

Impact of Rail Infrastructure Maintenance Conditions on the Vehicle-Track Interaction Loads

Naim Kuka⁽¹⁾, Caterina Ariaudo⁽²⁾*, Riccardo Verardi⁽²⁾, João Pombo^(3,4)

⁽¹⁾ Railway Dynamics & Mechatronics - ALSTOM Transport
naim.kuka@alstomgroup.com

⁽²⁾ Railway Dynamics - ALSTOM Ferroviaria S.p.A.
caterina.ariaudo@alstomgroup.com; riccardo.verardi@alstomgroup.com

⁽³⁾ Institute of Railway Research, University of Huddersfield, UK
j.pombo@hud.ac.uk

⁽⁴⁾ IDMEC, Instituto Superior Técnico, Universidade de Lisboa, Portugal & ISEL, IPL, Portugal
*Corresponding Author: caterina.ariaudo@alstomgroup.com

Abstract

The rail infrastructure and the track components are expensive assets with long life spans and high maintenance costs. The cost efficiency, performance and punctuality of train operations heavily depend on the track conditions. Ideally, the railway track would be completely smooth providing continuous support to the rolling stock running on it. In practice, however, the infrastructure cannot be installed without irregularities. These defects will increase over time due to the service loads imposed by the railway vehicles. The aim of this work is to use advanced computational tools to predict how the vehicles will respond to changing levels of track defects. For this purpose, the track and its maintenance conditions are characterized in realistic operation scenarios and modelled with detail in order to enable studying the interaction loads that are imposed to the vehicles by the track conditions. The presented methodology enables to identify the track health indexes that have higher influence on the dynamic loads transmitted to the rolling stock. It was observed that the track layout, track irregularities and degradation of the rails have the larger influence on the vehicle-track interaction loads with consequences in terms of safety and maintenance costs. In this way, this work contributes to the development of solutions with technological relevance, giving answer to the industry's most recent needs in terms of reducing the maintenance costs and decreasing the incidents that cause traffic disruptions, contributing to improve the competitiveness of the railway transport.

Keywords: Railway dynamics, Multibody systems, Vehicle–infrastructure interaction, Track geometry, Track maintenance.

1 Introduction

The health and long-time performance of the railway infrastructure is critical not only due to safety aspects but also owing to the high maintenance costs involved and the damage that it can cause to the vehicles. Despite its importance, the maintenance management of the track is, scientifically, one of the least understood and least

predictable elements of railway systems. In reality, the maintenance of rail tracks generally relies on empirical data and on the return of experience, together with needs or conveniences linked to day-by-day operation, more than on the studies that analyse all the factors of damage, such as the characteristics of vehicle and track, the mission profile, the route traffic, to cite only the most important.

The complexity and multidisciplinary of the vehicle-track interaction is mainly due to a reciprocal process of causes and effects [1], as illustrated in Figure 1. The dynamic behaviour of the rail vehicles and the forces exchanged between vehicle and track depends on the macro-geometry and micro-geometry of the track (further to other boundary conditions), but they depends also by the vehicle design parameters and the health state of various components. Hence, the vehicle design and its health state affect the damage of track components and, vice-versa, the track condition affects the dynamic behaviour and the deterioration of the vehicle parts. In this view, the Figure 1 indicates the framework and the key parts of the vehicle/track system to be considered in simulation models and in the investigations dealing with performances and vehicle/track damage.

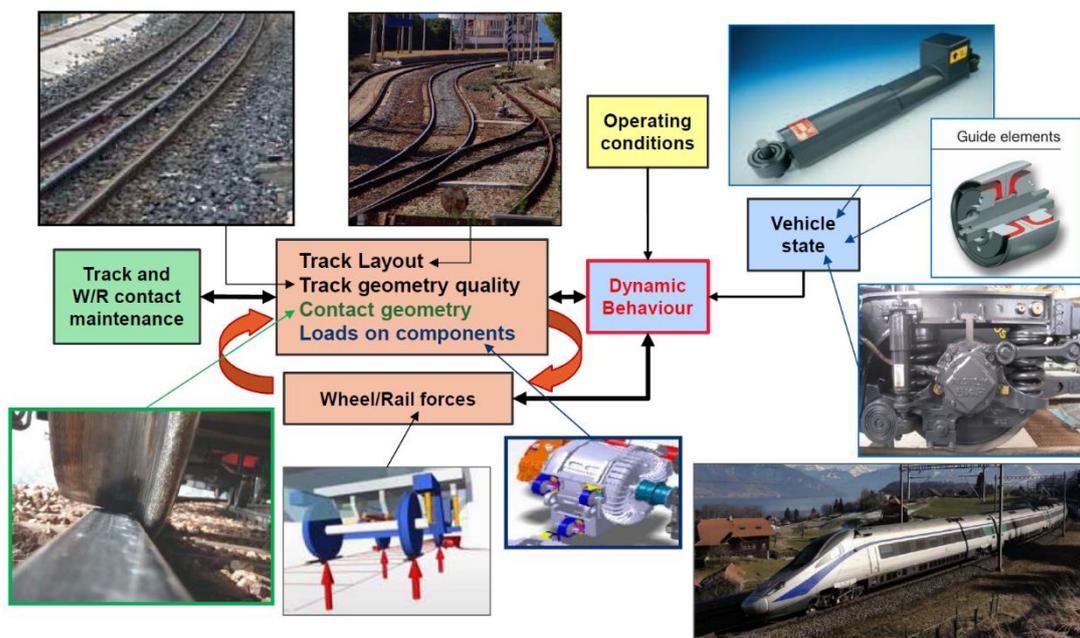


Figure 1: Overview of vehicle-track interaction and damage

Such studies require the use of advanced computational tools and experiments to validate models and de-risk the development of new technology. This involves using Multi-Body (MB) systems methodologies for railway vehicle dynamics to study different type of problems [2–5], including virtual homologation [6,7], study of the vehicle performance for selected tracks [8,9], derailments prevention [10–15], design of suspensions [16–18], tracks with complex geometries [19–22], traction or braking systems [23,24], pantograph-catenary interaction [25–35], just to mention a few.

The analysis of the track dynamic response has been object of many research works [36–42]. For this purpose, complex numerical models have been developed using MB

and Finite Element (FE) methodologies for flexible tracks in co-simulation [43–46]. Other authors have dedicated their experimental research activities to investigate the performance of various parts of the railway track structure. For example, the dynamic performance and long-term behaviour of concrete slab and ballasted tracks were studied through full-scale model tests with simulated train moving loads [47–55].

The study of the interaction between the vehicle and the track consists of the solution of the contact between the wheels and the rails [56–63]. This issue plays a key role since it not only affects the motion of the vehicle but also allows to study the effect of track irregularities [64–67], the prediction of some damaging phenomena such as wear [68–75], rolling contact fatigue [76,77] or corrugation [78–80] and other track singularities [81–84].

In [1] a methodology to study how the varying characteristics of the vehicle components can impact on the vehicle-track interaction loads and on the track damage was illustrated. The present work aims to study how the rail infrastructure maintenance conditions can influence the vehicle-track interaction loads. The innovative scope of this work is related to the studies that are performed in realistic operations conditions, allowing to trace a path towards the genuine definition of load mission profiles to assess the structural fatigue of the vehicle and track components. The evaluation criteria and the quantities assessed in this work are defined according to EN 14363 [85] and supported by the comparison with available field data, which also represents an innovative feature of this work.

It is known that the track design parameters are not the same for all railway lines. The radii of the curves, the installed cant, the slope of the transition curves, type of rail, type of sleeper, type of fastening, ballast characteristics, the track gauge differ according to whether they are high-speed lines, regional or suburban lines. Over time, and also according to the type of rolling stock circulating on the lines, the various parameters change, giving rise to different conditions of wheel-rail contact, track irregularities with different wavelengths, isolated defects, etc. These conditions influence the dynamic behavior, i.e. vibrations and shocks and, ultimately, the dynamic loads on the track and on the vehicle components [6,86–91]. Therefore, to avoid failures and damages to the various components of the vehicle-track system, it is important to consider loads that are representative of the operating conditions of service, for example for the design of the truck fatigue in accordance with EN 15827 [92] or in the application of IEC 61373 [93] for random vibration and shock testing of pneumatic, electrical and electronic rail components. The study presented here addresses this problem in an innovative way by analysing the influence of the maintenance state of the track on the loads transmitted to the vehicle running on it. It is also demonstrated that a good maintenance strategy will bring benefits, in terms of reliability, availability and costs to the infrastructure managers, vehicle manufacturers and rail operators.

The study proposed here starts with the assessment of the vehicle-track interaction loads, varying the track conditions from new to degraded, with particular attention to the track quality and to the rail wear status, although remaining in the range of the admissible maintenance limits. The assessment is based on the vertical wheel-rail contact forces, the lateral track shift forces and the accelerations on bogie and carbody, as defined in the EN 14363 [85].

2 Methodology

The study mainly uses dynamic MB simulations that allow to consider the different track conditions, combined with the vehicle's mission profile, and to evaluate their impact on the loads exchanged between the vehicle and the track. Advanced computational tools are necessary to represent the characteristics of the track and of the vehicle, thus reproducing the real physical phenomena.

The study of the influence of the track maintenance state on the loads transmitted to the vehicles and vice-versa, can be performed following two different approaches, based mainly on tests or on simulations:

1. Analysis of the data measured by test vehicles, running on different levels of track quality and wear of the rails, provided that all the other boundary conditions, like the speed, friction, vehicle parameters, etc. remain unchanged;
2. Simulation of the dynamic behaviour of the trains under different conditions of wheel-rail contact and track maintenance state and compare the results, mainly in terms of wheel-rail contact forces and accelerations on the vehicle.

The second approach is the one followed here, having the advantage of a full control of the simulation conditions. It gives the possibility to investigate the relevant vehicle-track scenario and service conditions foreseen by the mission profile in any design phase. The main issue remains the availability of good and representative data, especially of the track, based not only on theoretical formulation, but possibly on real measurements. The approach used here is the same that was proposed in [1], i.e., to obtain realistic vehicle-track loads representative of the service conditions and useful for the fatigue dimensioning of the railway vehicle components.

2.1 *Multibody Tools*

The studies described here were conducted using the commercial MB codes SIMPACK and VAMPIRE [94], thanks to the potentialities offered by these two codes. In particular, the following characteristics have been exploited: (i) the use of full non-linear models with a number of degrees of freedom necessary and sufficient to represent the dynamic behavior of the train; (ii) the wheel-rail contact model representative of the wheel-rail forces in all simulation conditions; (iii) the possibility of introducing the track geometry (track layout and track irregularity) and the speed profile corresponding to the real mission profile of the train; (iv) calculation times not too much penalizing and a user-friendly post-processing.

2.2 *Track Model*

For the aim of this study, much care has been put in the model of the track, with a full characterization of the track layout, track irregularities, rail profiles and track stiffness/damping characteristics, as represented in Figure 2. The assessment of the importance of all these aspects, on the vehicle-track loads, will be the focus of this work.

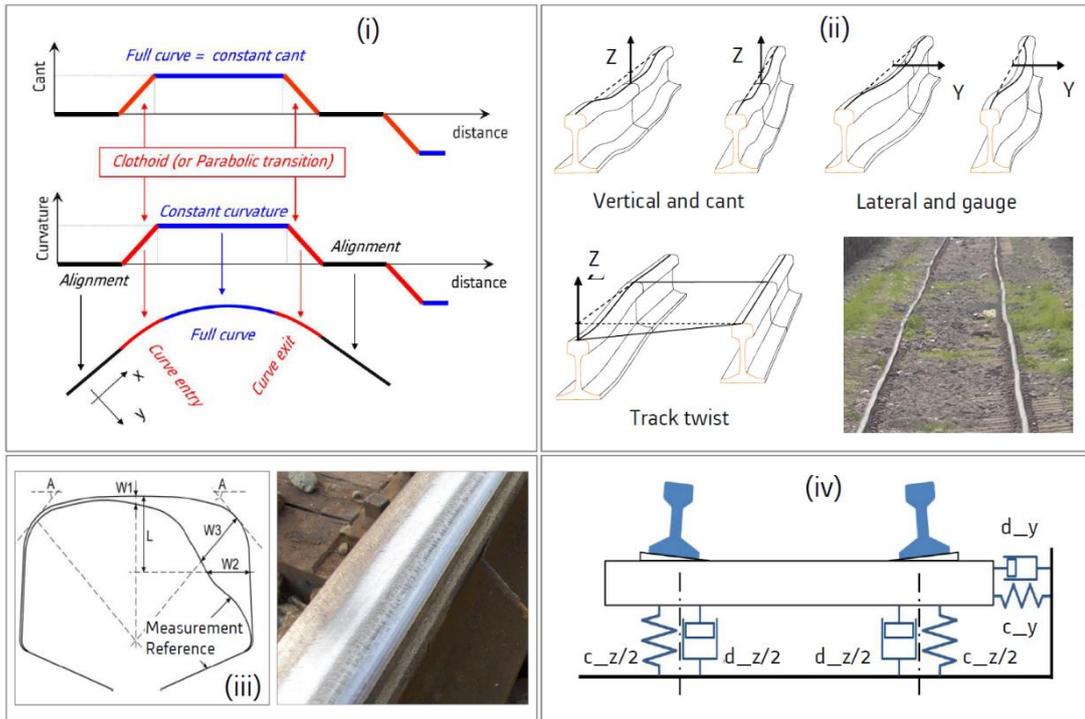


Figure 2: (i) Track layout, (ii) track irregularities (iii) rail profiles (iv) track stiffness/damping

2.3 Identification of Relevant Case Studies

In [1], the attention was mainly devoted to the impact that the maintenance conditions of the vehicle components can have on the loads transmitted to the infrastructure. Now, the study is enlarged by analysing the consequences of the maintenance state of the track on the vehicle components.

Dynamic studies can help to identify all the track features that have significant impact on the track and vehicle loads. Nevertheless, a preliminary analysis and selection of the key components/parameters to be considered is the first challenge in creating reliable simulation scenarios. The aspects of the track related to the contact conditions, such as the track layout, track irregularities and rail profiles, that can have a meaningful influence on the dynamics of the vehicle and, consequently, on the loads and on the wear in general, are analysed here.

First of all, an attempt is made to discriminate the effect of irregularities and the construction characteristics of the track from the effect of wear of the rail profile on the dynamic behaviour of the train and on the track damage. The application of the criteria required by the normative related to running safety, ride quality and track fatigue, assessed on a case-by-case basis, allow to evaluate if the loads are more impacting the vehicle's or the track's damage. The interested reader can find more details in references [1,71,95,96]. In the following some examples of impact analysis are provided.

As the aim of this work is to analyse the variation of vehicle-track loads as a function of the track parameters, the assessment quantities considered here are:

- The track loads, i.e., track shift forces and the vertical wheel-rail loads according to EN 14363 [85];
- The track damage.

Figure 3 highlights, qualitatively, the different impact that the three main types of track parameters, i.e. track geometry, rail profiles and track design, can have on the analyzed output.

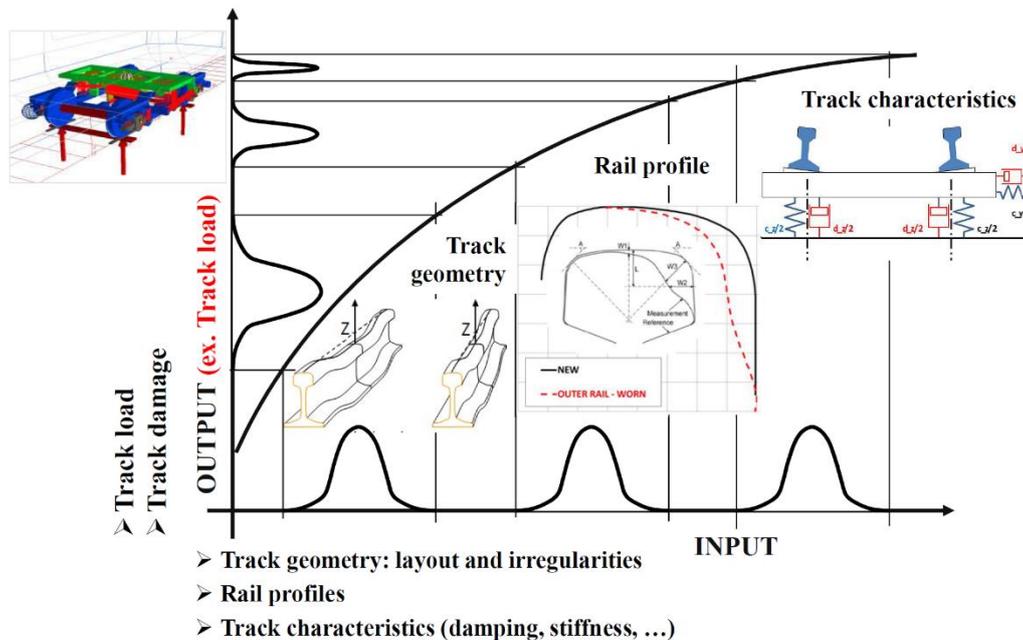


Figure 3: Approach for the identification of parameters impact on running behaviour

Each parameter can vary in a certain range of values, with a consequent effect on the outputs considered. For example, the figure shows that the geometry of the track and the state of wear of the tracks affect more than the elasticity of the track itself, which can vary depending on whether it is made by wooden or concrete sleepers or on the maintenance status of the track substructure.

Therefore, in order to obtain a reliable evaluation of the track loads, it will be necessary to model the real track layout, the measured irregularities and the measured rail profiles, while the track stiffness and damping values used could be those proposed by default by the MB codes, or scaled values, based on best practises and field experience.

3 Studies on the Maintenance Conditions of the Track

To correctly design a railway vehicle, the operating condition of the train shall be specified, including the maximum speed, the minimum radius curve (reverse curve), the maximum cant and maximum cant deficiency, and the following track characteristics:

- Track gauge (including any variation due to track widening in small curves);
- Rail head profile (new and typical worn);

- Inclination of the rail;
- Track twist;
- Track quality (track geometry deviation relative to line speed).

Other parameters, like the vertical stiffness and damping of the track or the rail material, that can have a direct effect on the wheel-rail contact forces, in different frequency ranges, are out of the scope of this study.

The focus here is on the impact on the track loads generated considering three main groups of characteristics: (i) the macro-geometry or track layout; (ii) the micro-geometry or track irregularities; (iii) construction parameters like the rail head profile.

3.1 Track Layout or Macro-Geometry

The railway tracks are made, in the horizontal plane, of alignment elements with infinite radius (straight track), constant radius (circular curve) or variable radius (transition curve between straight track and circular curve), as schematically illustrated in Figure 2 (i). In the vertical plane, each alignment element can have a vertical curvature, while the cant is the amount by which one running rail is raised above the other, to partially compensate the centrifugal force applied on a vehicle running at a certain speed in curve.

Figure 4 shows an example of the track macro-geometry inputs for simulation in SIMPACK: (i) a cartographic plane view of the line (layout); (ii) the curvature variation along the line, as inverse of the radius, so that in straight track the curvature is equal to zero, and; (iii) the corresponding cant in curve.

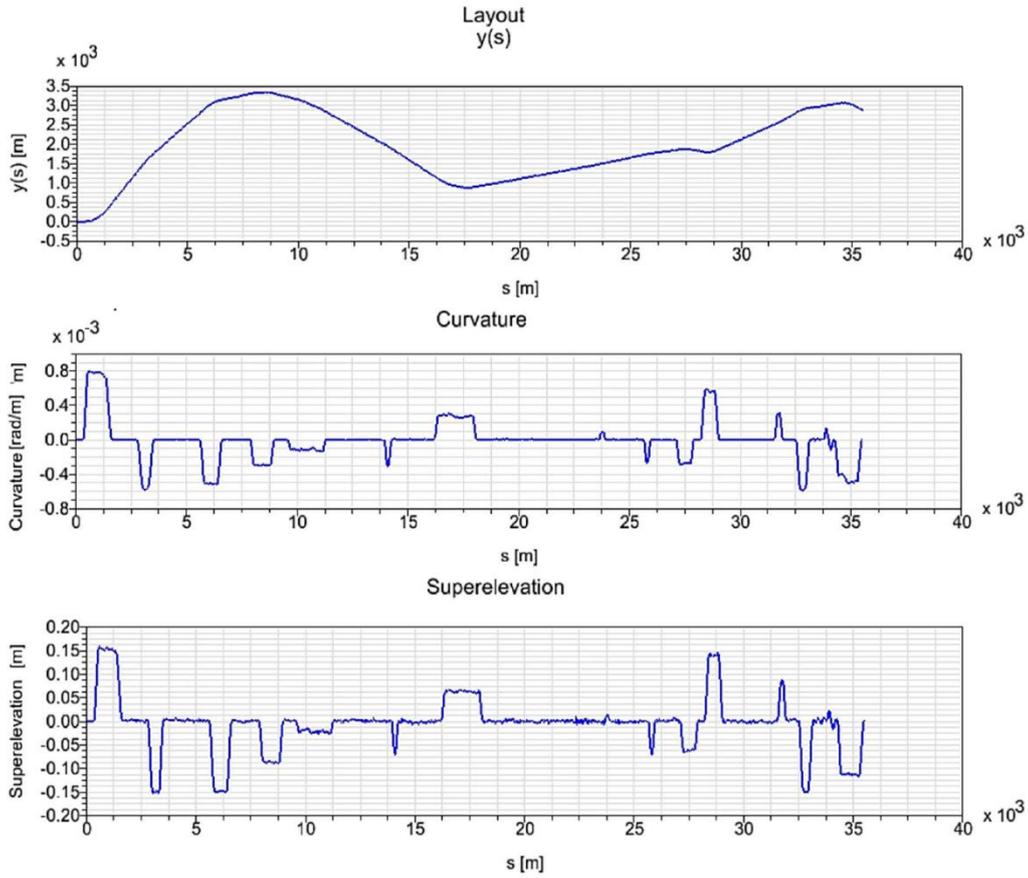


Figure 4: Model of track macro-geometry in MB environment

The wheel-rail forces in curve are influenced by the geometry of the curve and by the running speed, that together give rise to the lateral acceleration at track level, not compensated by the track cant, usually called Non-Compensated Acceleration (NCA), with the following expression:

$$NCA = V^2/R \cdot \cos\beta - g \cdot \sin\beta \approx V^2/R - g \cdot h/b \quad [\text{m/s}^2] \quad (1)$$

where V (m/s) is the train speed, R (m) is the curve radius, g (m/s^2) is the constant of gravity, h (mm) is the cant in curve, b (mm) is the distance between the contact points of the wheels on the rails and β is the track angle, as shown in Figure 5.

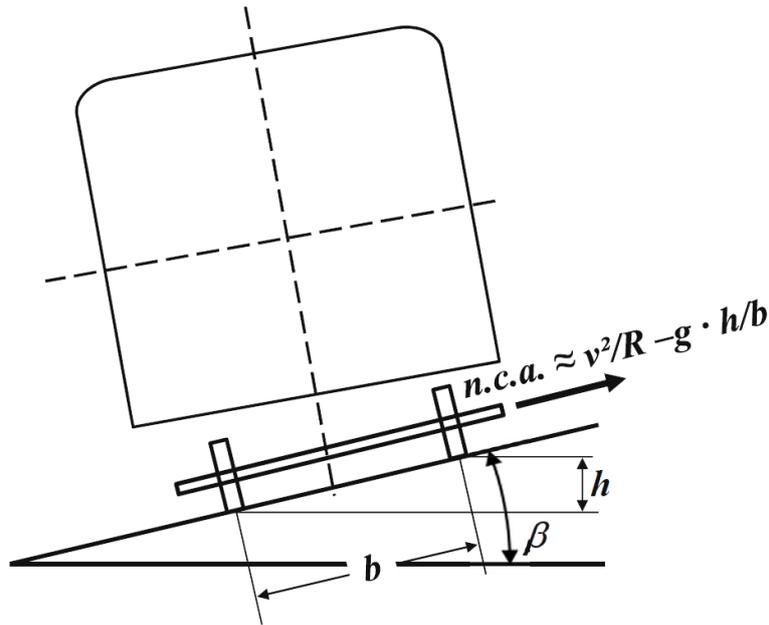


Figure 5: NCA definition

The cant deficiency (CD), a quantity commonly used in railway for practical reasons, is the amount of rail cant missing to annul the NCA and is given by:

$$CD = NCA \cdot b / g \quad [\text{mm}] \quad (2)$$

In the ideal case of a single wheelset running in a curve with CD, the quasi-static track shift forces (ΣY_{qst}), i.e the sum of lateral forces acting on the two wheels, is mainly function of the axleload and of the CD.

In real bogies, when running with a constant CD in all the curves (e.g. CD = 150 mm), the mean value of the ΣY_{qst} on the two wheelsets is constant, but normally not equally distributed between the two wheelsets, as shown in Figure 6 (left). The quantity on the first wheelset (WS1) and on the second (WS2) depends also on the design characteristics of the vehicle and varies as function of the radius of the curve. By increasing the cant deficiency in the same curves (e.g. CD = 300 mm), typical of a tilting train, not only an increase in forces is observed but a still different distribution according to the curve radius, as shown in the same figure (full lines in comparison to dotted lines). Finally, a further contribution to the wheel-rail contact forces is due to the track defects (dynamic effect), as it will be shown better below, depending, to a varying extent, from the speed, CD and dynamic properties of the vehicle. In Figure 6 (right) the two bars, as function of the CD, give an idea of the different component of ΣY_{max} . The mean quasi-static part due mainly to the CD, a certain scatter due to the radius of the curves and the dynamic effect due to the track irregularities.

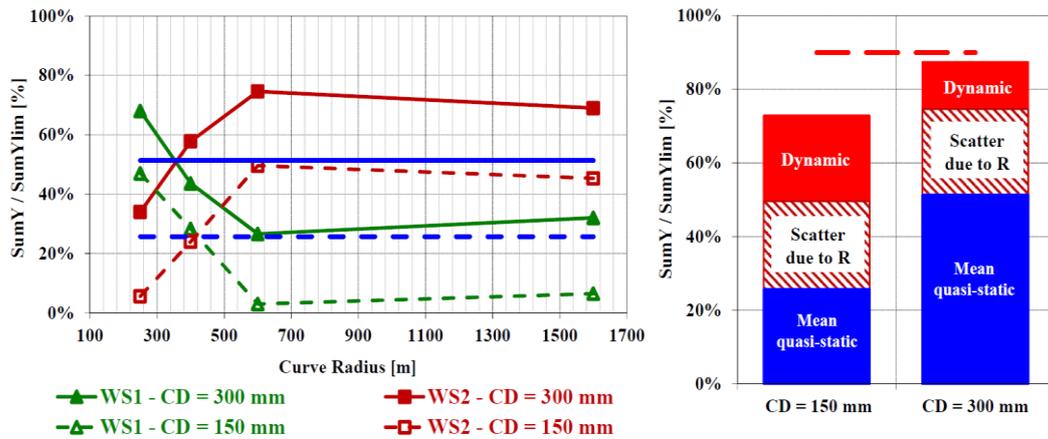


Figure 6: Variation of $\Sigma Y_{qst} / \Sigma Y_{lim}$ in function of curve radius (left); Contributions to track shift force ΣY_{max} (right)

As regards the maximum values of the forces that govern the running safety, and of the loads exchanged between the vehicle and the track, other parameters such as the Rate of Change of Cant (related to the length and gradients of the transition curves), as the track stiffness and damping values, as the transitions between open track and tunnel/bridges, should be taken into account.

The macro-geometry is therefore important and should be well known in the design phase of the railway vehicle. In

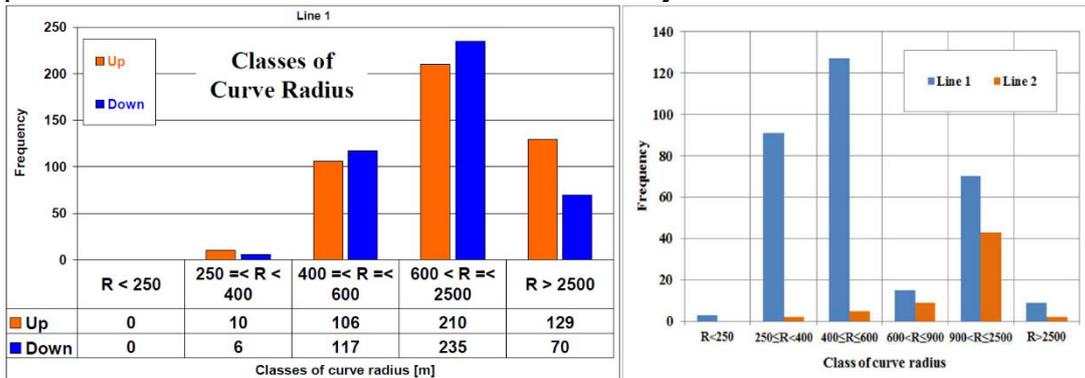


Figure 7 (left) an example of analysis of a service line, about 500 km long, in the two running directions, considering the classes of curve radius is shown.

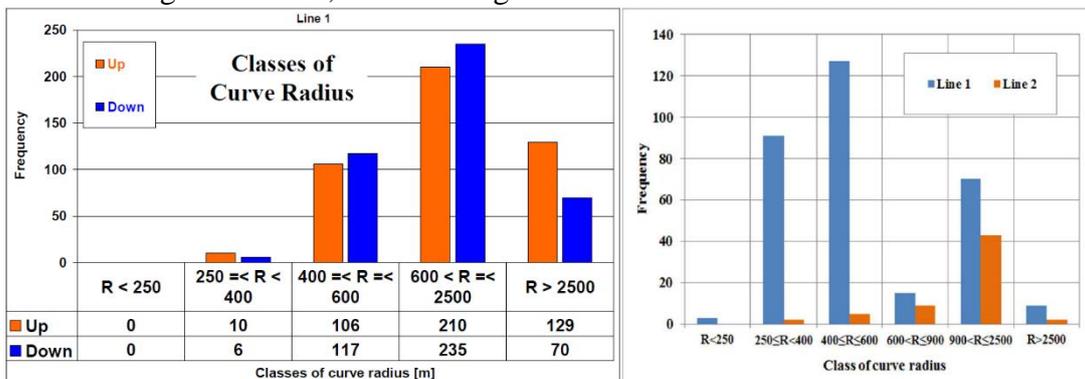


Figure 7 (right) the same type of analysis has been made on two different lines, dedicated to regional service, but with a quite different distribution of curves.

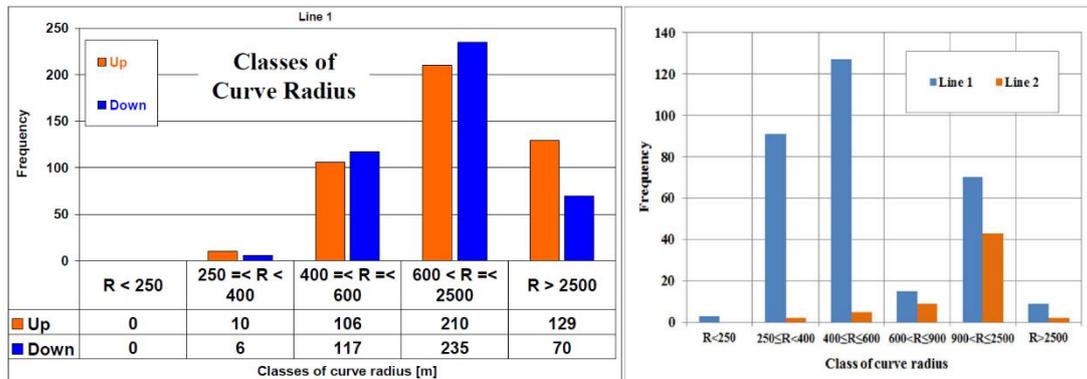


Figure 7: Examples of macro-geometry analysis - curve radii classification and distribution

In similar way, also other geometric parameters, like the cant in curve or the transition slope, can be analysed and a correlation between the track layout and the vehicle-track exchanged forces can be defined.

3.2 Track Irregularities or Micro-Geometry

Track irregularities or defects, named also as track micro-geometry, are deviations from the average value or from the design value of some geometric parameters of the track, defined in the vertical or horizontal plane, related to safety and ride quality. Their classification in different levels is known as analysis of the the track geometry quality.

The track irregularities that characterise the state of maintenance of a track, are evaluated in terms of:

- Type: longitudinal level, alignment, cross level, track gauge, twist, as shown in Figure 2 (ii);
- Amplitude of each defect;
- Wavelength (λ) content of each defect.

For the railway dynamic studies, wavelengths greater than 1 m are usually considered, while typical range for the track quality assessment, according to EN 13848-1 [97], are $3 \text{ m} < \lambda \leq 5 \text{ m}$, $25 \text{ m} < \lambda \leq 70 \text{ m}$ and, for line speeds greater than 250 km/h, $70 \text{ m} < \lambda \leq 150 \text{ m}$ for longitudinal level and $70 \text{ m} < \lambda \leq 200 \text{ m}$ for alignment.

Figure 8 shows an example of the track micro-geometry inputs for simulation in SIMPACK. The measured irregularities (lateral, vertical, cross level and track gauge) are analysed in terms of Power Spectral Density, in function of the spatial frequency, which is useful to identify the wavelength content of the measured data and, consequently, the exciting frequencies in function of the running speed.

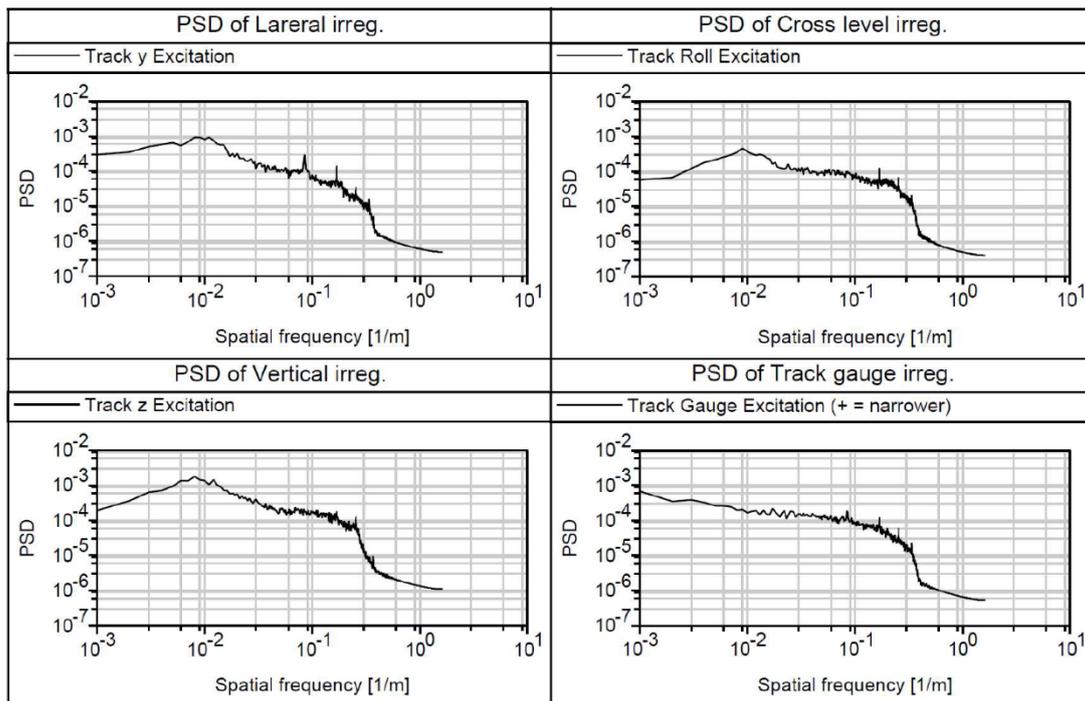


Figure 8: Analysis of wavelength content through Power Spectral Density function

The international standards and the directives of the railway administrations use the standard deviation or the peak values of the measured irregularities to judge the status of maintenance of the track. The EN 13848-5 [98] defines three levels of track conditions to which correspond actions of maintenance:

- Immediate Action Limit (IAL): Refers to the value which, if exceeded, leads to the infrastructure manager taking measures to reduce the risk of derailment to an acceptable level. This can be done either by closing the line, reducing speed or by correcting the track geometry.
- Intervention Limit (IL): Refers to the value, which, if exceeded, requires corrective maintenance in order that the immediate action limit shall not be reached before the next inspection.
- Alert Limit (AL): Refers to the value which, if exceeded, requires that the track geometry condition is analysed and considered in the regularly planned maintenance operations.

Thus, the infrastructure manager can assess the status of maintenance of the track by measuring the track irregularities and comparing the results with the limits of the following parameters:

- Lateral alignment: standard deviations (alert limit only)
- Longitudinal level: standard deviations (alert limit only)
- Lateral alignment (isolated defects): mean to peak values
- Longitudinal level (isolated defects): mean to peak values
- Track twist (isolated defects): zero to peak value
- Variation of gauge (isolated defects): nominal gauge to peak value
- Mean track gauge over any 100 m length: nominal gauge to mean value

The track quality is determined by the presence of the above defects with certain amplitudes that are repeated with several wavelengths.

To understand the effect of a single defect, let's consider the simple model illustrated in figure 9 consisting of a single wheelset, connected to a body of mass M through a suspension stage, which advances with speed V on a sinusoidal irregularity of amplitude A and wavelength λ . The simple equation that determines the variation of vertical force P at the wheel-rail contact is the following:

$$P = P_0 - (m_0/2) \cdot a_z \quad (3)$$

where P_0 is the vertical load at the wheel-rail contact point, m_0 is the mass of the wheelset and a_z the vertical acceleration of the suspended mass, directly proportional to the square of the speed V and the amplitude A and inversely proportional to the square of the wavelength λ . Therefore the vertical load P_0 and the related wheel unloading $\Delta P/P_0$, linked with overturning and derailment phenomena, depends not only by the speed but also by the shape of the track defects. In Figure 9 on the right at the top and bottom, on graphs with double-axis of the ordinates, the vertical acceleration and the wheel unloading $\Delta P/P_0$ with its limit are diagrammed: at the top, keeping the speed constant, as the wavelength of the defect changes; at the bottom, on the contrary, by setting a certain wavelength and by making the speed vary.

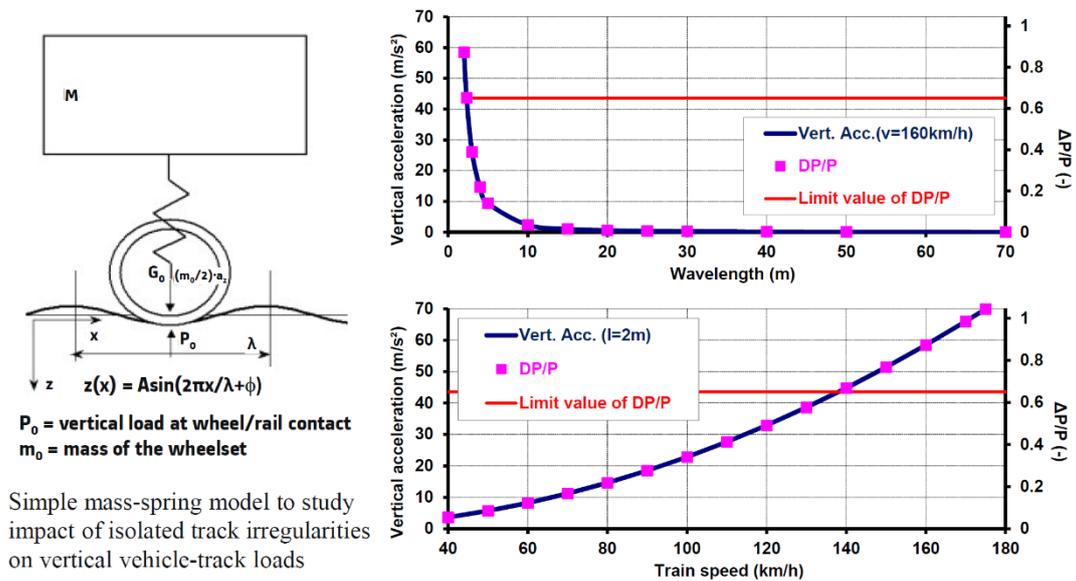


Figure 9: Simple model of track excitation (left); Vertical maximum acceleration and $\Delta P/P_0$ vs. wavelength at constant speed (right up); Vertical maximum acceleration and $\Delta P/P_0$ vs. speed at constant wavelength (right down)

It is evident that the running speed and the shape of the defect, characterised by amplitude and wavelength, are of great concern in determining the vehicle response to track irregularities.

Moving from the simple model to the complex model that considers the combination of multiple defects, Figure 10 on the left shows an example of how the

wheel unloading ($\Delta P/P_0$) increases when the track excitation is composed of multiple components. The graph in Figure 10 on the right shows how the vertical acceleration of the wheel, due to the combination of track irregularities, is amplified by about 15-20% (in the example considered) compared to the acceleration caused by the single vertical irregularity.

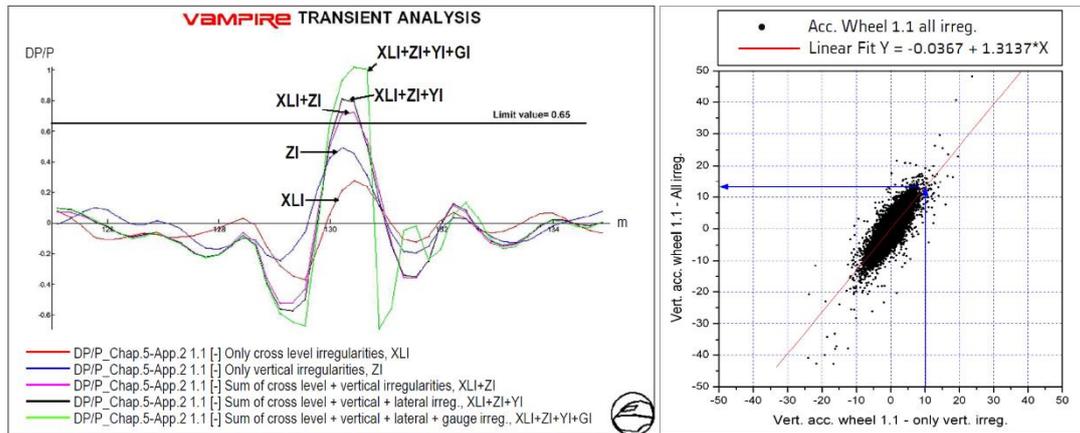


Figure 10: Combined track irregularities: Effect on $\Delta P/P_0$ (left), Effect on vertical acceleration of the wheel (right)

When considering all the track irregularities, the amplitude and the wavelength (λ) (or the spatial frequency ($1/\lambda$)) contents are very important in determining the vehicle behaviour. To study the effect of different amplitudes, simulations with the same model, changing only the track irregularities amplitude by a scaling factor are performed. For this type of analyses, the use of theoretical irregularities, like the small and big defects defined in ORE B176 Rp1 Annexe 6 [99], and shown in Figure 11, is more appropriate than the measured data. In the following plots, red and green curves indicate results for big and small defects, respectively.

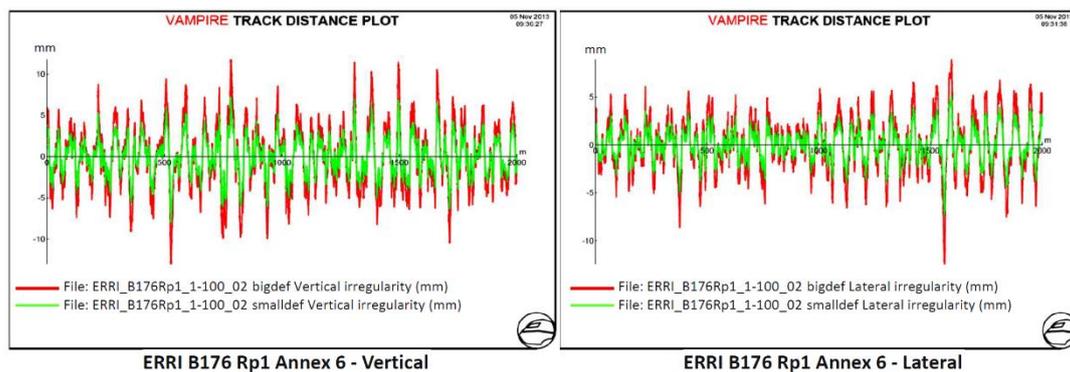


Figure 11: ERR1 B176 Rp1 Annex 6 small and big defects comparison: Space history of vertical defect (left), Space history of lateral defect (right)

The simulation results of the same vehicle model, running on the tracks with two levels of irregularities, can be compared to assess the influence of the different defect

amplitudes. Figure 12 compares the space history of the bogie and carbody accelerations, while the PSD analysis of the same signals is compared in Figure 13.

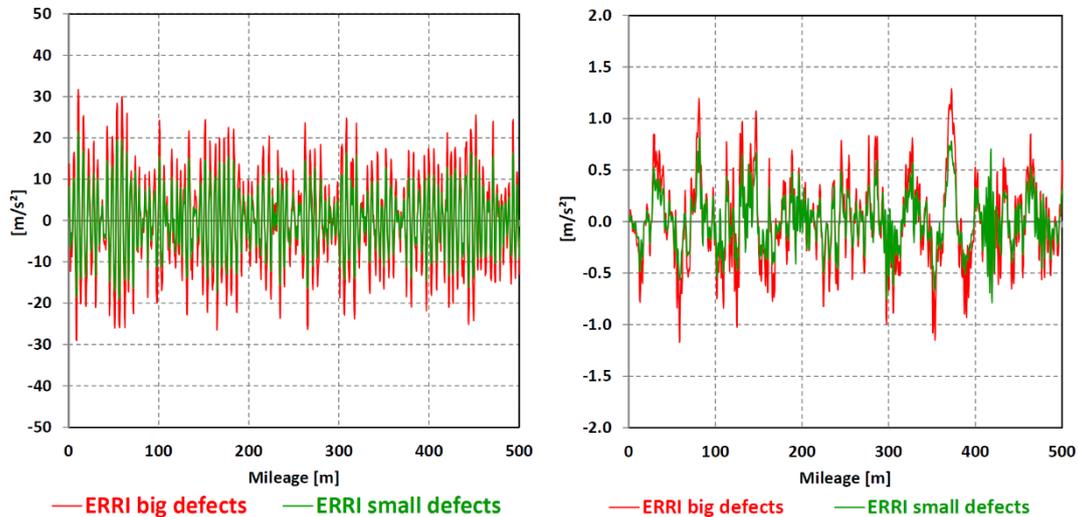


Figure 12: Space history of vertical acceleration on: Bogie (left); Carbody (right)

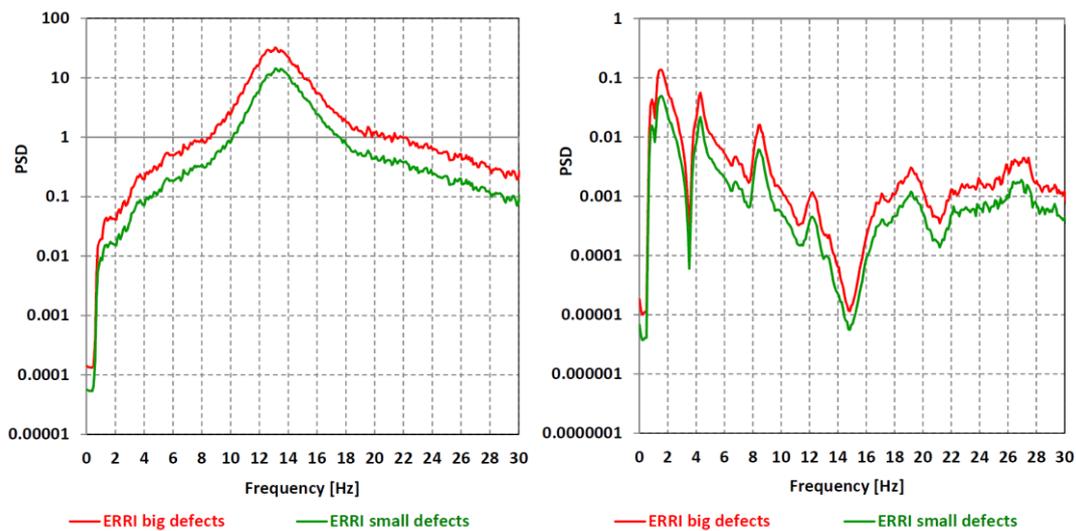


Figure 13: PSD of vertical accelerations: Bogie (left); Carbody (right)

These simulations are performed with an high speed train with flexible carbody running at 250 km/h. The same type of analyses can be done for lateral accelerations.

Accelerations and forces generated on the wheel/rail contact are transferred from the contact point to the different vehicle components and sub-systems. Figure 14 shows the response, to track excitation, of the wheelset (non-suspended mass), of the bogie (one level of suspension) and of the carbody (two levels of suspension). The typical frequency ranges of primary and secondary suspensions are highlighted in the dot-line rectangles, while the red rectangle put in evidence a range of frequencies

associated with the resonance of flexible modes of the carbody. It is observed that the level of vibrations on different parts of the vehicle and of the suspended equipments, varies in function of the frequency and of the track excitation.

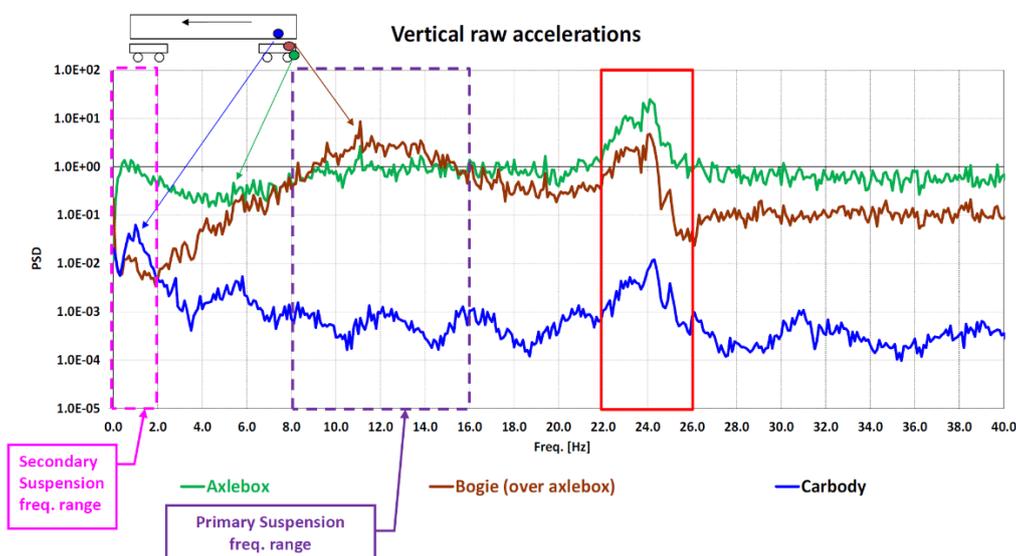


Figure 14: Frequency response to track excitation from axlebox to carbody structure

It is observed that the loads on the vehicle depend on the track excitation amplitudes and frequencies, on the different level of suspension of the detecting points, on the natural frequencies of the system and on the running speed. Moreover, the propagation of the vibrations on the vehicle induced by the track irregularities demonstrate that, in some cases, the filtering effect of the vehicle suspensions can be partly by-passed or even that the response of the vehicle-track system can be amplified with respect to the initial excitation.

3.3 Rail Profiles and Equivalent Conicity

Another indication of the maintenance status of the track is given by the wear of the rail profile, an example of which is shown in the upper part of figure 15. The variation in the shape of the profile, with different patterns on the inner (lower) rail and the outer (higher) rail, modifies the geometry of the wheel-rail contact and consequently the dynamics of the vehicle and in particular the loads on the vehicle and on the track. A derived and synthetic parameter for the evaluation of the conditions of the wheel-rail contact is the equivalent conicity γ_{eq} .

Figure 15 (top) shows the difference of between new (red) and worn (blue) rail profiles in a measured section of a curve and Figure 15 (down-left) the consequent equivalent conicity γ_{eq} calculated with new S1002 wheel profile.

The effect of the two contact conditions on the track-shift forces is shown in Figure 15 (down-right), through the Power Spectral Density analysis of the track-shift forces: the higher level of track-shift forces in case of worn rails, that is to say in case of higher equivalent conicity, is much evident. Therefore the maintenance status of the

rails and the equivalent conicity in service shall be monitored, to ensure safe operations and to limit as much as possible the dynamic loads.

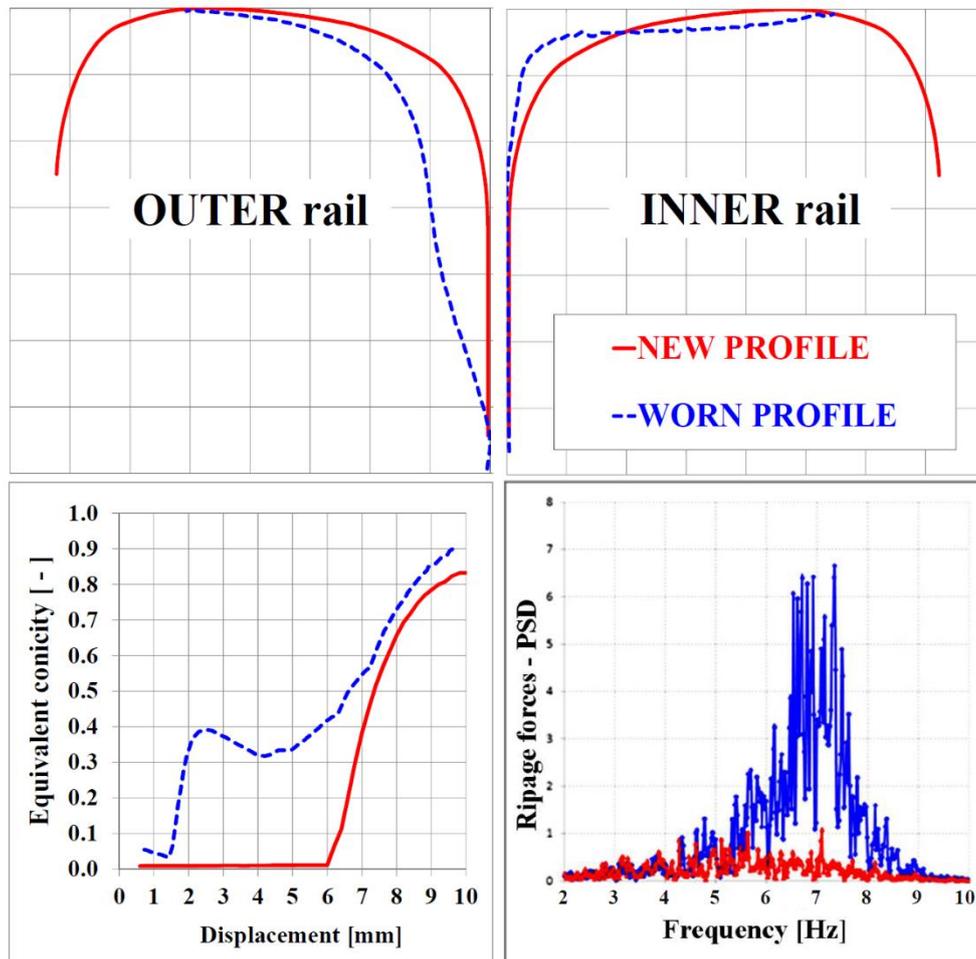


Figure 15: New and worn rail profiles patterns (top); Equivalent conicity γ_{eq} function (down-left); Track-shift forces analysis PSD (down-right)

4 Conclusions and Further Developments

The track degradation shall be kept under control to minimise safety risks and operational costs. This work analyses the influence of different states of track maintenance on vehicle-track loads and on the dynamic behaviour of the rail vehicles.

The track geometry, track design characteristics and wear of rail profiles are important factors in the design and life cycle cost of the rolling stock. The results presented here allow to conclude:

- The track layout is a key parameter affecting the wheel-rail contact loads and their distribution on different wheelsets and consequently the wheel-rail wear and profile evolution. Therefore, it should be known since the beginning and taken into account for the vehicle design optimization.

- The track quality, seen as amplitude and wavelength of the track irregularities on the inner and outer rail, is the main source of the vibrations and shocks acting on vehicle and track parts. Their frequencies are related with the irregularity wavelength and the running speed, while the amplitude of the accelerations and loads depends on the amplitude of the irregularities, together with the proper frequency and inertia of the vibrating components.
- The condition of degradation of the rails causes a significant increase in the loads exchanged between the vehicle and the track. Track maintenance criteria are therefore a key factor in minimizing safety risks and operating costs, both for the infrastructure managers and the service providers.

For all these reasons, the analyses presented in this work are important but need to consider the real health state of the track components and the service tolerances allowed for the various parameters of the track sub-system.

Future steps in this work should go towards (i) the enhancement of the vehicle MB models for the simulation of the vehicle-track response in the whole range of the frequencies of interest, and; (ii) the extension of the methodology proposed here to determine the loads mission profile on the vehicle parts and on the mounted equipments.

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ORCID iD

Naim Kuka: 0000-0002-0707-0445

João Pombo: 0000-0002-5877-1989

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Acronyms

CD	Cant Deficiency
$\Delta P/P_0$	Wheel unloading, $\Delta P = P - P_0$
γ_{eq}	Equivalent conicity according to EN15302
MBS	Multi-Body Simulation
NCA	Non Compensated Acceleration (by the cant in curve)
P	Dynamic vertical wheel force
P_0	Nominal static vertical wheel force
P_{F0}	Nominal static vertical wheelset force (EN 14363)
PSD	Power Spectral Density function
RCC	Rate of Change of Cant according to EN13803-1
S1002	Wheel tread profile according to EN13715 and in conformity with UIC Leaflet 510-2
ΣY_{qst}	Sum of quasi-static values (50 th percentile) of guiding forces of left and right wheel
ΣY_{max}	Sum of maximum guiding forces of left and right wheel used for assessing compliance with regard to the safety against track shifting
ΣY_{lim}	= (10 kN + $P_{F0}/3$) Maximum admissible value of ΣY_{max} according to EN14363