaR review) and should be familiar to the various financial audiences interested in forecasting VaR of Cryptocurrencies (e.g. Trucíos, 2019) or other commodities (e.g. Laporta et al., 2018).

⁵We actually perform computations of parameters for nonparametric methods using accordingly modified binned estimators to speed up our evaluations of the unknowns as per binning details discussed in Semeyutin & O’Neill (2019).

We define EWMA VaR forecasts as

$$\widehat{\text{VaR}}_{i,t+1,\alpha} = \widehat{F}_{t+1}^{-1}(1 - \alpha), \text{ for } \alpha \in (0, 1).$$

For example, for reflected gamma in (21), one-step-ahead VaR forecasts are therefore outlined by

$$\widehat{\text{VaR}}_{t+1,\alpha} = \widehat{F}_{t+1}^{-1}(1 - \alpha \mid \mathbf{F}_{t-1}; \mathbf{f}_t, \theta) = \begin{cases} -\hat{\sigma}_{t+1} \cdot Q^{-1}(\hat{\nu}_{t+1}, 2[1 - \alpha]) & \text{if } \alpha \leq 0.5 \\ \hat{\sigma}_{t+1} \cdot Q^{-1}(\hat{\nu}_{t+1}, 2\alpha) & \text{if } \alpha > 0.5 \end{cases}, \quad (26)$$

where $Q(\alpha, \hat{\nu}_{t+1}) = \int_{\alpha}^{\infty} z^{\hat{\nu}_{t+1}-1} \exp(-z) dz / \Gamma(\hat{\nu}_{t+1})$. Other quantile functions for VaR estimation with specifications listed in Section 2 can be found in the relevant EWMA literature. For instance, to obtain nonparametric EWMA VaR forecasts, we employ the empiric algorithm described in A. Harvey & Oryshchenko (2012) and do not describe it for brevity reasons.

[Figures 2, 3, and 4 around here.]

We illustrate some of the possible shapes for reflected gamma PDF and also perform its shapes comparisons with Student's t PDF in Figure 2. Though we do not categorise G-GAS as a direct competitor to t -GAS EWMA specification, our goal here is to demonstrate that reflected gamma can take varied PDF forms and that dynamics of its scale and tails shape parameters can be found similar to the t -GAS(ν_t). From Figure 2, setting $\nu = 1$ with reflected gamma we obtain the shape of Laplace distribution, while with $\nu = 1.5$ we can easily observe two reflected gamma PDFs combination forming our joint distribution for estimations. If we compare it to Student's t shapes in Figure 2, it is easy to note that reflected gamma PDF approximates body domain of the data generating process differently. However in the context of VaR, we are mostly interested in the tails approximation and it is easy to pick up that reflected gamma offers a range of tail decays within the functionality of its shape parameter (e.g. also see Chen & Gerlach, 2013, employing more elaborate two-sided Weibull distribution in the GARCH setting for VaR).

Employing reflected gamma may have a very straightforward rationale. Laplace EWMA schemes as in Guermat & Harris (2002) are usually expected to provide conservative estimates (e.g. Lucas &

Zhang, 2016) due to limited tail functionality. Double GPD as in Nadarajah et al. (2013) also allows for different tails, has Laplace as a special case and may offer attractive Extreme Value Theory links; however, its EWMA extension of Dupuis et al. (2014) is restricted to $\xi > 0$ assumption⁶ and implies heavier than Laplace tails for estimations. This restriction is still valid for VaR modelling of Cryptocurrencies as we may well expect financial returns to be heavy-tailed. On the other hand, unlike Dupuis et al.’s (2014) specifications, our G-GAS EWMA setting also allows for time-varying tails and does not require an iterative solution for its time-varying scale parameter. Therefore, it can be argued as more straightforward at the implementation stage. RiskMetrics type models are commonly expected to be parsimonious and easy to implement, since the more technically or computationally involved the forecasting scheme becomes, the less it is reasonable to restrict one’s portfolio of methods to the exponential decay weighting given other more elaborate and effective methods for modelling VaR of Cryptocurrencies (e.g. Peng et al., 2018).

Figure 3 illustrates BTC in-sample volatility and time-varying tails parameters for t -GAS(ν_t) EWMA and similar parameters for G-GAS(ν_t) EWMA. It is straightforward to note that volatility and scale dynamics of these specifications are not identical but have a similar pattern and common trends. For BTC, t -GAS(ν_t) produces tails parameter often close to 2, its lower tail bound limit keeping variance of the Student’s t defined, while G-GAS(ν_t) tail parameter fluctuates around 1, its special Laplace distribution case. This evidence highlights our key G-GAS EWMA motivations; it is straightforward to implement, can take robust Laplace form as well as offers a range of shapes to avoid potential conservatism of the Laplace EWMA based specification. We also visualise BTC in-sample G-GAS(ν_t) VaR estimates and their violations in Figure 4 for EWMA parameters in Figure 3. Figure 4 illustrates that this scheme provides adequate tail quantiles evaluation and is valuable for our Cryptocurrencies competition of RiskMetrics type models. Now we proceed to our next section, where we begin describing tests we use to backtest our one-step-ahead Cryptocurrencies VaR forecasts.

⁶Otherwise its EWMA driven scale/volatility is undefined and is no longer robust as highlighted by Dupuis et al. (2014).

5 Value-at-Risk Backtesting Framework

For the out-of-sample VaR violations denoted with $N = \sum_{t=1}^T \mathbb{I}_t$, where \mathbb{I}_t is an indicator function taking the value of 1 every time there is a larger realised loss than the VaR forecasts for the period T , Kupiec (1995) suggests employing the following Likelihood Ratio (LR) test for VaR backtesting:

$$\text{LRuc}_d(\alpha) = 2 \left(\log \left[\left(\frac{N}{T} \right)^N \cdot \left(1 - \frac{N}{T} \right)^{T-N} \right] - \log [(1 - \alpha)^{T-N} \cdot \alpha^N] \right). \quad (27)$$

The statistic in (27) is a $\chi^2(1)$ distributed, quantifies how well VaR exceedances' rate matches expectations and is commonly known as the unconditional coverage LR (LRuc) test. On the other hand, Christoffersen (1998) builds upon LRuc test idea and suggests a more inclusive LR procedure outlined by:

$$\text{LRcc}_d(\alpha) = \text{LRuc}_d(\alpha) - \text{LRin}_d(\alpha), \quad (28)$$

where LRuc_d is computed LRuc test distance in (27), and LRin_d is independence LR test distance outlined by:

$$\text{LRin}_d(\alpha) = 2 \left(\log \left[\pi_{00}^{T_{00}} \pi_{01}^{T_{01}} \pi_{10}^{T_{10}} \pi_{11}^{T_{11}} \right] - \log [(1 - \alpha)^{T_{01}+T_{11}} \cdot \alpha^{T_{00}+T_{10}}] \right), \quad (29)$$

where $\pi_{ij} = P(\mathbb{I}_t = j \mid \mathbb{I}_{t-1} = i) = \frac{T_{ij}}{T_{i0} + T_{i1}}$ for the first-order Markov chain transition matrix:

$$\nabla = \begin{pmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{pmatrix}$$

with T_{ij} accounting for transitions from states i and j . LRin statistic in (29) is a $\chi^2(1)$ distributed and in combination with (27) forms the conditional coverage LR (LRcc) test in (28). Therefore, LRcc test follows $\chi^2(2)$ distribution and jointly tests the first-order Markov independence of VaR violations and how their number matches our expectations hypothesis.

As pointed out by Berkowitz et al. (2011), Engle & Manganelli (2004) construct a more powerful and simple test (DQ) for evaluating VaR forecasts. If our VaR violations are i -th order independent and match our expected number of occurrences, for a demeaned VaR violations function $\lambda_t = \mathbb{I}_t - \alpha$,

all coefficients in the below regression setting:

$$\lambda_t = \beta_0 + \sum_i^{\pi} \beta_i \lambda_{t-i} + \sum_j^g \beta_{\pi+j} Z_{j,t} + \epsilon_t \quad (30)$$

should be zero. Wald test based VaR violations statistic for the setting in (30) is $\chi^2(\pi + g + 1)$ distributed, while it is common to set $\pi = 4$ and $Z_{j=1,t} = \widehat{\text{VaR}}_{i,t,\alpha}$ (e.g. Novales & Garcia-Jorcano, 2019).

For the quantile tests outlined by (27), (28) and (30), a typical significance threshold is set up at the 5% level (e.g. Laporta et al., 2018) as it is in our next section, where we provide backtesting results and VaR analysis for BTC, LTC and ETH with these tests.

6 Value-at-Risk Backtesting Results and Discussion

Results for the backtesting procedure described in Section 5 are provided in Tables 2, 3, 4 and 5, while we comply actual VaR forecasts with different EWMA VaR methods for the considered Cryptocurrencies into box-plots in Figure 5. Compiling our VaR one-step-ahead forecasts by different EWMA specifications into box-plots allows us to analyse relative consistency of the obtained estimations and evaluate their degree of conservatism. This evidence not only provides a compact presentation but is also valuable for comprehensive and insightful analysis with quantile results reported in Tables 2, 3, 4 and 5.

[Tables 2, 3, 4, 5 and Figure 5 around here.]

We begin with analysing 99.5% VaR confidence level results in Table 2. First, the only EWMA specification passing all three tests in Table 2 at the 5% significance level is L-GAS(p_t). Moreover, it is the only approach comfortably passing DQ test for BTC and ETH. For LTC, D-GAS, D-GAS(w), G-GAS(ν) and G-GAS(ν_t) also pass all tests at the 5% significance level. However, it is notable that for LTC RM and kCDF approaches pass LRuc and LRcc tests. Though kCDF may be expected to struggle at this VaR confidence level, it yields a slightly better outlook than RM for BTC and ETH when evaluating AE ratios of these models. AE ratios for kCDF are closer to one than those of RM

but are still above the expected level. This evidence highlights the insufficiency of RM Gaussian assumptions for Cryptocurrencies. t -GAS specifications provide a modest performance similar to our reflected gamma and Dupuis et al.'s (2014) Pareto approaches. However, it is worthwhile to highlight that t -GAS(ν_t) AE ratios are notably below one. GAS based applied recommendation of Zumbach (2007) provides the worst performance. Evaluating box-plots for this VaR level in Figure 5, it may be noted that D-GAS of Dupuis et al. (2014) yields the most conservative VaR projections. This is the most notable for LTC and can be explained by the time-invariant shape parameter of this approach. Accounting for positive and negative volatility with D-GAS(w) provides improvements similar to our G-GAS(ν_t) specification; however, for reflected gamma we achieve these results by varying the shape parameter over time. Despite its low AE ratio in Table 2, we observe that t -GAS(ν_t) does not yield the most conservative estimates. This evidence may be a valuable observation on t -GAS(ν_t) fit for Cryptocurrencies and we may expect a positively different outlook for this specification with the MCS procedure. To clarify, MCS is typically based on the VaR violations function of González-Rivera et al. (2004) and is designed to not only account for frequency of the losses but also for their magnitude.

Now we proceed to analyse 99% VaR confidence level results in Table 3. Again, the only EWMA specification passing all three tests for this risk level is L-GAS(p_t). It is also the only specification passing all three tests for BTC. For the methods passing all three tests for ETH, it is now joined by D-GAS, D-GAS(w), G-GAS(ν) and G-GAS(ν_t). It is worthwhile to highlight that D-GAS, D-GAS(w), G-GAS(ν) and G-GAS(ν_t) are the specifications also passing LRuc and LRcc tests for BTC at this level. These results suggest that these approaches meet expectations on the number of violations for Cryptocurrencies consistently, which are also the first-order independent; however, fail ensuring higher order independence levels for BTC. For LTC, we generally observe that most of the EWMA specifications meet all our quantile expectations with RM providing quite an appealing outlook for this Cryptocurrency at the 99% level. It outperforms both kCDF and eCDF approaches when taking DQ results for LTC into consideration. However, for ETH and BTC, we observe a generally more expected performance outlook for kCDF and RM. t -GAS specifications provide a very similar performance outlook to 99.5% level, while results in Figure 5 again point out the drawbacks of D-GAS in our estimation setting and the context of Cryptocurrencies. This is the most notable when evaluating box-plots for LTC at this level.

Analysing 97.5% VaR level results in Table 4, we again observe that L-GAS(p_t) continues to provide the most appealing VaR backtesting results. It is the only specification which passes all tests at this level. D-GAS, D-GAS(w), G-GAS(ν) and G-GAS(ν_t) provide similar performances to each other without a leading specification in the group. However, only G-GAS approaches consistently meet our expected number of violations and first-order independence criteria. 97.5% risk level is a domain where we may begin to expect the dominance of the nonparametric approaches. In Table 4, we observe that kCDF improves its previous performances at higher risk confidence levels. However, LRuc for BTC and DQ for BTC and ETH results are still notably behind the best performing specification of Gerlach et al. (2013).

Finally, we expect nonparametric EWMA specifications to take the key role at the 95% VaR confidence level in Table 5. However, both approaches struggle to provide first and higher-order independence of VaR violations for BTC and do not pass the DQ test for ETH and LTC. Moreover, kCDF and eCDF do not provide improved performances over their previous risk level results. On the other hand, based on the eCDF VaR backtest results we point out empirical evidence on the modelling value of kernel functional form and bandwidth parameters for the VaR of Cryptocurrencies estimations. It is not straightforward to select the best performing model for this risk confidence level. L-GAS(p_t) struggles to outperform RM consistently and only both G-GAS specifications ensure that number of VaR violations match expectations and are also at least first-order independent for all considered Cryptocurrencies.

7 Model Confidence Set

From the VaR backtesting in the previous section, L-GAS(p_t) repeatedly passes the 5% significance threshold at most of our VaR levels. However, tests we consider in Section 6 do not allow directly discriminating among the models which jointly pass our selected significance level and only target testing the frequency of VaR violations. Therefore to complement our analysis in Section 6, we also describe and apply the MCS procedure of Hansen et al. (2011) to our VaR forecasts. MCS is designed to construct a “superior set of models” (SSM) and allows explicitly ranking forecasting performances of our RiskMetrics variations for each sample at the specified VaR level. MCS backtesting results are bootstrap based, robust, relatively straightforward to interpret and therefore,

are valuable for practitioners in the applied context of our RiskMetrics estimations.

To yield an SSM, $\hat{M}_{1-\delta}^*$ at a confidence level $1 - \delta$, we consider an asymmetric quantile loss function of González-Rivera et al. (2004)

$$\mathbb{L}\left(x_t; \hat{\text{VaR}}_{\iota,t,\alpha}\right) = \begin{cases} (\alpha - 1) \cdot \left(x_t - \hat{\text{VaR}}_{\iota,t,\alpha}\right) & \text{if } x_t < \hat{\text{VaR}}_{\iota,t,\alpha} \\ \alpha \cdot x_t - \alpha \cdot \hat{\text{VaR}}_{\iota,t,\alpha} & \text{if } x_t \geq \hat{\text{VaR}}_{\iota,t,\alpha} \end{cases}, \quad (31)$$

designed to heavily penalise extreme exceedances of the VaR forecasts. Further, for the loss function in (31), we construct an Equal Predictive Ability (EPA) test. EPA test can be based on the loss differentials $d_{ij,t}$ between model i and model j ,

$$d_{ij,t} = \mathbb{L}_{i,t} - \mathbb{L}_{j,t},$$

and the average loss differential $d_{i,\cdot,t}$ between model i and any other competing model in the generic set of models M , so that $i, j \in M$ and

$$d_{i,\cdot,t} = \frac{1}{m-1} \sum_{j \in M} d_{ij,t},$$

where m denotes the dimensions of the initial participating models set M^0 . Null and alternative hypotheses for the EPA test are typically outlined by:

$$\begin{aligned} H_0 &: \mathbb{E}[d_{i,\cdot,t}] = 0, \quad \text{for all } i \in M \\ H_1 &: \mathbb{E}[d_{i,\cdot,t}] \neq 0, \quad \text{for some } i \in M \end{aligned} \quad (32)$$

and are constructed upon the “if a model i is preferred to the alternative model j when $d_{ij,t} < 0$ ” testing rationale. For the hypotheses in (32), Hansen et al. (2011) suggest the following statistic:

$$t_{i,\cdot} = \frac{\bar{d}_{i,\cdot}}{\sqrt{\hat{v}\text{ar}(\bar{d}_{i,\cdot})}}, \quad \text{for all } i \in M, \quad (33)$$

where $\bar{d}_{i,\cdot} = \frac{1}{m-1} \sum_{j \in M} \bar{d}_{ij}$ and $\bar{d}_{ij} = \frac{1}{n} \sum_{t=1}^T d_{ij,t}$, while $\hat{v}\text{ar}(\bar{d}_{i,\cdot})$ is the bootstrapped variance estimate of $\bar{d}_{i,\cdot}$. similar to the well-known tests for comparing two forecasts by Diebold & Mariano

(1995) and D. Harvey et al. (1997) among others. Finally, a coherent model elimination rule is required for the MCS procedure with (33) and is typically given by:

$$\varepsilon_M = \mathbf{arg\,max}_{i \in M} t_{i,..} \quad (34)$$

Elimination rule we set up in (34) concentrates on the standardised VaR exceedances relative to the computed average across other participating models since the greater are the computed statistic values in (33), the more distant are the actual realisations from the model’s forecasts. Overall, the MCS procedure begins with EPA test on some initial set of models. If the null is accepted at the first iteration, it reports ranked models in the initial set with a statistic in (33). On the other hand, if the null is rejected, a model with the highest computed statistic is eliminated, and the procedure is repeated until the null in (32) is accepted at the δ significance level, yielding an SSM. Similar to Laporta et al. (2018) and Caporale & Zekokh (2019) in our MCS estimations we aim to construct 5000 bootstrap samples for each VaR level and set $\delta = 0.2$. We report the computed SSM for Cryptocurrencies at each VaR level in Table 6.

[Table 6 around here.]

In Table 6, each entry indicates the ranking of EWMA specifications within $\hat{M}_{1-\delta}^*$, while no ranking implies that the model was eliminated at the chosen VaR confidence level. From Table 6, it is straightforward to observe that L-GAS(p_t) EWMA specification receives the highest ranking consistently at 99.5%, 99% and 97.5% VaR levels. At the 95% level it also provides an attractive performance; however, gets eliminated for LTC forecasts. These results are in line with our analysis and conclusions in Section 6 for L-GAS(p_t). The only remaining model for LTC forecasts in the SSM at the 95% level is t -GAS(ν_t) specification. Overall from Table 6, t -GAS(ν_t) EWMA can be classified as the second-best performing model in our setting for other Cryptocurrencies and VaR levels. It gets eliminated from the SSM only three times, second-lowest after L-GAS(p_t), and typically receives good ranking among remaining models. Though in Section 6, it struggles to meet the expected frequency of violations, when we account for the magnitude of the losses, it

provides a more appealing performance. On the other hand, it is also worthwhile to acknowledge the standard RM scheme performance. As may be expected from the analysis conducted in Section 6, it provides relatively good results for LTC at 99.5%, 99% and 97.5% VaR levels. Moreover, from Table 6, it is found superior to t -GAS(ν) specification and GAS version of the Zumbach's (2007) applied recommendation for t -GAS(5). These results highlight the importance of tails time variation for Cryptocurrencies.

In Table 6, RM also receives a higher ranking than L-GAS and outperforms D-GAS and D-GAS(w) specifications of Dupuis et al. (2014) at 99% and 97.5% VaR levels. Moreover, it outperforms G-GAS(ν) and G-GAS(ν_t) at 97.5% level as well as provides quite competing ranking to our gamma based models at the higher VaR confidence levels. These results for RM, L-GAS, D-GAS, D-GAS(w), G-GAS(ν) and G-GAS(ν_t) EWMA specifications highlight that considering special cases of the Laplace distribution with a shape parameter provide little modelling gains for VaR of Cryptocurrencies than special cases relying on the skewness parameter as in Gerlach et al. (2013). Therefore, we can conclude that for modelling VaR of Cryptocurrencies time-varying skewness parametrisation is more valuable than time-varying shape under the exponential weighting scheme. Finally, we also note less expected results for the nonparametric EWMA specifications we consider. Both eCDF and kCDF EWMA models get eliminated at 95% risk confidence and receive far from the highest ranking at the 97.5% VaR level in Table 6. Usually, we expect nonparametric specifications to capture relatively data abundant domains quite well, while from the results in Section 6 and here, these specifications struggle to provide an appealing modelling outlook at these levels. On the other hand, nonparametric EWMA provide very attractive box-plots in Figure 5; however, unlike for t -GAS(ν_t), their AE ratios reported in Tables 2, 3, 4 and 5 are typically above one. Therefore, the results we observe in Table 6 for these specifications shall be expected. Generally, this may be rationalised by the sharp and unprecedented fluctuations in Cryptocurrencies' value and corresponding bouts of extreme volatility due to the unique issues in the Cryptocurrencies market as discussed and summarised by Corbet et al. (2019) and Eross et al. (2019) among others. Therefore, specifications relying on the parametric assumptions may still outperform entirely data-driven nonparametric methods at the lower risk confidence levels as Cryptocurrencies' market is not mature yet. On the other hand, one may increase the size of the rolling window for better forecasting outcomes with nonparametric specifications; however, at the

current stage of Cryptocurrencies' market development, this may be still problematic due to the relatively small/reduced sample sizes of Cryptocurrencies for backtesting.

8 Concluding Remarks

In this work, we empirically tested whether VaR of Cryptocurrencies can be forecasted with EWMA models similar to the well-known RM approach of J.P. Morgan (1996) for the downside risk evaluations. To achieve our aim, we compiled approaches that are built upon J.P. Morgan's (1996) RM criticism (e.g. McMillan & Kambouroudis, 2009; Lucas & Zhang, 2016) as well as suggested our specification under this scheme. Employing LRuc, LRcc and DQ tests as well as MCS procedure, we identify that VaR of Cryptocurrencies can be successfully forecasted with parsimonious EWMA models. We also find that L-GAS(p_t) of Gerlach et al. (2013) performs the best at most of the considered VaR levels and is a valuable addition to the portfolio of methods used for Cryptocurrencies' VaR forecasting as in Peng et al. (2018). Besides, similar to Trucíos (2019) our EWMA results highlight good general performance of GAS framework in the Cryptocurrencies setting. The framework allows every parameter behind the data generating process to contribute to the conditional volatility estimates for our VaR modelling and therefore, also provides good forecasts with simple exponential weights. For example, with our MCS results, we observe this for t -GAS(ν_t) EWMA of Lucas & Zhang's (2016). Our G-GAS specifications do not achieve as positive outlook in the MCS procedure as t -GAS(ν_t); however, they provide modest and competing performance to other models with LRuc, LRcc and DQ results. On the other hand, our reflected gamma and GAS based specifications cannot be regarded as the exhaustive contribution. We aimed to enhance common Laplace distribution with a time-varying shape parameter similar to Lucas & Zhang's (2016) approach and complement our VaR investigation. With overall G-GAS results for Cryptocurrencies, we conclude that in the Laplace related specifications, time-varying skewness asymmetric volatility responses may be preferred over the responses driven by the time-varying symmetric tails parameter.

Extreme Value Theory linked double Pareto EWMA specifications of Dupuis et al. (2014) also provide a competing performance at LRuc, LRcc and DQ backtesting stage and can be worthwhile considering; however, if not accounted for positive and negative volatility, D-GAS tends to provide

an excessive conservative outlook in our estimations setting. This evidence does not necessarily mean that conservative estimates are not valid for applied use. Pérignon et al. (2008) point out that the six largest commercial banks in Canada prefer to overestimate their exposure to avoid additional financial penalties (e.g. see McMillan & Kambouroudis, 2009; McAleer et al., 2013, for examples of the number of violations and capital penalties under the Basel II and III standards) and thus, indicate that conservative exposure strategies may be valid for practitioners. On the other hand, similar to Gkillas & Katsiampa (2018), we also find that LTC is not the riskiest Cryptocurrency, however from our results, it is shown with the most basic J.P. Morgan’s (1996) RM rather than more elaborate Extreme Value Theory setting.

Future researches are encouraged to replicate our results as more observations for BTC, LTC and ETH become available or consider a wider pool of Cryptocurrencies to challenge our findings on the EWMA schemes. Indeed, it is also worthwhile to consider a more comprehensive portfolio of methods for VaR estimations similar to Caporale & Zekokh (2019). In addition, Laporta et al. (2018) find that quantile regressions outperform several common GARCH and GAS specifications in the setting of energy commodities and therefore, parametric and nonparametric quantile regressions may be an excellent addition to the set of models used for our estimations. EWMA based nonparametric quantile regressions as in Taylor (2007) are of particular interest, since in our setting their direct nonparametric competitors for entire distribution modelling as in A. Harvey & Oryshchenko (2012), struggle at the comfortable domains for nonparametric specifications. Other future investigations may also add a skewness parameter to our reflected gamma model and perform comparisons to the skewed Student’s t version of Lucas & Zhang’s (2016) EWMA specification.

We scripted all computations for producing this work and performed them in R version 3.5 by R Core Team (2013). For all replication scripts and data-related questions, one can contact the corresponding author. We have no conflict of interest to disclose and would like to thank the Editor, the Associate Editor, and the referee for careful reading, and for their comments, which greatly improved the paper. We also would like thank participants of the Cryptocurrency Research Conference 2019 at the University of Southampton for their helpful comments and suggestions on the earlier version of the work.

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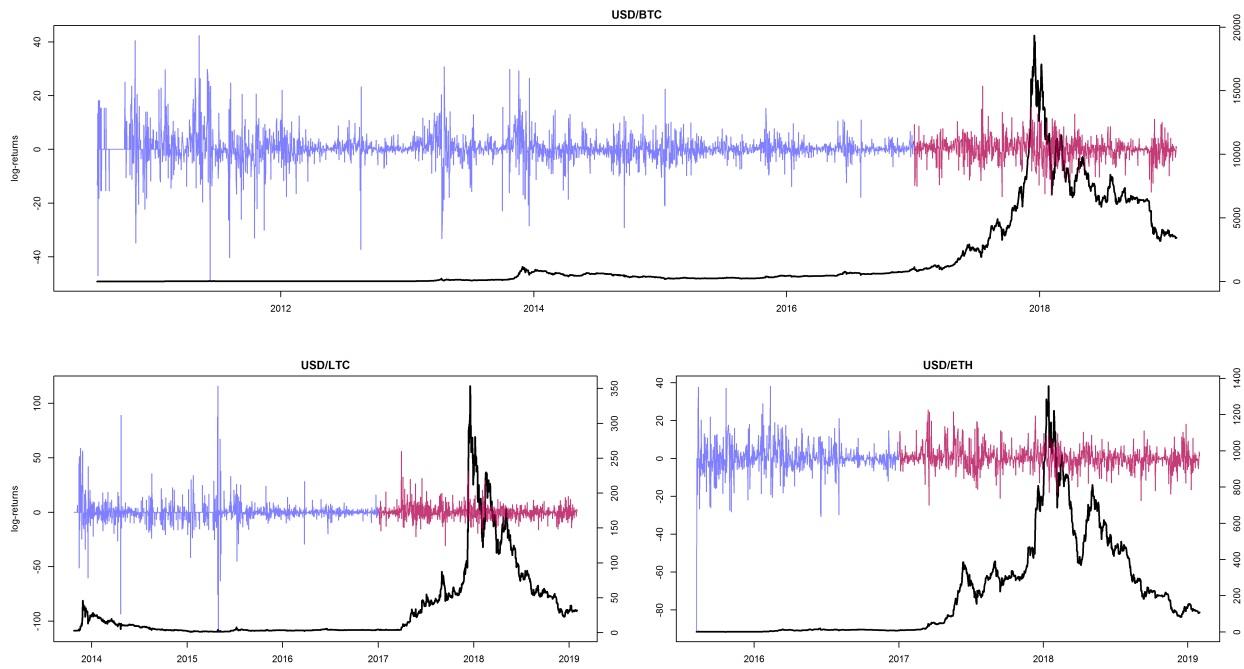
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Table 1: Descriptive Statistics for the Specified Log-returns.

sample	date	log-returns	T	Mean	St.Dev.	Skew.	Kurt.	LB(12)	LB ² (12)	AH(12)
training	18.07.2010-13.04.2013	BTC/USD	1000	0.6941	7.6671	-0.4954	11.7097	0.0205	0.0000	0.0000
	24.10.2013-20.07.2016	LTC/USD	1000	0.0322	11.9282	0.4896	34.0967	0.0000	0.0000	0.0000
	07.08.2015-19.12.2016	ETH/USD	550	0.2435	9.0502	-1.7315	24.0564	0.0033	0.0349	0.0000
testing	14.04.2013-31.01.2019	BTC/USD	2119	0.1706	4.5922	-0.0195	10.3352	0.0000	0.0000	0.0000
	21.07.2016-31.01.2019	LTC/USD	925	0.2209	6.4906	1.5119	13.4090	0.0107	0.0000	0.0000
	20.12.2016-31.01.2019	ETH/USD	723	0.3116	6.4064	0.2657	4.9713	0.1034	0.0000	0.0000

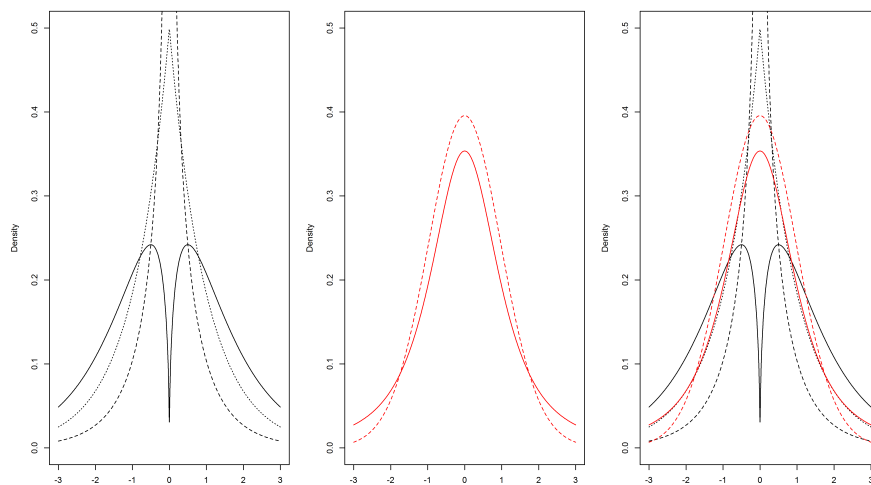
Notes: LB(12) and LB²(12) are the 12th order Ljung-Box no serial correlation probabilities in log-returns and squared demeaned log-returns respectively; AH(12) is 12th order Lagrange Multiplier no autocorrelation, normality, and homoscedasticity probabilities.

Figure 1: BTC/USD, LTC/USD and ETH/USD Exchange Rates and Corresponding Log-returns.



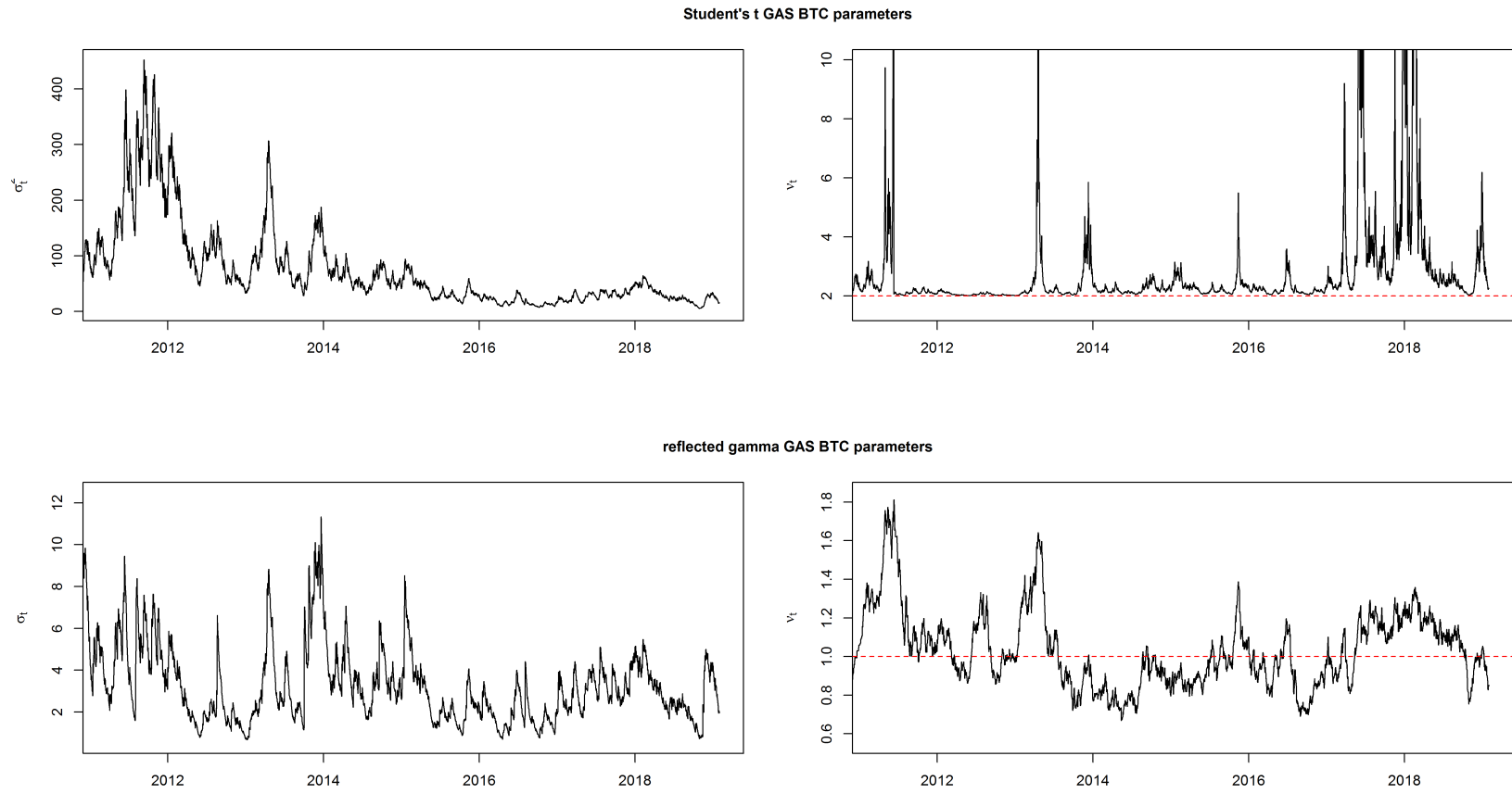
Notes: Log-returns from the beginning of 2017 are highlighted in red.

Figure 2: Reflected Gamma and Student's t PDFs Illustration for Different Shape Parameters.



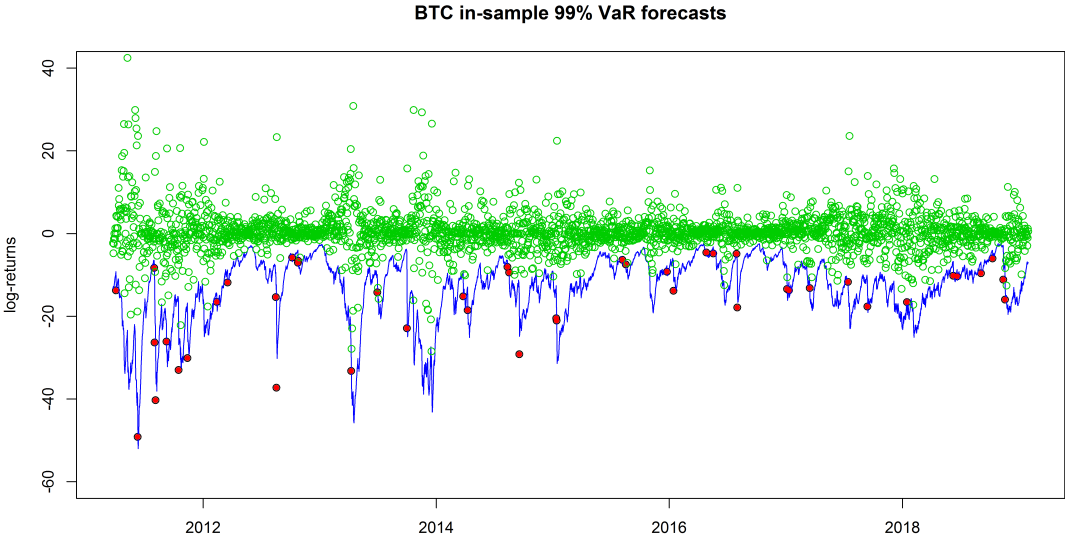
Notes from left to right: reflected gamma PDF with $\nu = 0.5, 1, 1.5$; Student's t PDF with $\nu = 2, 30$; reflected gamma PDF and Student's t PDF together.

Figure 3: In-sample Time-varying Parameters of Reflected Gamma and Student's t Distributions for BTC.



Notes: dashed horizontal lines indicate $\nu = 2$ for Student's t and $\nu = 1$ for reflected gamma respectively.

Figure 4: Reflected Gamma In-sample VaR Estimates.



Notes: 99% VaR in-sample violations are highlighted in red.

Table 2: VaR Backtesting Results: 99.5% Level.

VaR level	Approach	BTC							LTC							ETH						
		LRuc _d	LRuc _p	LRcc _d	LRcc _p	DQ _d	DQ _p	AE	LRuc _d	LRuc _p	LRcc _d	LRcc _p	DQ _d	DQ _p	AE	LRuc _d	LRuc _p	LRcc _d	LRcc _p	DQ _d	DQ _p	AE
99.5%	<i>t</i> -GAS(5)	42.6153	0.0000	44.3787	0.0000	129.1057	0.0000	3.5866	10.1967	0.0014	10.5678	0.0051	37.6194	0.0000	2.8108	7.6367	0.0057	10.0236	0.0067	58.7092	0.0000	2.7663
	<i>t</i> -GAS(ν)	1.6292	0.2018	4.4358	0.1088	35.6611	0.0000	1.4158	0.0890	0.7655	0.1238	0.9400	4.3328	0.7407	0.8649	1.3179	0.2510	5.7370	0.0568	45.2161	0.0000	1.6598
	<i>t</i> -GAS(ν_t)	0.6981	0.4034	6.0039	0.0497	51.8777	0.0000	0.7551	4.2013	0.0404	4.2035	0.1222	3.4347	0.8421	0.2162	0.4761	0.4902	0.5459	0.7611	27.1461	0.0003	1.3831
	L-GAS	15.9840	0.0001	16.9430	0.0002	57.3190	0.0000	2.4540	3.2543	0.0712	3.4313	0.1798	16.1188	0.0241	1.9459	2.4974	0.1140	2.6345	0.2679	23.5661	0.0014	1.9364
	L-GAS(p)	32.7373	0.0000	33.0524	0.0000	84.9182	0.0000	3.2091	3.2543	0.0712	3.4313	0.1798	16.0896	0.0243	1.9459	3.9664	0.0464	4.1457	0.1258	30.0041	0.0001	2.2130
	L-GAS(p_t)	0.0154	0.9013	0.1302	0.9370	0.2867	0.9999	1.0382	0.0890	0.7655	0.1238	0.9400	4.0199	0.7775	0.8649	0.0398	0.8418	0.0844	0.9587	2.0712	0.9558	1.1065
	RM	53.3523	0.0000	53.3851	0.0000	130.4122	0.0000	3.9641	3.2543	0.0712	3.4313	0.1798	18.2734	0.0108	1.9459	12.1237	0.0005	12.5293	0.0019	59.0638	0.0000	3.3195
	D-GAS	1.6292	0.2018	1.8432	0.3979	23.9153	0.0012	1.4158	0.0298	0.8630	0.0842	0.9588	5.2619	0.6280	1.0811	1.3179	0.2510	1.4185	0.4920	15.3020	0.0323	1.6598
	D-GAS (w)	0.1795	0.6718	0.3163	0.8537	15.1539	0.0341	1.1326	0.6557	0.4181	0.6752	0.7135	5.4624	0.6037	0.6486	3.9664	0.0464	4.1457	0.1258	27.1988	0.0003	2.2130
	G-GAS(ν)	3.2857	0.0699	3.5608	0.1686	27.6175	0.0003	1.6045	1.0582	0.3036	1.1651	0.5585	9.6376	0.2101	1.5135	2.4974	0.1140	2.6345	0.2679	21.8796	0.0027	1.9364
	G-GAS(ν_t)	5.4177	0.0199	5.7617	0.0561	31.6495	0.0000	1.7933	2.0298	0.1542	2.1696	0.3380	12.6240	0.0818	1.7297	1.3179	0.2510	1.4185	0.4920	15.5801	0.0292	1.6598
	kCDF	25.9516	0.0000	26.4594	0.0000	86.9111	0.0000	2.9259	3.2543	0.0712	3.4313	0.1798	20.2179	0.0051	1.9459	3.9664	0.0464	4.1457	0.1258	25.3632	0.0007	2.2130

Notes: LRuc_d, LRuc_p, LRcc_d and LRcc_p are likelihood ratio statistic and probabilities for Kupiec (1995) and Christoffersen (1998) tests respectively, while DQ_d and DQ_p outline associated regression output of Engle & Manganelli's (2004) test. AE outlines standard actual/expected number of violations ratio. Tests' probabilities exceeding standard 5% backtesting confidence threshold are highlighted in grey.

Table 3: VaR Backtesting Results: 99% Level.

VaR level	Approach	BTC							LTC							ETH						
		LRuc _d	LRuc _p	LRcc _d	LRcc _p	DQ _d	DQ _p	AE	LRuc _d	LRuc _p	LRcc _d	LRcc _p	DQ _d	DQ _p	AE	LRuc _d	LRuc _p	LRcc _d	LRcc _p	DQ _d	DQ _p	AE
99%	<i>t</i> -GAS(5)	35.9250	0.0000	39.4803	0.0000	121.9983	0.0000	2.5484	4.0848	0.0433	5.2741	0.0716	24.5046	0.0009	1.7297	5.0271	0.0250	6.2574	0.0438	35.6679	0.0000	1.9364
	<i>t</i> -GAS(ν)	3.2771	0.0703	6.4949	0.0389	54.5064	0.0000	1.4158	0.0598	0.8067	0.2787	0.8699	9.3982	0.2253	1.0811	0.9577	0.3278	3.3445	0.1878	25.9731	0.0005	1.3831
	<i>t</i> -GAS(ν_t)	1.4027	0.2363	9.3344	0.0094	32.1818	0.0000	0.7551	3.8234	0.0505	3.8582	0.1453	3.7423	0.8089	0.4324	0.2244	0.6357	4.6434	0.0981	35.9044	0.0000	0.8299
	L-GAS	17.4672	0.0000	18.5906	0.0001	53.5391	0.0000	2.0293	0.7551	0.3849	1.0709	0.5854	8.7970	0.2676	1.2973	1.7123	0.1907	3.7523	0.1532	27.1497	0.0003	1.5214
	L-GAS(p)	25.2206	0.0000	25.8682	0.0000	73.2828	0.0000	2.2652	0.7551	0.3849	1.0709	0.5854	8.8721	0.2620	1.2973	3.7611	0.0525	5.2281	0.0732	36.8344	0.0000	1.7981
	L-GAS(p_t)	0.8976	0.3434	1.1727	0.5563	1.3698	0.9865	0.8023	0.0069	0.9339	0.1839	0.9121	2.4085	0.9338	0.9730	0.7789	0.3775	0.8487	0.6542	1.4990	0.9823	0.6916
	RM	26.9028	0.0000	26.9200	0.0000	61.0309	0.0000	2.3124	0.3153	0.5744	0.5804	0.7481	8.2100	0.3144	1.1892	9.6631	0.0019	10.4830	0.0053	47.2609	0.0000	2.3513
	D-GAS	0.3609	0.5480	0.9111	0.6341	14.3079	0.0460	1.1326	0.6035	0.4372	0.7104	0.7010	3.3766	0.8481	0.7568	0.4061	0.5239	0.6333	0.7286	13.0473	0.0710	1.2448
	D-GAS (w)	0.6542	0.4186	1.7262	0.4218	26.3462	0.0004	1.1798	2.3678	0.1239	2.4222	0.2979	4.9428	0.6669	0.5405	0.0801	0.7772	0.2593	0.8784	11.1420	0.1325	1.1065
	G-GAS(ν)	1.0285	0.3105	1.6748	0.4328	18.1186	0.0114	1.2270	0.0069	0.9339	0.1839	0.9121	5.4229	0.6085	0.9730	0.4061	0.5239	0.6333	0.7286	12.7644	0.0781	1.2448
	G-GAS(ν_t)	0.3609	0.5480	0.9111	0.6341	15.5639	0.0294	1.1326	0.0598	0.8067	0.2787	0.8699	7.6388	0.3655	1.0811	0.4061	0.5239	0.6333	0.7286	12.5996	0.0825	1.2448
	kCDF	18.9270	0.0000	18.9352	0.0001	49.9084	0.0000	2.0765	2.1288	0.1446	2.5596	0.2781	19.7606	0.0061	1.5135	3.7611	0.0525	4.2379	0.1202	23.3552	0.0015	1.7981
eCDF	23.5810	0.0000	26.1772	0.0000	71.9639	0.0000	2.2180	3.0390	0.0813	3.5341	0.1708	25.8975	0.0005	1.6216	6.4390	0.0112	7.0755	0.0291	36.9904	0.0000	2.0747	

Table 4: VaR Backtesting Results: 97.5% Level.

VaR level	Approach	BTC							LTC							ETH						
		LRuc _d	LRuc _p	LRcc _d	LRcc _p	DQ _d	DQ _p	AE	LRuc _d	LRuc _p	LRcc _d	LRcc _p	DQ _d	DQ _p	AE	LRuc _d	LRuc _p	LRcc _d	LRcc _p	DQ _d	DQ _p	AE
97.5%	<i>t</i> -GAS(5)	18.8328	0.0000	27.5977	0.0000	70.2020	0.0000	1.6423	1.9195	0.1659	2.8201	0.2441	17.0593	0.0170	1.2973	6.7529	0.0094	6.8096	0.0332	42.6748	0.0000	1.6598
	<i>t</i> -GAS(ν)	2.2090	0.1372	4.0321	0.1332	36.7795	0.0000	1.2081	0.4535	0.5007	3.7017	0.1571	19.1895	0.0076	0.8649	1.8091	0.1786	1.8601	0.3945	34.2983	0.0000	1.3278
	<i>t</i> -GAS(ν_t)	1.2966	0.2548	4.2646	0.1186	20.4879	0.0046	0.8495	8.0659	0.0045	8.3309	0.0155	8.4554	0.2942	0.4757	1.6174	0.2035	3.0844	0.2139	13.3249	0.0646	0.7192
	L-GAS	3.0505	0.0807	6.3269	0.0423	21.2816	0.0034	1.2459	0.4535	0.5007	1.0274	0.5983	9.0903	0.2462	0.8649	0.4618	0.4968	0.6846	0.7101	18.7388	0.0090	1.1618
	L-GAS(p)	9.8242	0.0017	14.7481	0.0006	31.7607	0.0000	1.4535	0.0570	0.8112	0.4194	0.8108	7.4732	0.3813	0.9514	3.1449	0.0762	3.1495	0.2071	28.3277	0.0002	1.4385
	L-GAS(p_t)	0.7180	0.3968	0.7199	0.6977	4.5958	0.7092	0.8872	3.3371	0.0677	4.7289	0.0940	6.1790	0.5190	0.6486	1.6174	0.2035	3.0844	0.2139	4.8037	0.6839	0.7192
	RM	11.4204	0.0007	15.8754	0.0004	31.6099	0.0000	1.4913	0.4535	0.5007	1.0274	0.5983	5.3239	0.6205	0.8649	6.7529	0.0094	8.7384	0.0127	37.3707	0.0000	1.6598
	D-GAS	0.0202	0.8869	0.2824	0.8683	10.6150	0.1563	1.0193	2.5197	0.1124	3.0836	0.2140	8.7016	0.2748	0.6919	0.0478	0.8270	0.4617	0.7939	18.3742	0.0104	1.0512
	D-GAS (w)	0.0784	0.7794	1.4823	0.4766	15.9231	0.0258	1.0382	4.2900	0.0383	4.7208	0.0944	11.3911	0.1224	0.6054	0.4618	0.4968	0.6846	0.7101	12.5775	0.0831	1.1618
	G-GAS(ν)	0.0765	0.7822	0.1238	0.9400	13.4682	0.0615	0.9627	0.4535	0.5007	1.0274	0.5983	9.0450	0.2494	0.8649	0.0669	0.7959	0.8868	0.6418	14.2830	0.0464	0.9405
	G-GAS(ν_t)	0.0000	0.9972	0.0920	0.9551	12.2984	0.0912	1.0005	0.4535	0.5007	1.0274	0.5983	9.3417	0.2290	0.8649	0.0003	0.9857	0.9209	0.6310	15.5535	0.0295	0.9959
	kCDF	4.0170	0.0450	5.3304	0.0696	31.4882	0.0001	1.2836	0.9887	0.3201	1.0159	0.6017	7.2598	0.4023	1.2108	3.1449	0.0762	3.1495	0.2071	14.6842	0.0403	1.4385
	eCDF	8.3358	0.0039	11.7473	0.0028	42.5498	0.0000	1.4158	0.9887	0.3201	1.0159	0.6017	8.0664	0.3268	1.2108	5.7406	0.0166	5.7671	0.0559	20.4051	0.0048	1.6044

Table 5: VaR Backtesting Results: 95% Level.

VaR level	Approach	BTC							LTC							ETH						
		LRuc _d	LRuc _p	LRcc _d	LRcc _p	DQ _d	DQ _p	AE	LRuc _d	LRuc _p	LRcc _d	LRcc _p	DQ _d	DQ _p	AE	LRuc _d	LRuc _p	LRcc _d	LRcc _p	DQ _d	DQ _p	AE
95%	<i>t</i> -GAS(5)	11.7217	0.0006	17.3663	0.0002	34.9348	0.0000	1.3403	0.7247	0.3946	0.7267	0.6953	11.6392	0.1131	1.1243	7.2740	0.0070	7.6164	0.0222	28.1210	0.0002	1.4661
	<i>t</i> -GAS(ν)	5.3759	0.0204	12.7111	0.0017	32.6440	0.0000	1.2270	0.4236	0.5151	2.3863	0.3033	14.0784	0.0498	0.9081	6.4796	0.0109	6.9280	0.0313	32.8349	0.0000	1.4385
	<i>t</i> -GAS(ν_t)	6.1591	0.0131	14.1651	0.0008	31.1613	0.0001	0.7739	13.5703	0.0002	13.7746	0.0010	13.1274	0.0691	0.5189	0.0209	0.8851	0.5801	0.7482	10.3622	0.1690	1.0235
	L-GAS	2.4454	0.1179	12.3101	0.0021	28.0000	0.0002	1.1515	2.5800	0.1082	4.0998	0.1287	8.2594	0.3103	0.7784	3.1445	0.0762	4.3553	0.1133	24.9369	0.0008	1.3001
	L-GAS(p)	8.8031	0.0030	15.7688	0.0004	35.1157	0.0000	1.2931	0.6512	0.4197	1.3643	0.5055	9.4892	0.2194	0.8865	5.0158	0.0251	5.7216	0.0572	23.3858	0.0015	1.3831
	L-GAS(p_t)	3.7895	0.0516	8.3188	0.0156	19.6083	0.0065	0.8211	4.4202	0.0355	6.5997	0.0369	7.2038	0.4080	0.7135	0.5208	0.4705	0.7549	0.6856	5.7920	0.5642	0.8852
	RM	0.0415	0.8386	8.5401	0.0140	21.2523	0.0034	1.0193	1.6451	0.1996	2.8009	0.2465	6.5857	0.4733	0.8216	2.1232	0.1451	3.7541	0.1530	20.6964	0.0042	1.2448
	D-GAS	0.4838	0.4867	7.4744	0.0238	22.2333	0.0023	1.0665	5.9585	0.0146	8.6614	0.0132	15.7068	0.0279	0.6703	0.6577	0.4174	0.8594	0.6507	23.2937	0.0015	1.1342
	D-GAS (w)	0.6289	0.4277	7.3387	0.0255	40.2225	0.0000	1.0760	1.2607	0.2615	2.2556	0.3237	10.9283	0.1418	0.8432	0.0007	0.9796	0.4583	0.7952	15.1663	0.0339	0.9959
	G-GAS(ν)	0.0380	0.8454	4.1970	0.1226	18.1159	0.0115	0.9816	3.7463	0.0529	5.6899	0.0581	8.9274	0.2579	0.7351	0.0209	0.8851	0.0271	0.9865	21.7389	0.0028	1.0235
	G-GAS(ν_t)	0.0380	0.8454	4.1970	0.1226	15.1080	0.0346	0.9816	3.7463	0.0529	5.6899	0.0581	9.0044	0.2523	0.7351	0.0209	0.8851	0.0271	0.9865	21.7652	0.0028	1.0235
	kCDF	0.0109	0.9168	11.0032	0.0041	25.8269	0.0005	1.0099	0.6512	0.4197	0.6703	0.7152	23.7487	0.0013	0.8865	1.6832	0.1945	1.7247	0.4222	12.0083	0.1003	1.2172
	eCDF	1.1751	0.2784	8.8257	0.0121	25.3553	0.0007	1.1043	0.0359	0.8498	0.0549	0.9729	23.0638	0.0017	0.9730	3.7239	0.0536	4.7501	0.0930	24.5046	0.0009	1.3278

Table 6: Superior Set of Models and Their Ranks as Provided by the MCS Procedure.

Approach	99.5% confidence panel			99% confidence panel			97.5% confidence panel			95% confidence panel		
	BTC	LTC	ETH	BTC	LTC	ETH	BTC	LTC	ETH	BTC	LTC	ETH
t -GAS(5)	11	8	8	-	9	8	-	5	7	-	-	-
t -GAS(ν)	7	-	-	-	-	-	-	-	9	-	-	-
t -GAS(ν_t)	2	6	3	2	3	4	-	2	2	-	1	-
L-GAS	5	3	7	7	7	9	-	8	8	-	-	-
L-GAS(p)	9	2	6	8	5	7	-	4	6	-	-	-
L-GAS(p_t)	1	1	1	1	1	1	1	1	1	1	-	1
RM	12	5	5	5	2	5	-	3	3	-	-	-
D-GAS	8	-	-	10	-	-	-	-	-	-	-	-
D-GAS(w)	10	-	2	6	-	6	-	-	10	-	-	-
G-GAS(ν)	4	4	-	4	8	-	-	-	-	-	-	-
G-GAS(ν_t)	3	9	-	3	10	-	-	-	-	-	-	-
kCDF	6	7	4	9	4	2	-	7	4	-	-	-
eCDF	-	-	-	-	6	3	-	6	5	-	-	-

Figure 5: BTC, LTC and ETH VaR Forecasts Spreads at the Specified Confidence Levels.

