

Handbook of Production Economics: Survey of Applications

Applications of Production Economics in Education

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Abstract

This chapter provides a comprehensive survey of the existing literature on production economics from the education perspective, bringing together findings from the education costs, production and efficiency contexts, and relating to all levels of education including primary, secondary (both compulsory and non-compulsory) and higher education.

Keywords: Education; higher education; costs; production; economies; elasticities; efficiency

JEL classification: D24; I20; I21; I23; L25

1. Introduction

This chapter focuses on production economics in the context of the education sector. This sector makes an interesting case study because of its particular characteristics, which derive from the fact that returns to education can be both private (accruing to the individual in terms of higher salaries) and social (accruing to society in terms of increased productivity and economic growth). As a consequence of the benefits accruing to society as a whole from individuals being educated, education institutions are often publicly funded, although the extent of the public funding likely varies by level of education (and country). The public funding aspect of education affects costs, production and efficiency in that sector, and these are all relevant in the production economics context.

It is worth considering two broad components of education which we will term: a) higher or tertiary education, encompassing non-compulsory education for post-18 year olds often in universities; and b) education, encompassing primary and secondary education. The latter is largely compulsory, at least up to the age of around 16 years, particularly in developed countries, and predominantly publicly funded. The former is not compulsory, but is also in receipt of substantial public funding, particularly in developed countries, since there are still considered to be some benefits of higher education accruing to society (as well as the individual). In addition to these categories, education can be provided at many levels to adults (typically 25 years plus). Adult education is provided very differently from the traditional primary, secondary and higher education levels, with provision often being in the form of modules offered through a blended or distance medium (Eurydice 2018). Given the frequent lack of data on adult education provision, there will be only little reference to this particular sector in the paper below.

To illustrate the importance of education, in 2015, across all OECD countries the average spending on education institutions across the spectrum of education levels is some 5% of GDP, with a variation from around 3% to 6%. On average in the OECD, the majority of around 70% of the spending on education is to non-tertiary education institutions (as might be expected), and this is equivalent to 3.5% of GDP, with a variation of 3% to 4.5% (see *Education at a Glance 2018* https://read.oecd-ilibrary.org/education/education-at-a-glance-2018_eag-2018-en#page260, accessed June 14th 2019). The largely publicly funded nature of organisations in the education and higher education sectors therefore makes this an interesting focus in the context of production economics. Such organisations are generally non-profit making. Yet the amount of public funds received by schools and universities, and the role these institutions play in driving growth in the economy, make it imperative for them to be run efficiently and effectively. An empirical knowledge of concepts in production such as the size of economies of scale or scope, efficiency levels, and possibilities for substitution between inputs (or, indeed, between outputs) are all important in the education context.

While the education and higher education sectors of many countries comprise largely publicly funded institutions, privately funded institutions also exist, to a greater or lesser extent, at all levels

of education. The focus of this chapter is generally on the non-profit, largely publicly funded provision, but private sector examples will be reviewed as appropriate.

This chapter is in six sections of which this introduction is the first. Section 2 focuses on cost functions in education including concepts, estimation, and findings from the literature. The section ends with some recent developments, policy implications and suggestions for future work. Output distance functions are the subject of section 3, which examines concepts, estimation and findings from the literature before concluding with policy implications and possible topics for future exploration. Section 4 turns to efficiency and productivity change including concepts, findings from the literature, recent developments, policy implications and future work. Level of analysis is the focus of section 5 which examines and reviews the literature on various possibilities including individuals, funding areas, and countries. Final conclusions are drawn in section 6, which also suggests areas for future applications of production economics in the education and higher education contexts.

2. Cost functions, economies of scale and scope

Schools and higher education institutions (HEIs) are multi-product organisations. School pupils, for example, are taught, and attain qualifications in, multiple subjects in schools; universities produce outcomes from teaching, research and third mission activities. This leads to the estimation of multi-product cost functions (Baumol *et al.* 1982) in the education and higher education contexts, and permits the testing of a number of key production economics concepts such as:

- Existence or otherwise of economies of scale
- Existence or otherwise of economies of scope
- Extent of substitution possibilities through evaluating elasticities of substitution

Full details on the theory underpinning cost functions and economies of scale can be found in chapters 16 and 17 (respectively) of volume I of this publication. Each of these concepts will be considered briefly in the context of the empirical literature in this section.

2.1 Background on cost concepts

In a multi-product production situation such as we have in education and higher education, the cost relationship is

$$C(y) = f(y; p) \tag{1}$$

Where:

y is the vector of outputs;

p is the vector of input prices.

In order to estimate this function empirically, the researcher must select a functional form which should:

- Be consistent with cost minimisation given outputs and input costs i.e. it must be a non-negative and non-decreasing function.
- Provide predictions of costs when the value of one or more outputs is zero. This is particularly needed in order to derive estimates of economies of scale and scope, and precludes cost functions in logarithms such as the Cobb-Douglas.
- Allow for the existence of scale or scope economies or diseconomies, without enforcing their existence.

Functional forms which fulfil these criteria and which have been used in empirical studies include the cross elasticity of substitution, quadratic, and hybrid translog. Each has been used in empirical studies, and has various advantages and disadvantages in terms of estimation, a brief overview of which can be found in Johnes *et al.* (2005), while a detailed comparison of the merits of the translog over the Cobb-Douglas can be found in Gronberg *et al.* (2011).

In this multi-product case there are two concepts relating to economies of scale (Johnes, J 2020). Ray economies of scale are the savings in costs occurring when all outputs increase (while holding the output mix constant). Product-specific economies of scale are the cost savings which occur when *one* output increases and all other outputs remain at fixed production levels (Johnes *et al.* 2008b). If we assume that we have k inputs ($k = 1, \dots, K$) and m outputs ($m = 1, \dots, M$), these concepts can be denoted for the general case as follows:

$$S_R = \frac{C(y)}{\sum_m y_m C_m(y)} \quad (2)$$

Where:

S_R represents ray economies of scale;

y_m is the m th output;

$C_m(y) = \partial C(y)/\partial y_m$ is the marginal cost of producing the m th output.

$$S_m(y) = AIC(y_m)/C_m(y) \quad (3)$$

Where:

$S_m(y)$ denotes product specific economies relating to product m (where $m = 1, \dots, M$);

$AIC(y_m) = [C(y_M) - C(y_{M-m})]/y_m$;

$C(y_M)$ is the total cost of producing all M outputs;

$C(y_{M-m})$ is the total cost of producing all M outputs except output m .

Values above (below) 1 indicate the presence of economies (diseconomies) of scale in the estimated long run cost equation. Evaluating these measures can be useful from a policy viewpoint in determining, for example, whether an expansion in provision is best effected through increasing

the size of existing providers (schools or universities) or, if diseconomies of scale are observed in the sector, by introducing entirely new providers.

Economies of scope, in contrast, occur when it is less costly to produce a number of outputs together rather than to produce each output independently in its own specialist production unit (Johnes, J 2020). As with economies of scale, in this multi-product case we have two concepts relating to economies of scope. Global economies of scope occur when the costs of producing all outputs together in a single firm are less than the sum of the costs of producing each output in a separate firm. Product-specific economies of scope for product m arise when the costs of producing all outputs together in a single firm are less than the sum of costs of producing output m in a separate firm and all outputs apart from m in another firm (Johnes *et al.* 2008b). These can be denoted in the general case as follows:

$$S_G = [\sum_m C(y_m) - C(y)]/C(y) \quad (4)$$

Where:

S_G denotes global economies of scope;

$C(y_m)$ is the cost of producing output m .

$$SC_m = [C(y_m) + C(y_{M-m}) - C(y)]/C(y) \quad (5)$$

Where:

SC_m denotes product-specific economics of scope for output m .

Economies of scope can arise if it is possible to spread the costs of central services across an array of outputs. It is likely that both schools and HEIs benefit from producing their outputs in one production unit as they may be able to spread the costs of capital and administration across their different outputs whether it is teaching across different disciplines (schools and universities), or teaching and research (universities). The degree of scope economies in universities depends on the extent to which the products (for example, research and teaching) are produced jointly as opposed to separately, and this issue is considered further below in the context of the empirical literature. The empirical evaluation of these measures can provide useful policy and managerial insights into the degree to which organisations should become more (or less) specialised in the outputs produced. In the higher education case, for example, economies of scope can indicate whether HEIs should be research-focused or teaching-focused, or even whether they should specialise in a specific discipline (such as arts or medicine).

2.2 Estimating cost functions in education and higher education: challenges and methodology

Knowing the parameters of the estimated cost function in an education context can clearly offer useful insights to managers and policy makers alike. But the implementation of the cost function methodology in education is not easy, and decisions regarding specification (of costs, outputs and functional form, for example) and estimation approach in particular can potentially affect

outcomes and conclusions drawn from any cost function analysis. The first major challenge in estimating education cost functions is identifying what is meant by 'costs'. Costs (or expenditure) can be allocated to various categories. For universities these might include administration versus academic expenditure; research versus teaching expenditures; recurrent versus capital expenditures. For schools these might be instructional and non-instructional expenditures, or total fee revenue. Many empirical studies are interested in total recurrent expenditure and this is the typical definition of costs. There are, however, exceptions. Some studies, for example, have focussed specifically on administration (rather than total recurrent) costs in the context of universities (Coelli *et al.* 2005; Casu and Thanassoulis 2006).

When estimating cost functions, there is an underlying assumption that education providers are seeking to minimize their costs. Given that such organisations are typically in receipt of public funds, this assumption is open to debate. Indeed, an early examination of costs of universities in the USA suggests that universities do not minimize costs but rather spend all the income they receive (Bowen 1981). This view is challenged (Lloyd *et al.* 1993) on the premise that providers with diverse sources of funding (such as universities) are more likely to adopt optimising behaviour than when receiving all funding from the public purse. More recently, the marketization of, and increasing competition in, higher education sectors across the world following the global financial crash and subsequent constraints on public funding have put increasing pressure on higher education providers to minimize their costs. Similar pressures can also be seen in the education sector where policies to increase competition amongst publicly funded schools in some countries would also lead to increasing cost minimization behaviour.

While the outputs of schools and HEIs may seem obvious, their precise measurement is not so clear-cut. Schools often use test or examination performance, or graduate numbers (by subject) to reflect teaching outputs and their quality (Bee and Dolton 1985; Gyimah-Brempong and Gyapong 1992; Bates 1993; Duncombe *et al.* 1995; James *et al.* 1996; Bates 1997; Mancebon and Bandrés 1999; Ruggiero and Vitaliano 1999; McEwan and Carnoy 2000; Mante 2001; Bowles and Bosworth 2002; Mante and O'Brien 2002; Ruggiero 2007; Burney *et al.* 2013; Chakraborty and Blackburn 2013). Where this is not available, student enrolment numbers might be substituted (Riew 1966; Kumar 1983; Jimenez 1986; Riew 1986; Callan and Santerre 1990; Jimenez and Paqueo 1996; Ray and Mukherjee 1998; Smet and Nonneman 1998; Mancebón and Mar Molinero 2000; Smet 2001; Banker *et al.* 2004; Zimmer and Buddin 2009; Burney *et al.* 2013). But the disadvantage of enrolment figures is that they fail to reflect output quality. Since schools' outputs are affected by their environment, the background of the pupils who attend the school, and the quality of teachers, as well as other contextual variables relating to pupils, families, school or the school location are often added to the cost equation to take these factors into account (Bee and Dolton 1985; Dougherty 1990; Barrow 1991; Gyimah-Brempong and Gyapong 1992; Duncombe *et al.* 1995; James *et al.* 1996; Jimenez and Paqueo 1996; Bowles and Bosworth 2002; Zimmer *et al.* 2009; Gronberg *et al.* 2011; Gronberg *et al.* 2012; Chakraborty and Blackburn 2013).

Universities produce outputs which can be categorised as teaching, research or third mission. Student numbers are commonly used to reflect teaching outputs (Beasley 1990; Johnes 1990; Ahn

and Seiford 1993; Beasley 1995; Hashimoto and Cohn 1997; Avkiran 2001; Abbott and Doucouliagos 2003; Johnes *et al.* 2005; Stevens 2005; Johnes *et al.* 2008b; Worthington and Lee 2008; Johnes and Johnes 2009; Margaritis and Smart 2011; Thanassoulis *et al.* 2011; Nemoto and Furumatsu 2014), often categorised by level (undergraduate or postgraduate, for example) and broad disciplines (such as science, non-science and medicine), but various problems arise not least of which is the issue of quality of teaching output. Graduate numbers have been used in preference to student numbers in order to try to capture quality (Athanasopoulos and Shale 1997), but this ignores the quality of degrees obtained by different graduates. Quality is addressed in various ways including adding variables to reflect 'quality' to the cost equation such as average entry qualifications of intake or a value-added measure (Verry and Davies 1976; Johnes *et al.* 2005; Stevens 2005; Johnes *et al.* 2008b).

These outputs can be seen as the short-term outcomes of higher education. Long term benefits from taking a higher degree might be measured using labour market metrics such as numbers of graduates achieving a job or graduates' starting salary (Agasisti 2011b; Kong and Fu 2012; Johnes 2013; Bogetoft *et al.* 2015; Lee and Johnes 2019)

Measuring research is also problematic. Nationally organised research rating exercises (such as the Research Excellence Framework in the UK) have measures of both quantity and quality (Glass *et al.* 2006), but these are available only at intervals and therefore not always a reflection of current position. Citations and publications counts can also be used (as in, for example, Johnes and Johnes 1993; De Witte and Hudrlikova 2013; Nazarko and Šaparauskas 2014) but can be difficult to obtain and may not reflect the current output. As a consequence, many studies resort to input measures, such as competitively won grant income, rather than output measures, in an effort to capture quality and quantity of current activity.

Outcomes from third mission activities are the most difficult to measure in the higher education context with many empirical studies not even attempting it, although its omission will inevitably lead to problems of bias in the estimated cost function (some exceptions include Johnes *et al.* 2005; Johnes *et al.* 2008b; Johnes and Johnes 2009; Thanassoulis *et al.* 2011, where university income from other services rendered is included to reflect third mission activities).

Input prices should also be included in the cost function if these vary across production units, with many studies including the price of capital and/or the price of labour (Cohn *et al.* 1989; Glass *et al.* 1995a; 1995b; Longlong *et al.* 2009). Average salary is commonly used to reflect the price of labour, but this, of course, may be more of a reflection of the distribution of staff across grades in their organisation than an indication of the price of labour. If prices are not known precisely but are known to vary by broad geographical location of the university, a location indicator can be incorporated in the function as a proxy (Johnes *et al.* 2005; Johnes *et al.* 2008b; Johnes and Johnes 2009; Thanassoulis *et al.* 2011). Capturing price variations is typically likely to be more important in inter-country rather than within-country studies.

Different organisations have different missions or objectives. In the higher education sector, some universities might choose to focus more on, for example, research as opposed to teaching, and vice versa. In the education context, some countries distinguish between schools with vocational as opposed to academic routes. An underlying assumption when estimating a cost function for an industry is that the firms within it all have similar objectives (Getz *et al.* 1991). If this assumption does not hold, then cost functions might be estimated separately for different mission groups (Cohn *et al.* 1989; Johnes *et al.* 2005; Johnes *et al.* 2008b; Johnes and Johnes 2009; Thanassoulis *et al.* 2011; Zhang and Worthington 2017; Zhang *et al.* 2017). Recent advances in estimation techniques also provide alternative approaches for this situation (see section 2.5 below).

A full and detailed examination of all these issues can be found in Johnes *et al.* (2005)

In practice, the translog cost functional form provides further opportunities for estimating additional quantities of interest namely elasticities of substitution. We can write the translog cost function as (Gyimah-Brempong and Gyapong 1992)

$$\ln C = \delta_0 + \sum_{m=1}^M \delta_m \ln y_m + \frac{1}{2} \sum_{m=1}^M \sum_{n=1}^M \delta_{mn} \ln y_m \ln y_n + \sum_{k=1}^K \mu_k \ln P_k + \frac{1}{2} \sum_{k=1}^K \sum_{l=1}^K \mu_{kl} \ln P_k \ln P_l + \sum_{m=1}^M \sum_{k=1}^K \gamma_{mk} \ln y_m \ln P_k + \varepsilon \quad (6)$$

We assume the following conditions:

a) Linear homogeneity of degree +1 in input prices

$$\sum_{k=1}^K \mu_k = 1 \quad (7a)$$

$$\sum_{l=1}^K \mu_{kl} = \sum_{k=1}^K \gamma_{mk} = 0 \quad (7b)$$

b) Symmetry

$$\delta_{mn} = \delta_{nm} \quad (8a)$$

$$\mu_{kl} = \mu_{lk} \quad (8b)$$

We can derive a set of input share equations (S_k) from equation (6) as follows:

$$\frac{\partial \ln C}{\partial \ln P_k} = S_k = \mu_k + \sum_{l=1}^K \mu_{kl} \ln P_l + \sum_{m=1}^M \gamma_{mk} \ln y_m \quad (9)$$

Two possible measures of elasticity are the Allen and Morishima elasticities of substitution, respectively, which can be estimated from this set of equations. The Allen elasticity of substitution measures the impact of a change in the price of the k th input on the demand for the l th input with output held constant. This is estimated in this cost function context (denoted by superscript C) by

$$A_{kl}^C = (\mu_{kl} + S_k S_l) / S_k S_l \text{ for } k \neq l \quad (10a)$$

$$A_{kl}^C = (\mu_{kl} + S_k (S_k - 1)) / S_k^2 \text{ for } k = l \quad (10b)$$

A positive (negative) value suggests that inputs k and l are substitutes (complements). The Morishima elasticity of substitution can be estimated in this cost function context (denoted by superscript C) by:

$$M_{kl}^C = S_l(A_{kl}^C - A_{ll}^C) \quad (11)$$

The two measures are different when there are more than two inputs and the production technology is represented by the translog as here (Gyimah-Brempong and Gyapong 1992). The Allen elasticities are symmetric whereas the Morishima elasticities are not. As such, the Allen and Morishima elasticities of substitution may not provide consistent conclusions (see Gyimah-Brempong and Gyapong 1992 for more details).

2.3 Estimation approach: SFA versus DEA

The idea that production might vary by mission group leads on to a more general concern regarding production and technology and their implications for estimating empirical cost functions. Clearly concepts such as scale economies require estimation of a long run cost function. The challenge for empirical researchers is that they must assume that their sample of production units (schools or HEIs, for example) are operating on the long run cost function in the time period under study (Getz *et al.* 1991). In reality, this is unlikely to be the case and the observations will be a mix of those operating in a long run equilibrium, those operating in a short run equilibrium, and those in either a short run or a long run position not operating efficiently. This might be a particularly pertinent consideration if the sector under study is going through a period of rapid change, in which case organisations may be in various short run equilibria as they move towards their long run positions (Brinkman and Leslie 1986).

Some of the earliest empirical cost functions in higher education are estimated using ordinary least squares (OLS) and a linear functional form (Verry and Layard 1975; Verry and Davies 1976); the latter therefore largely precludes the existence of economies of scale and scope, and the former simply estimates a line of best fit through all the data regardless of position (short run versus long run; efficient versus inefficient). The seminal work of Cohn *et al.* (1989) incorporates the multi-product nature of production drawing on the work of (Baumol *et al.* 1982), but the estimation method (OLS applied to cross section data) still does not address the issue of observations at different production points. This is also the case for many subsequent studies (see section 2.4 below).

Early school cost functions use OLS and a quadratic function, permitting estimation of optimum size and scale economies, but precluding economies of scope (Osburn 1970; Bee and Dolton 1985; Butler and Monk 1985). The multi-product nature of school production is recognised in later studies by using a translog functional form, with applications in Bolivia, the USA and Flanders (Jimenez 1986; Callan and Santerre 1990; Smet and Nonneman 1998). But the estimation methods do not allow for inefficiencies or data points being at different production points (short run versus long run).

There has been a growing recognition that using methods which preclude the possibility of inefficiency is a problem (Costrell *et al.* 2008), especially as the non-profit nature of education does not naturally provide the incentives for efficiency which prevail in a private sector setting. The more widespread availability of frontier estimation methods which allow for inefficient operations, however, has led to frontier estimation methods increasingly being the customary approach when estimating empirical cost functions in education. A cost function estimated using frontier techniques envelopes the data; thus its position is determined by the outermost data points which are, in turn, likely to be those in a long run equilibrium and/or most efficient in the sector.

Frontier estimation techniques can be parametric, such as the family of estimation methods falling under the umbrella of stochastic frontier analysis (SFA) (Aigner *et al.* 1977), or non-parametric, with data envelopment analysis (DEA) being a common approach in this context (Charnes *et al.* 1978; 1979). Full details of SFA and DEA can be found in chapters 12 to 14 of volume I of this publication.

SFA assumes an error comprising two components – one a normally-distributed random error, and the other a one-sided term, often following a half-normal or exponential distribution, and attributed to inefficiency. The basic SFA model produces estimates of the cost function parameters which are identical for all organisations in the data set. The advantage of the approach is that the significance of the parameters can be tested (Schmidt 1985-1986; Cohn and Rossmiller 1987), and they can be used to produce estimates of economies of scale and scope as well as elasticities of substitution. As a consequence, SFA has been used in many empirical cost function studies (see section 2.4).

DEA is a non-parametric approach often used to derive estimates of organisations' efficiencies, and makes no assumptions regarding functional form. This means that there are no problems with misspecification. Moreover, the linear programming method of estimation means that DEA easily accommodates multiple inputs and multiple outputs (Mancebon and Bandrés 1999; Mante 2001). Furthermore, it provides weights of inputs and outputs which vary by each organisation (or decision making unit – DMU) in the sample, and which maximise the efficiency score subject to weights being positive and universal (Coelli *et al.* 2005). The inter-institutional variation in weights can be particularly advantageous in the context of education and higher education where we have noted that mission can vary by production unit. DEA can also provide useful benchmarking information for managers to help them improve performance. But estimates of economies of scale and scope, and of elasticities of substitution, are more difficult to derive in the non-parametric context (see Thanassoulis *et al.* 2011 for an example).

In the context of cost functions where there is a single input (expenditure) and multiple outputs, a parametric technique is typically preferred over a non-parametric one, although there are exceptions as will be discussed in section 2.4 below. As already noted, however, expenditure itself can be divided into different categories, thereby leading to a multi-input multi-output situation. DEA can therefore be advantageous where the underlying cost components are known and of

interest (Ahn *et al.* 1989; Beasley 1990; Ahn and Seiford 1993; Beasley 1995; Athanassopoulos and Shale 1997).

2.4 Findings from the literature

We will focus in this section largely on literature which incorporates the multi-product nature of education into the estimated cost functions (Baumol *et al.* 1982). This literature can be divided into parametric and non-parametric studies, and the former can be further divided into those using frontier estimation methods, and those not doing so.

Parametric, non-frontier estimation

The school context is complex with some studies examining scale at a funding area level (such as school districts in the USA, or local education authority in the UK), others looking at the school level, and others still considering both. There is evidence of scale economies in funding areas (Butler and Monk 1985; Andrews *et al.* 2002) and in funding areas up to a certain size (Duncombe *et al.* 1995; Zimmer *et al.* 2009); and a study of only large school districts finds no evidence of scale economies (Robertson 2007). There is also evidence of scale economies in schools (Bee and Dolton 1985; Jimenez 1986; Dougherty 1990; Bowles and Bosworth 2002). When the two are considered together (schools and funding districts), economies of scale are found in schools (Lewis and Chakraborty 1996), and in both schools and school funding areas (Chakraborty *et al.* 2000). In a rare study of secondary schools which incorporates multiple outputs (relating to education clusters), ray economies of scale are observed along with product-specific economies in 6 of the 7 fields considered, and also global economies of scope (Smet and Nonneman 1998). Economies of scale are confirmed in a study of school districts which are assumed to produce two outputs, namely primary and secondary education, but in this case there are no economies of scope (Callan and Santerre 1990), and this aligns with findings from an earlier multi-product cost study by Jimenez (1986).

In the context of higher education, and across various developed countries, cost functions estimated using parametric non-frontier estimation methods tend to find that there are ray economies of scale, but that the evidence on economies of scope is more mixed (Cohn *et al.* 1989; de Groot *et al.* 1991; Dundar and Lewis 1995; Glass *et al.* 1995a; 1995b; Koshal and Koshal 1999; 2000; 2001; Sav 2011; Worthington and Higgs 2011).

Parametric, frontier estimation

Much of the literature on estimates of scale economies in the context of schools is based on non-frontier estimation. There are some exceptions where both non-frontier and frontier methods are applied (Duncombe *et al.* 1995), and these find that the coefficients for the frontier model are similar to those estimated using non-frontier methods. More recently, Gronberg *et al.* (2015) use SFA in the school context to investigate the economies deriving from consolidation of school districts in Texas. They find that economies of scale can be gained from consolidating very small school districts (producing primary and secondary education). Other studies which use frontier

methods have largely focused on efficiency aspects rather than scale economies, and so will be reviewed in section 4.

There are many studies of economies of scale and scope in higher education using parametric, frontier estimation methods to estimate the multi-product cost function (for example, Johnes 1996; Izadi *et al.* 2002; Abbott and Doucouliagos 2003; Johnes *et al.* 2005; Stevens 2005; Johnes *et al.* 2008b; Agasisti and Johnes 2009b; Johnes and Johnes 2009; Thanassoulis *et al.* 2011; Johnes and Johnes 2013; Nemoto and Furumatsu 2014; Agasisti 2016; Johnes and Johnes 2016). While there is some variation in findings, typically the studies using these methods find that ray economies of scale are exhausted; an exception relates to Japanese private universities where many HEIs enjoy economies of scale (Nemoto and Furumatsu 2014). Some also find there are product-specific scale economies relating to research and/or postgraduate outputs (Johnes and Salas Velasco 2007; Agasisti and Johnes 2010; Johnes and Schwarzenberger 2011; Nemoto and Furumatsu 2014; Agasisti and Johnes 2015). Most of the evidence on scope economies, however, points to diseconomies of scope both globally and (where calculated) for individual products (Johnes, G 1997; Izadi *et al.* 2002; Johnes *et al.* 2005; Johnes *et al.* 2008b; Johnes and Johnes 2009; Nemoto and Furumatsu 2014).

Non-parametric frontier estimation

The need for cost function parameter estimates with which to derive measures of scale and scope economies means that cost function studies using non-parametric frontier estimation methods typically examine efficiency rather than the specific cost function concepts discussed above. As such, a review of these studies will be presented in section 4. An exception is Thanassoulis *et al.* (2011) who examine costs in English higher education and find opportunities for expanding student numbers are possible through currently unexploited scale and scope economies (Thanassoulis *et al.* 2011; Thanassoulis *et al.* 2016).

Summary

Aside from the fact that the findings provide some mixed messages, there are some additional caveats. In the context of schools, most studies do not include externality costs of increasing school size. As a school increases its size, for example, student discipline issues increasingly arise and the crime and violence which this may engender impose external costs on pupils, families and society more generally, which are not taken into account in a standard school cost function (Ferris and West 2004). This suggests that care should be taken when interpreting the results of standard cost functions. An area of future research which would be particularly useful to managers and policy makers alike would therefore revolve around developing empirical cost functions for schools which incorporate these externalities.

Another caveat is that if recommendations regarding scale economies are derived from cost functions which inadequately measure quality of outputs, there may be a detrimental effect on pupil outcomes from increasing school size. The relationship between size and outcomes can be

examined more closely using a production function approach and we will consider this further in section 3, while the issue of increasing school size and pupil outcomes is investigated in depth by Schiltz and De Witte (2017). Caveats regarding inadequate measurement of quality apply equally in the higher education context.

2.5 Recent developments in estimating cost functions

Education providers, whether at primary, secondary, or tertiary levels, can vary widely in terms of, for example, their mission, size and history. Such diversity, if not taken into account, can potentially affect parameter estimates and hence the estimated economies of scale and scope. Inclusion in the cost function of contextual variables to reflect defined characteristics, such as mission or region, is one approach to addressing the diversity issue (Zhang and Worthington 2017; Zhang *et al.* 2017); another approach is to estimate cost functions within pre-defined groups based on perceptions about what characteristics ought to affect cost function parameters. These characteristics might be type of institution such as public or private (Cohn *et al.* 1989; James *et al.* 1996), or mission group (Johnes *et al.* 2005; Johnes *et al.* 2008b; Johnes and Johnes 2009; Thanassoulis *et al.* 2011). Such an approach can identify differences in estimated parameters across the specified groups.

Thus, known characteristics of institutions affect costs, but there may also be unobserved characteristics which also affect costs. Random parameter (RP) SFA (Tsionas 2002; Greene 2005), and latent class (LC) SFA (Lazarsfeld and Henry 1968; Orea and Kumbhakar 2004) allow both observable and unobservable characteristics to be taken into account in the estimation of parameters, and efficiency scores – explored further in section 4 (Johnes and Johnes 2016). In particular, RP SFA, which requires panel data for estimation, allows parameters to vary by each individual provider, while LC SFA, which can be applied to cross section data, permits parameters to be derived for groups of HEIs – however the groups are not pre-defined by the analyst but rather they are determined by the data. These methods not only lead to different parameters across (groups of) institutions, but also allow the calculation of scale and scope economies by individual provider or by group. Studies adopting RP SFA have found evidence that ray economies of scale are typically exhausted or decreasing, and diseconomies of scope are observed. There are some product-specific economies of scale, but these vary from study to study (Johnes and Salas Velasco 2007; Johnes *et al.* 2008b; Johnes and Johnes 2009; Agasisti and Johnes 2010; Johnes and Schwarzenberger 2011; Agasisti and Johnes 2015). When LC SFA methods are used, the findings on economies of scale and scope vary from group to group (Johnes and Johnes 2013; Johnes and Johnes 2016). Whilst the LC approach is attractive in defining groups based on the data, the disadvantage is that the results can be difficult to interpret if the composition of the resulting groups does not align with any obvious patterns.

2.6 Policy implications and future work

Empirical estimations regarding economies of scale and scope whether at school or university level are policy relevant as they can feed into considerations regarding potential consolidation – this

might be at funding unit level (such as districts) or at organisation level. Thus policies to amalgamate schools or to merge universities can be developed from such empirical work.

However, the findings reported above at both school and university levels are often mixed or conflicting, and it is difficult to develop coherent policies on such a basis. Future work which examines the reasons why findings on economies of scale and scope vary by type of organisation might therefore help to better inform policy (Hemelt *et al.* 2018). A useful contribution to the literature undertakes a meta-analysis of cost function studies to identify reasons for the mixed findings in the higher education context (Zhang and Worthington 2018). It seems that estimates of scale efficiency vary according to model specification and functional form assumed, and whether or not managerial efficiency is taken into account (a quadratic cost function in particular seems to lead to a conclusion of diseconomies of scale, as does a model which accounts for inefficiency). Estimates of scope efficiency, are affected not just by model specification but also by period covered by the study, sample size and type of data. In particular, estimates derived from older, cross section data from small or developing country samples of universities are likely to lead to the conclusion that scope economies exist (Zhang and Worthington 2018). At school level, a meta-regression analysis of optimum school size based on 10 studies with 22 estimates finds the optimum school size to be around 1543 pupils (Colegrave and Giles 2008). These studies provide useful insights, and much more work of this kind would be welcome, particularly at the school level.

A less explored area of research concerns the derivation of elasticities from parametric cost functions. Both Allen and Morishima elasticities (defined in section 2.2 above) can provide useful details about substitution possibilities between inputs. One example in higher education can be found (Worthington and Higgs 2008) and suggests that within universities it is easier to switch into capital inputs than into academic or non-academic labour; indeed the substitution possibilities between the two types of labour seem limited. Examples from the schools context suggest that instructional, support and administrative inputs are generally substitutable (Jimenez 1986; Callan and Santerre 1990; Gyimah-Brempong and Gyapong 1992), although Allen elasticities imply less ability to substitute between the non-instructional input and others (Gyimah-Brempong and Gyapong 1992). Much more work could be undertaken in this context to provide useful policy insights.

3. Production functions, distance function, shadow prices and elasticities

Concepts relating to the multi-product nature of education and higher education can also be examined in a production function context. Given both the multi-input and multi-output nature of production, estimation of production-related concepts leads to the output distance function approach which has numerous advantages: it does not assume any particular optimizing behaviour on the part of the firms, which is an advantage in the non-profit context in which schools and universities often operate; it does not require a knowledge of prices of either inputs or outputs, the latter being particularly useful in education and higher education where teaching outputs, for example, are difficult to value; and it does not require prices to be exogenous (Coelli and Perelman

1999; Coelli 2000; Uri 2003). In the context of education and higher education, the concepts of interest in the production setting include

- Existence or otherwise of returns to scale
- Existence or otherwise of returns to scope
- Extent of substitution possibilities in the production relationship through evaluation of elasticities of substitution (between inputs)
- Extent of complementarity or substitutability between the outputs through evaluation of elasticities of substitution (between outputs)

Full details of the theory underpinning production and the related concepts (including elasticities) can be found in chapters 3 and 22 of volume I of this publication. Each of these concepts will be considered briefly in the context of the empirical literature in this section.

3.1 Background on production concepts

We assume that schools or universities produce multiple outputs from a variety of inputs. Let us assume, as in section 2, that providers – be they schools or HEIs – use a vector of inputs $x \in \mathbb{R}_+^K$ to produce a vector of outputs $y \in \mathbb{R}_+^M$. We assume that providers focus on producing outputs relative to given inputs (an output-oriented approach), and hence define the production technology for a provider as

$$P(x) = \{y \in \mathbb{R}_+^M : x \text{ can produce } y\} \quad (12)$$

Where:

y is already defined;

x is the vector of inputs.

The output distance function (Shephard 1970), denoted by $D(x, y)$, is defined on the output set $P(x)$ as:

$$D(x, y) = \min_{\theta} \{\theta : (y/\theta) \in P(x)\} \quad (13)$$

The output distance function is non-decreasing, convex, positively linearly homogeneous of degree +1, and can be used to derive shadow prices and substitution properties. We define shadow prices of inputs as

$$\partial D(x, y) / \partial x_k \quad (14)$$

The marginal rate of technical substitution between inputs k and l ($MRTS_{kl}$), reflects the slope of the isoquant, provides a measure of substitutability between inputs k and l , and is derived from the ratio of input shadow prices:

$$MRTS_{kl} = \frac{\partial D(x,y)/\partial x_k}{\partial D(x,y)/\partial x_l} \quad (15)$$

This statistic is affected by the units in which inputs are measured, and so it is conventional to calculate a normalized $MRTS_{kl}$:

$$sub_{kl} = \frac{\partial D(x,y)/\partial x_k}{\partial D(x,y)/\partial x_l} \cdot \frac{x_k}{x_l} \quad (16)$$

If $sub_{kl} > 1$ ($sub_{kl} < 1$) it is difficult (easy) to substitute out of input k into input l (Paul *et al.* 2002). An alternative measure of substitutability is the Allen elasticity defined in this output distance function context as

$$A_{kl}(x, y) = \frac{D(x,y)D_{kl}(x,y)}{D_k(x,y)D_l(x,y)} \quad (17)$$

If $A_{kl}(x, y) > 0$ (< 0) it is difficult (easy) to substitute between the two inputs. When the number of inputs exceeds two, there are many directions in which the curvature of the isoquant can be measured, and thus sub_{kl} and $A_{kl}(x, y)$ can be unsatisfactory in reflecting substitutability in this situation. The Morishima elasticity of substitution appears to be a more satisfactory measure of substitutability in the multiple input case (Blackorby and Russell 1989). The (indirect) Morishima elasticity of substitution is defined in this output distance function context as (Paul *et al.* 2002):

$$M_{kl}(x, y) = -\frac{d \ln[D_k(x,y)/D_l(x,y)]}{d \ln[x_k/x_l]} = x_k \frac{D_{kl}(x,y)}{D_l(x,y)} - x_l \frac{D_{lk}(x,y)}{D_k(x,y)} \quad (18)$$

This gives the percentage change in the slope of the MRTS brought about by a percentage change in ratio of inputs. If x_k and x_l are highly substitutable values will be small (less than or equal to zero); and the elasticity rises if substitutability possibilities between the inputs x_k and x_l are limited. The Morishima elasticity (in contrast to the Allen elasticity) is asymmetric such that $M_{kl}(x, y)$ will not normally equal $M_{lk}(x, y)$ (Grosskopf *et al.* 1995).

3.2 Estimating distance functions in education and higher education: challenges and methodology

Many of the challenges of empirical estimation in the production context are the same as those already discussed in section 2.2 in the costs context and will not be discussed further here. In addition, just as there are problems in specifying output measures in education and higher education, there are also challenges in identifying satisfactory measures of inputs to the education production process.

Inputs to schools are broadly defined as labour, capital and pupils. Inputs to higher education are similarly defined as labour, capital and students. Labour inputs can be categorised by type of labour (teaching or support), and might be standardised by number of students to give a staff to student ratio (Breu and Raab 1994; McMillan and Datta 1998; Ray and Jeon 2008; Ouellette and Vierstraete 2010; Zoghbi *et al.* 2013). Staff quality might be incorporated by using qualifications,

experience indicators or salary data (Sengupta and Sfeir 1986; Ahn *et al.* 1989; Sinuany-Stern *et al.* 1994; Chalos and Cherian 1995; Gronberg *et al.* 2012; Johnson and Ruggiero 2014). Capital normally encompasses buildings, computers and library facilities (Ahn *et al.* 1989; Ahn and Seiford 1993; Arcelus and Coleman 1997; Johnes 2008; 2014b). Measuring pupil or student input is possibly the biggest challenge as pupils vary by ability, socio-economic, personal and family circumstances, and these can all affect their success in school or higher education, and so should be incorporated in some way. Finally, the environment in which a provider is located can impact its outcomes, and so variables to reflect the economic characteristics of the location are typically included, particularly at school level (see De Witte and López-Torres 2017 for a full overview of all possible inputs).

It is worth noting that providers, particularly in education where catchment areas are relatively local, have varying amounts of control over the different inputs. Thus, they can certainly control how much labour or capital is employed, but they cannot control the economic or demographic characteristics of the environment in which they operate. It is therefore important to take account of these non-discretionary inputs as they might impact the estimated parameters and hence measures of returns to scale and scope.

The distance function assumes constant technology across all providers. As in the case of costs, differences in mission or other potentially relevant attributes (such as size) can be accommodated in various ways, the most straightforward one being inclusion of additional variables to reflect these differences.

Perhaps the main challenge in the production context is the estimation of the output distance function. If we assume that the outputs in the multi-output, multi-input case are entirely separately produced, then we can undertake estimation separately for each output. This means that for each output, estimation of the distance function simply reverts to estimation of the production function in a single output production context. This is the approach taken in many studies of production in the education context (Sengupta and Sfeir 1986; Cooper and Cohn 1997; McEwan and Carnoy 2000; Kang and Greene 2002; Adkins and Moomaw 2003; Engle *et al.* 2007; Conroy and Arguea 2008) and in early production studies in the higher education context (Johnes and Taylor 1989b; 1989a; 1989c; 1990b; 1990a; Johnes, J 1997). This approach, however, does not capture the obvious ‘jointness’ of production potentially observed in education and higher education, for example, academic staff in universities undertaking research may also feed into their research-led teaching (De Witte *et al.* 2013), and therefore precludes an examination of substitutability (Chizmar and Zak 1983; Chizmar and McCarney 1984; Chizmar and Zak 1984; Gyimah-Brempong and Gyapong 1991).

The multi-input, multi-output nature of production raises potential difficulties of estimation. The non-parametric approach underpinned by linear programming techniques, can accommodate both multiple inputs and multiple outputs but the derivation of the concepts of interest (such as elasticities or returns to scale or scope) can be more difficult. The ease of application, however,

makes the non-parametric approach a popular choice in the estimation of education and higher education distance functions with multiple inputs and multiple outputs.

The parametric approach allows for stochastic errors, provides estimates from which returns to scale and scope and elasticities can be derived, but the estimation is more problematic as it is typical to have a single output related to multiple inputs, or a single input (often costs as explored in section 2.1 above) related to multiple outputs. As a consequence, there is limited application of parametric methods to the empirical estimation of multi-input multi-output distance functions in education, although there is a growing literature. The disadvantages of the parametric approach are that results may be sensitive to the choices of functional form and error distribution.

The practical application of a parametric distance function requires a functional form that is a) flexible; b) easy to estimate; and c) permit the imposition of homogeneity (Coelli and Perelman 2000). The translog fulfils all three criteria and is the functional form of choice in many education applications. We assume as before that providers use inputs x_k ($k = 1, \dots, K$) to produce outputs y_m ($m = 1, \dots, M$). The translog distance function is defined as

$$\ln D(x, y) = \alpha_0 + \sum_{m=1}^M \alpha_m \ln y_m + \frac{1}{2} \sum_{m=1}^M \sum_{n=1}^M \alpha_{mn} \ln y_m \ln y_n + \sum_{k=1}^K \beta_k \ln x_k + \frac{1}{2} \sum_{k=1}^K \sum_{l=1}^K \beta_{kl} \ln x_k \ln x_l + \sum_{k=1}^K \sum_{m=1}^M \delta_{km} \ln x_k \ln y_m + \varepsilon \quad (19)$$

Distance function restrictions require that the following conditions hold:

a) Homogeneity of degree +1 in outputs

$$\sum_{m=1}^M \alpha_m = 1 \quad \text{and} \quad (20a)$$

$$\sum_{n=1}^M \alpha_{mn} = 0 \quad m = 1, 2, \dots, M \quad \text{and} \quad (20b)$$

$$\sum_{m=1}^M \delta_{km} = 0 \quad k = 1, 2, \dots, K \quad (20c)$$

b) Symmetry:

$$\alpha_{mn} = \alpha_{nm} \quad m, n = 1, 2, \dots, M \quad \text{and} \quad (21a)$$

$$\beta_{kl} = \beta_{lk} \quad k, l = 1, 2, \dots, K \quad (21b)$$

The homogeneity in outputs restriction means that $D(x, \omega y) = \omega D(x, y)$ and hence the M th output can be chosen arbitrarily such that $\omega = 1/y_M$. Thus equation (19) can be written as:

$$-\ln y_M = \alpha_0 + \sum_{m=1}^{M-1} \alpha_m \ln \left(\frac{y_m}{y_M} \right) + \frac{1}{2} \sum_{m=1}^{M-1} \sum_{n=1}^{M-1} \alpha_{mn} \ln \left(\frac{y_m}{y_M} \right) \ln \left(\frac{y_n}{y_M} \right) + \sum_{k=1}^K \beta_k \ln x_k + \frac{1}{2} \sum_{k=1}^K \sum_{l=1}^K \beta_{kl} \ln x_k \ln x_l + \sum_{k=1}^K \sum_{m=1}^{M-1} \delta_{km} \ln x_k \ln \left(\frac{y_m}{y_M} \right) + \varepsilon \quad (22)$$

Where the error $\varepsilon = -\ln D(x, y)$

The output distance function can be used to provide estimates of returns to scale as follows. The output elasticity for input k is

$$-\varepsilon_{D,x_k} = -\partial \ln D / \partial \ln x_k = \partial \ln y_M / \partial \ln x_k = \varepsilon_{y_M,x_k} \quad (23)$$

and measures the percentage change in output y_M if x_m changes by 1 percent, holding the output ratios constant (Paul and Nehring 2005).

If we sum these output elasticities over k we get

$$\varepsilon_{y_M,x} = \sum_{k=1}^K \varepsilon_{y_M,x_k} \quad (24)$$

which is analogous to a returns to scale estimate. If $\varepsilon_{y,x} > 1$ we have increasing returns to scale as a 1% increase in x_k results in a more than 1% increase in output expansion (with proportional changes in all outputs) (Paul and Nehring 2005).

3.3 Findings from the literature

Much of the literature regarding output distance functions focuses on deriving estimates of efficiency (the subject of section 4) rather than on the production concepts referred to in this section. Early efforts to model the higher education production function in a framework where multiple outputs are produced from multiple inputs employ canonical correlation estimation methods and find a degree of substitutability between inputs based on data on individual university students (Chizmar and Zak 1983; 1984). Most recently a SFA tranlog output distance function suggests that returns to scale appear to be exhausted across the English higher education sector. Based on estimates of Allen and Morishima elasticities, substitution is difficult between academic and non-academic staff (a similar result to the schools context reported in section 2.5), and much easier between academic staff and capital inputs (Johnes 2014a).

Returns to scale can also be established using the non-parametric DEA approach. Where this has been used in the higher education context, the findings generally point to the prevalence of constant or decreasing returns to scale (Bayraktar *et al.* 2013; Johnes 2014a; Clermont *et al.* 2015).

In the context of schools, the parametric approach taken has been to estimate a single output production functions with multiple inputs typically to have a better understanding of returns to scale and optimal school size. The literature is somewhat simplistic in its approach with a surprisingly limited focus on functional form (Andrews *et al.* 2002) especially compared to cost function studies (at school and higher education level) and production function studies in the higher education context. The output measures used are largely based on average test score, and efficiency is typically not included (exceptions are Deller and Rudnicki 1993; Lee and Smith 1997). The limited evidence from this arena suggests that returns are more often constant or decreasing (Summers and Wolfe 1977; Fetler 1989; Fowler and Walberg 1991; Deller and Rudnicki 1993) than increasing (Kenny 1982; Ferguson 1991).

3.4 Policy implications and future work

The work on elasticities of substitution reveals some interesting differences in terms of opportunities to substitute between inputs between HEIs which subsequently merge, those which do not merge at all, and post-merger institutions mergers (Johnes 2014a). Greatest opportunities for substitution are generally observed for HEIs which will subsequently merge. Institutional merger is sometimes considered as a policy initiative by governments (Cai and Yang (2016) summarise merger activity across countries), and so this observation is important as it suggests that institutions which do not have the appropriate initial characteristics prior to merger may not reap the potential rewards (see section 4.4 for more on the efficiency effects of mergers in higher education). More work is needed to investigate these findings further and to confirm whether initial characteristics of providers are indeed important in determining success following merger.

4. Efficiency, productivity change and analyses of factors underlying efficiency

A by-product of the frontier estimation techniques applied in the costs or production contexts is that they also lead to the derivation of measures of efficiency for providers in the sample. By choosing a frontier estimation method, the researcher is therefore also able to undertake a detailed examination of efficiency and, if panel data is available, productivity of organisations. Such analysis is particularly important in the education and higher education contexts where the non-profit nature of the sector makes traditional financial ratios inappropriate for performance measurement (Berkner 1966), but yet the public-funding aspect makes it crucial to understand that resources are being used efficiently. The interest in efficiency, and the availability of a wealth of data on inputs and outputs, in education and higher education sectors around the world has led to a large literature on education efficiency and productivity, a review of which can be found in Johnes, G (2020).

Efficiency should not be confused with effectiveness: the latter relates to doing the right things – in education it means having the right quantity of outputs – while the former relates to doing things right – in education it means using scarce resources to produce the highest possible outputs (Førsund 2017; Cherchye *et al.* 2019). Typically, efficiency receives the greater attention in the literature, and this will be the focus of this section. It should be noted, however, that one novel publication looks at, distinguishes, and provides comparative measures of both concepts (efficiency and effectiveness), with an application in the secondary schooling context, and this will be reviewed further below (Cherchye *et al.* 2019)

4.1 Background on efficiency concepts

Efficiency work is rooted in the seminal contribution of (Farrell 1957), and the two main approaches used to derive and examine efficiency are SFA and DEA (already discussed in section 2). These methods can be used to derive various measures of efficiency based on cost (or input distance) functions and output distance functions (Jondrow *et al.* 1982). From a cost point of view,

the parametric measure of efficiency is derived from the error term of, for example, equation (6), i.e. as $\varepsilon = v + u$ where v is a stochastic error and u is the one-sided efficiency term. In the production context, the parametric estimate of efficiency is derived from, for example, equation (22). The distance measure, $\ln D(x, y)$, is the quantity of interest in equation (19) as this provides a measure of efficiency, and this is derived from the error term in equation (22), which is typically assumed to be split into two components i.e. $\varepsilon = v - u$ where v is a stochastic error and u is the one-sided efficiency term.

The non-parametric measure of efficiency is often derived from the DEA approach such that $D(x, y)$ is defined as (Charnes *et al* 1978; 1979):

$$D(x, y) = \frac{\sum_{m=1}^M a_m y_m}{\sum_{k=1}^K b_k x_k} \quad (25)$$

where y_m and x_k are as already defined; a_m is the weight applied to output m and b_k is the weight applied to input k . For each DMU, the weights are found by maximizing efficiency subject to the constraints that weights must be non-zero and universal. DEA can be applied in the context of constant returns to scale (CRS) or variable returns to scale (VRS). A DMU is fully efficient if $D(x, y) = 1$.

In establishing the efficiency of an organisation, we therefore examine its observed production/costs relative to best practice in the entire industry. As such, the frontier methodology provides a benchmark which an inefficient provider can use to help it to become more efficient, and ultimately to move on to the best practice frontier.

When we have a panel of data, bringing in a time dimension (denoted by t and by $t + 1$), we are able to perform an analysis of productivity change which can be measured using the Malmquist productivity index (Malmquist 1953), developed by Caves *et al.* (1982) and further by Färe *et al.* (1994) is derived as follows for the output distance function (where superscripts and superscripts denote the time period of the distance function):

$$M(x_{t+1}, y_{t+1}, x_t, y_t) = \left[\left(\frac{D^t(x_{t+1}, y_{t+1})}{D^t(x_t, y_t)} \right) \left(\frac{D^{t+1}(x_{t+1}, y_{t+1})}{D^{t+1}(x_t, y_t)} \right) \right]^{1/2} \quad (26)$$

Notation is as defined earlier, and $D^t(x_{t+1}, y_{t+1})$ denotes the distance of the period $t+1$ observation from the period t frontier. If the Malmquist productivity change index exceeds unity, there has been an improvement in productivity between periods t and $t+1$. Values less than 1 suggest the converse.

The change in the production position of a provider over the two time periods has two underlying determinants: first, the provider can produce more because the output distance frontier for the sector has moved outwards, and therefore the potential for production across all providers is expanded; second, the provider's position relative to the time-relevant frontier can change. The

Malmquist productivity index can be decomposed two components as follows (Färe *et al* 1989, 1992):

$$M(x_{t+1}, y_{t+1}, x_t, y_t) = \left(\frac{D^{t+1}(x_{t+1}, y_{t+1})}{D^t(x_t, y_t)} \right) \left[\left(\frac{D^t(x_{t+1}, y_{t+1})}{D^{t+1}(x_{t+1}, y_{t+1})} \right) \left(\frac{D^t(x_t, y_t)}{D^{t+1}(x_t, y_t)} \right) \right]^{1/2} \quad (27)$$

The first component, $\left(\frac{D^{t+1}(x_{t+1}, y_{t+1})}{D^t(x_t, y_t)} \right)$, measures the change in technical efficiency over the two periods (i.e. whether or not the unit is getting closer to its efficiency frontier over time), and the second component, $\left[\left(\frac{D^t(x_{t+1}, y_{t+1})}{D^{t+1}(x_{t+1}, y_{t+1})} \right) \left(\frac{D^t(x_t, y_t)}{D^{t+1}(x_t, y_t)} \right) \right]^{1/2}$, measures the change in technology over the two time periods (i.e. whether or not the frontier is shifting out over time). Values of either of these components of greater (less) than unity suggest improvement (deterioration) in the measure.

4.2 Findings from the literature

Efficiency

There is a huge literature reporting findings on efficiency in both education and higher education, and various reviews can be found (see, for example, Bradley *et al.* 2001; Worthington 2001; Johnes 2004; 2015; Thanassoulis *et al.* 2016; Johnes and Johnes 2019) including a particularly detailed one (De Witte and López-Torres 2017). This section provides a brief overview of that literature.

In the context of schools and further education institutions, and taking a production perspective, mean efficiency varies from just under 0.6 to well over 0.9 using parametric estimation methods (Deller and Rudnicki 1993; Cooper and Cohn 1997; Grosskopf *et al.* 1997; Chakraborty *et al.* 2001; Grosskopf *et al.* 2001; Kang and Greene 2002; Mizala *et al.* 2002; Smith and Street 2006; Kirjavainen 2007; Conroy and Arguea 2008; Fieger *et al.* 2016). A similar spread of mean efficiency scores is observed when using non-parametric methods (Bessent and Bessent 1980; Bessent *et al.* 1982; Färe *et al.* 1989; Ray 1991; Bonesrønning and Rattsø 1994; Chalos and Cherian 1995; Ruggiero *et al.* 1995; Chalos 1997; Kirjavainen and Loikkanen 1998; Noulas and Ketkar 1998; Mancebón and Mar Molinero 2000; Ruggiero 2000; Bradley *et al.* 2001; Ramanathan 2001; Mizala *et al.* 2002; Muñiz 2002; Chakraborty 2003; Gstach *et al.* 2003; Borge and Naper 2005; Oliveira and Santos 2005; Afonso and St. Aubyn 2006; Kantabutra and Tang 2006; Primont and Domazlicky 2006; Smith and Street 2006; Rassouli-Currier 2007; Mancebón and Muñiz 2008; Tyagi *et al.* 2009; Al-Enezi *et al.* 2010; Bradley *et al.* 2010; Haelermans and De Witte 2012; Johnes *et al.* 2012; Mancebón *et al.* 2012; Portela *et al.* 2012; Burney *et al.* 2013; Haelermans and Ruggiero 2013; Thieme *et al.* 2013; Harrison and Rouse 2014; Podinovski *et al.* 2014). For most of these studies, values are typically at the higher end of the range, but depend on model specification, context of the sample, type of schools, for example, public or private), and (in the case of DEA, whether constant or variable returns to scale are assumed, with the latter providing higher mean estimates. An exception to these studies is in the context of Australian schools

(Chakraborty and Blackburn 2013) where mean efficiency is around 0.4 for primary schools and 0.5 for secondary schools – these results are discussed further later in this section.

When a costs perspective is taken, mean efficiency is found to be relatively high with a range of 0.83 to 0.96 using parametric methods (Barrow 1991; Ruggiero and Vitaliano 1999; Gronberg *et al.* 2012), and 0.664 to 0.95 using non-parametric methods (Bates 1993; 1997; Mancebon and Bandrés 1999; Ruggiero 1999; Ruggiero and Vitaliano 1999; Mancebón and Mar Molinero 2000; Harrison and Rouse 2002; Banker *et al.* 2004; Ruggiero 2007).

Most studies at university level use non-parametric methods (often DEA) in a production context to estimate efficiency. Such studies which cover an array of university sectors, find average efficiency to be relatively high. Mean values tend to fall in the range 0.5 to 0.97 (Ray and Mukherjee 1998; Abbott and Doucouliagos 2000; Abbott and Doucouliagos 2003; Fandel 2007; Agasisti and Pérez-Esparrells 2010; Kong and Fu 2012; Bayraktar *et al.* 2013; Johnes 2014b; Papadimitriou and Johnes 2018), but there are some models which yield mean efficiency below 0.5 (Warning 2004; Kuah and Wong 2011; Duh *et al.* 2014; Mikušová 2015). Parametric estimation methods applied in a production context yield relatively low mean efficiency scores of the order 0.5 to 0.8 (Johnes 2014b). Mean efficiency derived from cost function studies falls in a similar range of around 0.5 upwards with smaller, specialist institutions more likely to exhibit lower average efficiency (Izadi *et al.* 2002; Johnes *et al.* 2005; Stevens 2005; Giménez and Martínez 2006; Johnes *et al.* 2008a; Johnes *et al.* 2008b; Johnes and Johnes 2009; Thanassoulis *et al.* 2011; Johnes and Johnes 2016).

Only a few studies have compared efficiency values of providers derived using alternative methods. While efficiencies from parametric and non-parametric estimations of cost or output distance functions are often significantly correlated (Johnes, G 1997; McMillan and Chan 2006; Kempkes and Pohl 2010; Johnes 2014b), these correlations are not always particularly strong suggesting that different estimation methods can lead to different conclusions.

These findings on efficiency levels are interesting insofar as they lead to questions as to why one provider is substantially more (or less) efficient than another. It should be remembered, however, that they are only estimates; the possibility of providing standard errors around the efficiency scores allows the researcher to establish whether there are significant differences between providers. Where this has been done, the conclusion is that there are significant differences only between the best and worst performers (Johnes 2006a; 2014b; Papadimitriou and Johnes 2018). It should be noted that the estimation methods assume that the units under examination are comparable – in terms of, for example, their production technology or environment. If such differences between institutions exist but are not allowed for, this might be captured in the efficiency score and hence these scores should be interpreted with caution (Johnes and Johnes 2013; Johnes and Johnes 2016).

The differences between institutions and the subsequent questions raised by efficiency analyses often lead to a second stage investigation as to what factors might actually influence how

efficiently an institution can operate. There is a considerable literature examining the determinants of efficiency at both school and higher education levels. Methods of analysis vary. Early studies typically use DEA followed by a Tobit approach to accommodate the contention that the dependent variable (efficiency score) is a censored variable taking values between 0 and 1 (McCarty and Yaisawarng 1993; Ruggiero *et al.* 1995; Kirjavainen and Loikkanen 1998; Ruggiero and Vitaliano 1999; Bradley *et al.* 2001; Chakraborty *et al.* 2001; Abbott and Doucouliagos 2002; Borge and Naper 2005; Kantabutra and Tang 2006; McMillan and Chan 2006; Rassouli-Currier 2007; Conroy and Arguea 2008; Bradley *et al.* 2010; Naper 2010; Kounetas *et al.* 2011; Johnes *et al.* 2012; Selim and Bursalioglu 2013). Later studies argue that the dependent variable is not censored but fractional (McDonald 2009) and that the appropriate second stage analysis should take an OLS estimation approach, with White heteroscedasticity consistent standard errors, which produces consistent estimators for large samples (Hoff 2007; McDonald 2009). A regression approach (or suitable panel data methodology) is used in the second stage in a number of studies (Ray 1991; McMillan and Datta 1998; Mancebón and Mar Molinero 2000; Ramanathan 2001; Harrison and Rouse 2002; Warning 2004; Burney *et al.* 2013; Harrison and Rouse 2014).

Separate second stage analyses, such as those referred to above, have been criticised. When using SFA to derive the efficiency scores, these scores are assumed to be independently and identically distributed. Yet in the second stage they are assumed to be affected by factors relating to, for example, the DMU. Models which address this issue have been devised for both cross section and panel data (Kumbhakar *et al.* 1991; Reifschneider and Stevenson 1991; Huang and Liu 1994; Battese and Coelli 1995) and such methods which simultaneously apply SFA *and* investigate the determinants of efficiency have been applied in the education context (see, for example, Stevens 2005; McMillan and Chan 2006; Zoghbi *et al.* 2013; Agasisti *et al.* 2016).

Analyses of efficiency have uncovered a vast array of determinants of efficiency. At the schools level, school-related determinants including per pupil expenditures on teachers, teacher salary, physical resource expenditure, and scale (school or class) have all been found to be important, although direction of effect can vary from study to study (Kirjavainen and Loikkanen 1998; Noulas and Ketkar 1998; Ruggiero and Vitaliano 1999; Bradley *et al.* 2001; Ramanathan 2001; Kantabutra and Tang 2006; Burney *et al.* 2013).

Pupil discipline record, absenteeism and having pupils with special educational needs also affect school efficiency (Lovell *et al.* 1994; Mancebón and Mar Molinero 2000; Borge and Naper 2005; Waldo 2007; Conroy and Arguea 2008) as does type of school such as selective and single sex girls' schools (Bradley *et al.* 2001; Burney *et al.* 2013). Factors relating to the pupils themselves are also highly important in determining the efficiency of schools. Such factors include ethnic background, socio-economic status, and parental education (Ray 1991; McCarty and Yaisawarng 1993; Kirjavainen and Loikkanen 1998; Noulas and Ketkar 1998; Afonso and St. Aubyn 2006; Rassouli-Currier 2007; Conroy and Arguea 2008; Harrison and Rouse 2014).

Variables relating to the wider region in which the school is located are also important in determining efficiency levels of schools. These include variables indicating the unemployment rate, and the wealth and educational attainment of inhabitants of the area (Noulas and Ketkar 1998; Ruggiero and Vitaliano 1999; Kang and Greene 2002; Afonso and St. Aubyn 2006; Rassouli-

Currier 2007; Bradley *et al.* 2010; Harrison and Rouse 2014). Direction of relationship between such variables and efficiency can vary according to study. Finally, political factors have also been found to play a part in determining school efficiency. These include the source of funding (particularly deriving from local sources) and political leaning of residents of an area, both of which can affect efficiency (Adkins and Moomaw 2003; Borge and Naper 2005; Waldo 2007).

The array of variables affecting efficiency is therefore vast (more information can be found in Burney *et al.* (2009)) and the variables vary in terms of what the school can control (such as resources) and what they cannot (such as characteristics of the pupils in the catchment area and the regional environment). Clearly this distinction is important in terms of developing policies to improve efficiency. As an initial step, the second stage analysis can be used to compute a revised efficiency score which takes into account the variables. One study which does this finds that mean efficiency in primary schools rises from 0.4 to 0.9 and in secondary schools from 0.5 to 0.9 (Chakraborty and Blackburn 2013). This demonstrates the effect these variables can have in explaining inter-institutional differences in efficiency, and managers and policy makers should be aware of this.

Similarly useful results are found in the higher education context. University related factors include provider size and composition, age, governance (such as public or private), source of funding, geographical location, as well as staff characteristics such as gender, age and ethnicity (McMillan and Datta 1998; Warning 2004; Stevens 2005; McMillan and Chan 2006; Kounetas *et al.* 2011; Wolszczak-Derlacz and Parteka 2011; Selim and Bursalioglu 2013; Zoghbi *et al.* 2013). The influence of student characteristics on efficiency is less well investigated (Stevens 2005).

It is worth ending this sub-section with a quick note on effectiveness. Cherchye *et al.* (2019) define a measure of effectiveness for organisations by assuming constant resources; in practice this means applying the CRS DEA framework with resources equal to unity for all DMUs in order to derive an effectiveness score. In applying this methodology to Flemish secondary schools, they find that performance can be improved more by improving efficiency (as there is unexploited production capacity) than effectiveness. It will be interesting to see this methodology applied to different sectors and countries.

Productivity

Measures of productivity have typically been undertaken using non-parametric approaches in education and higher education. In applications to higher education sectors as diverse as the UK, Italy, Spain, China, Australia, Australasia and Iran, productivity growth is found, and this appears to be more a consequence of technology change (the frontier shifting out) than of efficiency change (inefficient units getting closer to the frontier) (Flegg *et al.* 2004; Johnes *et al.* 2008a; Johnes *et al.* 2008b; Johnes 2008; Worthington and Lee 2008; Ng and Li 2009; Margaritis and Smart 2011; Barra and Zotti 2013; Johnes 2014b). There are, however, some exceptions where productivity has increased but due to efficiency rather than technology change (Rayeni and Saljooghi 2010; García-Aracil 2013). When samples are split, for example by mission group, findings are more nuanced with some groups experiencing productivity decline, and this too is a consequence of shifting frontier (Thanassoulis *et al.* 2011; Thanassoulis *et al.* 2016). In the context of productivity

improvement, it is hypothesised that recent innovations to higher education such as e-learning support for teaching and digital support enabling and supporting research networks may well be reasons for the frontier being pushed out. The inefficient universities may find it difficult to keep pace with the changing technologies.

In the context of schooling, we find similar results regarding productivity change and the underlying cause being technology improvements at both the post-compulsory (Bradley *et al.* 2010) and secondary school levels (Ouellette and Vierstraete 2010; Essid *et al.* 2014) in the UK and Canada. Where productivity is found to decline (Thieme *et al.* 2013; Podinovski *et al.* 2014), this is also related to technological performance rather than efficiency decline. Johnson and Ruggiero (2014) take the Malmquist decomposition one step further by adding in a component relating to environmental harshness. In a practical application to Ohio school districts, the approach reveals that while technological progress drives productivity change in top-performing school districts, it is the environmental harshness which is the most important driver for low-performing districts. A similar approach is applied to Dutch schools and also provides useful insights (Brennan *et al.* 2013).

4.3 Recent developments in efficiency measurement

Many developments covered in earlier sections are relevant here. Heterogeneity amongst providers, and how it is addressed, is an important factor in efficiency studies. Some researchers choose to divide their sample based on a known characteristic, such as public or private funding (Mancebón and Muñiz 2008; Kong and Fu 2012; Duh *et al.* 2014), or by mission group. More recently, developments in the methodological approaches are used to address heterogeneity in the efficiency context. Thus LC and RP SFA, whilst providing different parameters by group or unit (respectively), also provide different efficiency scores by group or unit.

We have referred throughout this chapter to the issue of institutional diversity in education and higher education sectors, and considered ways in which diversity has been handled. Another emerging approach in the efficiency context (based on cost functions) is one which distinguishes between transient and permanent efficiency (Vittadini *et al.* 2011; Colombi 2013; Colombi *et al.* 2014; Kumbhakar *et al.* 2014; Tsionas and Kumbhakar 2014; Filippini and Greene 2016). The underlying premise is that some differences between organisations arise from a historical and geographical context which the education provider cannot alter. Inefficiency differences arising from such structural variations should be addressed differently from those arising from transient (or short-term) factors. There are some subtle differences in the precise approach, in this context. An SFA approach which allows for unobserved heterogeneity and incorporates the premise of transient and permanent inefficiency (Kumbhakar *et al.* 2014; Tsionas and Kumbhakar 2014) has been applied in the higher education context (Gralka 2018; Agasisti and Gralka 2019). It seems that for German and Italian universities, transient efficiency is relatively high, while persistent efficiency is much lower. Papadimitriou and Johnes (2016) use an approach developed by Filippini and Greene (2016) and also find that persistent efficiency is lower than transient efficiency in the English higher education sector. Clearly policies for improving efficiency are likely to need to be

adapted in light of this finding: a low persistent efficiency value, for example, suggests a need for structural changes.

An aspect of production analyses which we have not yet explored is that of complexities in the production process. So far we have assumed that all inputs go into a ‘black box’ at the start of production, and all outputs come out of it at the end point. In reality, the ‘black box’ may be hiding a more complex production process whereby some inputs may produce a set of outputs at one stage, and then (some of) these outputs, possibly along with other inputs, then become inputs into a second stage of production which produces more outputs. Where a production process can be divided into a series of sub-processes, a standard DEA fails to account for the efficiency of each sub-process. By ignoring such complexities, the standard DEA might lead to bias in efficiency estimates (Kao and Hwang 2008; Kao 2014), and conceals useful information about efficiency of each of the stages. Network DEA (NDEA) (Färe 1991; Tone and Tsutsui 2009) takes into account such complexities of production and provides estimates of efficiency at each stage. A number of studies have applied a network DEA approach mainly in the higher education context (Johnes 2013; Yang *et al.* 2018; Lee and Johnes 2019).

A network approach, whereby outputs such as student satisfaction and student achievement are assumed to happen in a first stage, while employment outcomes happen in a second stage (where student achievement is an input into that second stage), reveals considerably more discrimination in terms of HEIs identified as efficient. Moreover, the second stage (production of student outcomes in the labour market) is less efficient than the first stage, thereby providing managers with useful information on where they should concentrate their efforts in terms of improving efficiency (Johnes 2013; Lee and Johnes 2019). Indeed, an analysis of the factors underpinning each of the sets of efficiencies (stage 1 and stage 2) indicates that there are different reasons for differential performance in each case, and hence provides more information for managers and policy makers (Lee and Johnes 2019). More work of this type at both school and higher education level would be useful.

We have noted in section 4.2 above the many studies which employ a second stage analysis to explore the variables which might impact efficiency scores. However, such studies are valid only if the separability condition between the input-output space of the first stage and the space of the external factors in the second stage holds. In the situation where the separability condition does not hold, then a conditional DEA model is the appropriate approach (Cazals *et al.* 2002; Daraio and Simar 2005; 2007). While it is important to check that the separability condition holds (Simar and Wilson 2007; Simar and Wilson 2011), and a test of the validity of the separability assumption is available (Daraio *et al.* 2018), studies which investigate the issue of separability and apply a conditional non-parametric approach are relatively rare to date (see, for example, Blackburn *et al.* 2014; Cordero *et al.* 2017; Cordero *et al.* 2018). The early indication is that academic or school-related variables may be less important than economic and cultural indicators. A particularly novel and interesting application of the conditional efficiency model investigates efficiency of the provision of adult education programmes in Flanders (Schiltz *et al.* 2019). This work suggests that characteristics of the adult learners and homogeneity among the teachers on programmes are

important determinants of managerial efficiency in the adult education context. Clearly more work using this approach is required at all levels of education.

The Malmquist approach has been extended to allow comparisons of performance between groups rather than time periods (Camanho and Dyson 2006), and this has further been extended to examine and compare patterns of change across groups over time (Aparicio *et al.* 2017). For example, in the context of schools in the Basque country in Spain, this approach establishes that privately run schools have consistently better performance and that this is because of superior technological performance. The methodology can also be applied when there are more than two groups. When Ohio school districts are assigned to 5 groups based on environmental harshness, the Malmquist decomposition shows that productivity is largely explained by environmental harshness, and that technological progress is also hampered by the harshness of the environment (Johnson and Ruggiero 2014). Distinctions are also found between public and private universities in Spain with private universities outperforming their public counterparts at the start of the study period, but the Malmquist decomposition reveals that the public universities catch up over the period (de la Torre *et al.* 2017).

4.4 Policy implications and future work

Whilst average efficiency is generally found to be high in many education studies, there is typically a spread of performance across providers, and this means that the results can potentially be useful at a policy level. Efficiency-based funding (Fandel and Gal 2001), for example, is one aspect where there has been relatively little work, but the applications that exist suggest some potential for efficiency improvements by distributing resources based on efficiency. Sexton *et al.* (2012) provide an example of an efficiency-based state funding scheme for HEIs underpinned by DEA. Such a scheme, which would encourage HEIs to behave in such a way as to be consistent with government or state objectives, would reap potential savings of 9% across the sector, with differential savings observed in each provider. A particular advantage of the approach is that, as efficiency improves relative to a given DEA frontier, any subsequent DEA will produce an improved frontier against which efficiency will be measured, and so there is a natural tendency of the approach for ongoing improvement (Sexton *et al.* 2012).

A drawback of the approach is that efficiency estimates based on annual estimations can fluctuate from one year to another meaning that there is potential for instability in resource allocations (Fandel 2007). A reduction in sensitivity might be achieved by using a moving average over several years. In addition, an efficiency-based funding scheme may not be appropriate if there is little significant deviation in efficiency across providers. In such cases, the studies should instead be used to provide institutions with useful information on benchmarking and examples of good practice (Johnes and Johnes 2016).

Even where efficiency does apparently vary substantially across providers, we know from the second stage analyses undertaken in previous studies that efficiency is affected by various factors, and some of these are outside the control of the institutions. Strategies to improve efficiency must

therefore be nuanced. For example, if efficiency is affected by the ethnic mix of pupils (Bradley *et al.* 2010), providers can do little to alter that. Instead, they must focus on ways in which to improve outcomes of the at-risk groups, and this may then impact on efficiency. The importance of variables reflecting the conditions in the wider environment means that local and government policies to improve economic conditions in a catchment area can also impact school efficiency.

The introduction of increased competition in school sectors has been a deliberate policy of some governments (in the UK, for example) to improve school performance and efficiency. There are various studies which have specifically examined the impact of increased competition on efficiency in various state school sectors (Bradley *et al.* 2001; Bradley and Taylor 2002; Agasisti 2011a; 2013; Harrison and Rouse 2014). With one exception (Grosskopf *et al.* 2001), these studies find that the larger the number of schools in a region, the higher the schools' efficiency. Some studies find that competition from private schools impacts on efficiency in publicly funded schools (Agasisti 2011b; Misra *et al.* 2012; Agasisti 2013), although the effect quickly diminishes as distance from the school decreases (Kang and Greene 2002). Competition has also been investigated as a driver of efficiency in higher education, where it has been found to have a positive effect in the Canadian higher education context, although not always significantly so (McMillan and Datta 1998; McMillan and Chan 2006).

A final example of how efficiency analyses might inform policy arises in the context of mergers. Theoretically, a merger might be expected to have benefits in terms of increased efficiency accruing from returns to scale or returns to scope where the merging providers have complementary offerings (Skodvin 1999; Harman 2000). A suite of papers utilising a sample of data relating to English higher education suggest that, typically, efficiency improves following merger, but that the benefits accrue in the years immediately following the merger and do not continue indefinitely (Johnes 2014b; Johnes and Tsionas 2014; Papadimitriou and Johnes 2018). There is scope for more work into the evolution over time of the effects of merger on subsequent efficiency.

5. Level of analysis

In the preceding sections, we have made little reference to the level of the analyses undertaken. In many cases, the estimations, be they cost functions or output distance functions, are at provider level. There are some exceptions in the schooling context, where the level might equally well be the funding region (such as school district in the US context or local education authority – LEA – in the UK context). The review of efficiency in education by De Witte and López-Torres (2017) confirms the provider (defined as organisation, school, department etc.) as the typical unit of analysis in such studies: of 223 papers relating to efficiency in the education context over the period 1977 to 2015, 147 are at the organisation level (with 89 relating to HEIs and 58 to schools); 44 focus on the funding district, county or city level; while 9 studies are at the level of the country, and 23 at the level of the individual student. A number of these studies are of note because they

focus on a particular discipline or department (Colbert *et al.* 2000; Kao and Liu 2000; Casu *et al.* 2005; Giménez and Martínez 2006; Kao and Hung 2008; Dehnokhalaji *et al.* 2010; Aziz *et al.* 2013; Selim and Bursalioglu 2013; Mayston 2014; Sîrbu *et al.* 2016), or a support service (Moreno and Tadepalli 2002; Kao and Hung 2003; Casu and Thanassoulis 2006; Ray and Jeon 2008; Simon *et al.* 2011). In this section we take a brief look at the studies undertaken at individual, funding area, and national level analyses to see what additional information they provide, and what challenges arise, in the context of production economics.

5.1 Individual level analyses

Individual level studies are not uncommon in the schooling literature relating to education production functions, which has long recognised that pupils are nested within schools and hence the data are hierarchical in nature. As such, multi-level modelling (MLM) has been developed to estimate such functions whilst allowing for within-unit variations (Goldstein 1987; Woodhouse and Goldstein 1988; Goldstein 1997). Recognition of the hierarchical structure avoids issues such as aggregation bias and mis-estimated parameters, and the MLM approach is sufficiently flexible that it can allow both intercept and slope coefficients to vary. An additional advantage of such an approach is that it is possible to disentangle the effects of both pupils and schools on their outcomes. The disadvantage is that MLM is not a frontier estimation technique, and so there is no allowance for inefficiency in the education production function.

An alternative approach which allows for inefficiency is to apply DEA to individual level data. Such an approach has been taken in a small number of studies in the schooling context (Thanassoulis 1999; Portela and Thanassoulis 2001; Thanassoulis and Portela 2002). By using a meta-frontier type of approach, it is possible to decompose overall efficiency for a pupil into that attributable to the pupil him/herself and that attributable to the school (assuming just pupil and school levels – additional levels are possible). By careful aggregation of the pupil efficiencies (Thanassoulis *et al.* 2016) schools derive more information as to the source of their shortcomings (pupil or school), and can devise appropriate initiatives accordingly.

Applications of individual level DEA in universities are also relatively rare. Findings from such studies suggest that efficiencies derived from aggregate university level analyses incorporate both individual and institution performance components; an individual level DEA, meanwhile, provides more detailed information about the source of the inefficiency i.e. student or university (Johnes 2006b). A comparison of MLM and individual level DEA applied to the same data set finds interesting differences in the performance rankings of universities based on the two approaches, and these are particularly relevant for the best- and worst-performing HEIs (Johnes 2006a). This is in contrast to findings at school level; De Witte *et al.* (2010) find more alignment between their results from MLM and an individual level non-parametric approach using a sample of school pupils.

5.2 Funding area analyses

Whilst not as prolific in number as organisation studies, papers focusing on efficiency within funding areas in education are nevertheless reasonably numerous. They mostly relate to school level education, and are based on both parametric and non-parametric approaches. One of the earliest such studies utilises maximum likelihood and corrected ordinary least squares to estimate efficiency amongst local education authorities in providing schooling in England, using a cost function approach (Barrow 1991). The level of estimated efficiency depends on whether the approach is deterministic (with efficiency levels around 83% to 89%) or stochastic (with efficiency levels much higher at well over 90%).

Experimentation with efficiency measurement continues in the context of funding areas with a comparison of ratios (comparing a single output to a single input, for example cost per student graduated) and efficiencies derived from a variety of DEA models (Engert 1996). There are significant inconsistencies between the ratios and DEA efficiency measures, which was not surprising as the ratios fail to take into account the multi-input multi-output nature of production. Subsequent studies largely use standard frontier techniques such as DEA and SFA (including conditional and network DEA), applied in cost or production settings, and generally establish similar levels of efficiency to the earliest studies (Grosskopf *et al.* 1999; Grosskopf and Moutray 2001; Fukuyama and Weber 2002; Kang and Greene 2002; Banker *et al.* 2004; Johnson and Ruggiero 2014; Grosskopf *et al.* 2015).

A non-frontier strand of literature employs a (modified) quadriform approach (Hickrod *et al.* 1989) to the identification of efficiency amongst school funding areas (Houck *et al.* 2010). The modified quadriform approach is a means whereby performance of units can be displayed in a two-dimensional depiction. Specifically, costs are regressed on a set of uncontrollable school characteristics, and school output (such as graduation rate) is regressed on the same set of characteristics. The resulting residuals from each regression equation are plotted for each school district, and performance is examined in quadrants ranging from efficient (described as low-input and high-output) through effective, ineffective and finally inefficient (described as high input and low output). Whilst interesting and easy to interpret, such an approach does not adequately account for the multi-dimensional nature of production, is non-frontier, and relies on regression residuals which contain both unexplained variation and random error. An adaptation to provide a buffer around residuals which are low in magnitude (and therefore such districts can be assumed to be performing as expected), addresses the latter point to some extent but other drawbacks remain. A comparison of the quadriform approach with frontier techniques can be found in Rolle (2004).

Higher education studies rarely feature in the funding area context, mainly because higher education is often a national (not regional) responsibility – hence national level analyses are more appropriate and these are discussed in the next section. An exception is a study of Chinese higher education at the level of Chinese provinces which takes a production function approach (Wu *et al.* 2020), and where efficiency levels are found to be relatively low (with mean technical efficiency of under 40%).

Such funding area studies can provide useful insights into efficiency or (in the rare cases where it is calculated, productivity (Ouellette and Vierstraete 2010)) for the funding providers. The relationship between the funding area and organisations within it is rarely utilised – a network approach by (Grosskopf *et al.* 2015) is an example where the relationship is adapted into the approach. A meta-frontier analysis of schools within funding areas might also provide a useful extension to this particular body of literature.

5.3 National level analyses

The benchmarking advantages of such tools as DEA are well known. As austerity measures have been introduced in various education and higher education sectors around the world in the last decade, there has been an increasing recognition that international comparisons are necessary to provide benchmarks of good practice which may be outside of national boundaries. Combined with this, the last decade has seen a constant improvement in the availability of data at all levels across countries meaning it is now increasingly possible to make such international comparisons, and to identify exemplars of good practice across countries for national governments to emulate.

Studies which make international comparisons – whether at school or higher education level – fall into two categories. There are those which use provider-level data across two or more countries and then frequently take a meta-frontier approach to make cross-country comparisons (Agasisti and Johnes 2009a; Agasisti and Pérez-Esparrells 2010; Wolszczak-Derlacz and Parteka 2011; Wolszczak-Derlacz 2017; Agasisti and Gralka 2019); and there are those which use national level data (i.e. the nation is the DMU) to derive their results (Giménez *et al.* 2007; Agasisti 2011b; Aristovnik and Obadić 2011; Agasisti 2014; Aristovnik and Obadić 2014; Bogetoft *et al.* 2015; Azar Dufrechou 2016). Interesting differences between countries can be found. Agasisti and Pérez-Esparrells (2010), for example, compare universities in Italy and Spain, and find, using the Malmquist productivity index approach, that productivity has been rising in both countries over the study period. In comparing the countries however, it appears that technological change underpins productivity increases in Italy whereas it is efficiency gains which underpin the observation for Spain.

There is no doubt that such studies will proliferate as more data becomes available, and that is beneficial so long as results are treated with caution. There are various problems with cross-country comparisons and in particular the latter approach. It is extremely difficult to get comparable data on costs or inputs and outputs at the national level. The assumption that production technology and environment are the same across diverse sets of countries is open to serious doubt. Thus if a national level study is to be undertaken, it is advisable either to use individual providers to seek useful insights into education provision across countries using a meta-frontier type of approach, or, if national level data are to be used, then a parametric estimation approach which allows for unobserved heterogeneity should ideally be adopted. There is scope for much more work in this context.

6. Conclusions

This chapter has examined empirical findings relating to production economics concepts in the context of education and higher education. Education is an important sector of any economy as the benefits (in terms of increased productivity) accrue to both the individuals who consume the education and also to society as a whole. This is particularly the case for primary and secondary education, which are typically compulsory in many countries, and to a limited extent of tertiary education as well. As such, education and higher education are in receipt of publicly allocated funds, potentially making the incentives for efficient operation less compelling than in a private sector. The public funding of all levels of education, combined with the incentives and pressures which that imposes on the providers operating in the sector, make education and higher education interesting sectors in which to examine concepts from production economics.

This chapter examines findings relating to costs, production and efficiency in education and higher education, and contributes to the production economics literature by bringing together the findings of these diverse literatures, at all levels of education, into one repository. The review has uncovered a number of key areas for future research.

The mixed findings emerging from all topics in both education and higher education contexts makes it difficult for managers and policy makers to take a consistent message on, for example, the existence (or otherwise) of economies of scale or scope, the degree of substitutability between different inputs, the extent of inefficiency in the sectors, and the identification of factors affecting efficiency. This points to a pressing need for more detailed analyses of the literature to provide a framework for why results vary and hence permit the users of the work to make informed decisions. A key contribution in this area is by Zhang and Worthington (2018) who undertake a meta-regression analysis of the empirical cost function literature in higher education. They are able to identify reasons why the findings on economies of scale and scope vary across the studies. More studies of this type in the education context, or relating to output distance functions and efficiency (at both schools and higher education levels) are also needed.

In terms of factors affecting efficiency, the conditional DEA approach offers a rigorous methodology for identifying those variables which are most important in affecting efficiency. This knowledge is essential in determining strategies for improving efficiency and hence getting more value for public funds, and in particular in revealing whether institution level or regional level or national level policies will be most effective.

While economies of scale and scope (and returns to scale and scope in the production context) are relatively well researched, there is much less empirical research into elasticities of substitution between inputs (or between outputs). In times of public funding constraints, such information could be particularly useful to managers and policy makers. Similarly, more work on the potential benefits of performance-based funding would be welcome.

Finally, there is considerably more scope for education studies which make comparisons across countries. These might use national level data, in which case appropriate methods which take into account unobserved heterogeneity should definitely be applied. But the increasing availability of large individual level data sets offers opportunities for findings from these sources. However, more work is required on the application of frontier methods to the individual level context, and using these results to derive insights into concepts, such as efficiency, relating to providers and even nations.

Empirical applications of production economics to education and higher education have a long and fruitful history and are set to continue to provide useful information to both managers and policy makers alike.

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