

# **THE EFFECTS OF COMPETITION AND COLLABORATION ON EFFICIENCY IN THE UK INDEPENDENT SCHOOL SECTOR**

## **ABSTRACT**

This paper tests whether competition and collaboration as well as a broad range of other factors lead to improved efficiency in the UK independent school sector. These schools have always operated in a competitive environment, but collaborative groupings are also observed. To answer our main aim, a robust conditional efficiency estimation is employed. Greater efficiency is associated with higher market share, at least at the low levels of market share observed in the sector. There is also some evidence of a positive effect on efficiency of collaboration. Findings regarding the efficiency of schools in the independent sector will be of interest to both independent and state schools as Government policy in the UK has over time encouraged schools in the state sector to become more competitive as an initiative designed to enhance efficiency. The Government is also encouraging greater collaboration between state schools such that they gain benefits from collaboration and sharing of good practice.

## **KEYWORDS**

Competition; collaboration; education; robust partial frontiers; conditional model.

## **JEL CLASSIFICATIONS**

C14; C23; C61; C67; D24; I21

## **1. INTRODUCTION**

The focus of this paper is the UK independent senior schools' sector, particularly those schools that educate pupils up to the age of approximately 18. The paper examines the multiple factors determining efficiency in this UK independent school sector, paying particular attention to the impact on efficiency of competition between schools (both independent and state schools) and cooperation between schools. It is intended that the results will be of interest to those (particularly managers) operating both in independent and state school sectors as well as to policy-makers, as the state school sector moves into a more competitive environment, and as collaboration is increasingly encouraged by policy-makers. The results are also likely to be of interest to education policy-makers internationally. A further feature of the analysis below is that it highlights that it is not appropriate to consider the independent and state education sectors as totally separate, and that there is competition between these education sectors.

The independent (or private) school sector in the UK has been in operation since 1394, functioning in a highly competitive environment. In 2019 there were 1,364 independent

schools in the UK<sup>1</sup>, educating over 530,000 pupils in that year. With average fees in 2019 for boarders of £34,695, and for day pupils of £14,289, independent schools face competition from within their own sector and also from schools in the state sector, where comparable fees are essentially zero, and competition is on the basis of (perceived) quality of education. Of new pupils entering the UK independent school sector in 2019, 27.4% came from the state sector, another indicator of the competition between these education sectors. School fees are the key source of income for independent schools. Since the financial crisis (2007/2008) independent schools have faced a particularly tough environment, and pupil numbers declined in the aftermath of the crisis between 2009 and 2011, presumably because some families faced greater difficulties affording the school fees. There are now signs of buoyancy: pupil numbers have been increasing in recent years, and are now more than 21,000 higher than in 2009. In addition, since 2010 annual fee increases have averaged 3.9%, whereas in the previous decade annual fee increases averaged 6.6% (Independent Schools Council 2019). The current economic situation caused by Covid-19, however, might impact negatively on the independent school sector once again.

Pupil composition in independent schools is diverse on a number of dimensions. Pupils coming from ethnic minority backgrounds, for example, have been on the increase: while 23% of pupils were from this group in 2009, this had risen to 33.8% in 2019, and this varies by region as in the UK population as a whole. Independent schools also attract pupils from across the social spectrum with around 34% of pupils receiving some level of fee reduction, and just over 1% paying no fees at all. The vast majority of pupils (87%) attend their independent school on a daily basis, and most (75%) are in coeducational (boys and girls taught together) schools. Independent schools have an international dimension not typically found in the state sector with 5.4% of pupils coming from overseas (Independent Schools Council 2019). To put the UK independent school sector into perspective, the state school sector in England (in particular, state-funded primary and secondary schools) is considerably bigger with 8.06 million pupils and over 20,000 schools. Over 30% of pupils are from ethnic minorities and 15.4% are eligible for and claim free school meals – an indicator of socio-economic status (Department for Education 2019).

The independent school sector provides an excellent context in which to investigate the effect of school competition on efficiency of operation as competition has been so long established. Many independent schools also operate under the umbrella of a coalition. Collaboration in the independent schools sector is here taken to include schools under common ownership such as the United Church Schools Trust (UCST – 12 schools) and Woodard group (20 independent schools plus others not in the independent school sector), and looser coalitions including the Eton Group (12 schools); the Rugby group (18 schools); Yorkshire Boarding Schools Group (YBSG – 16 schools); Girls Day School Trust (GDST – 26 schools). These

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<sup>1</sup> Of these 1,364 schools, 645 are junior schools (up to age 11), 251 senior schools (from age 11) and 468 are mixed age schools.

coalitions can share good practice and may be able to make cost savings that translate into greater efficiency. Yet coalitions can also be detrimental to competition: the Sevenoaks cartel comprised 50 independent schools that, in the early 2000s, were investigated and found guilty of operating a fee-fixing cartel. The range of school groups which we observe in the independent school sector therefore provides an ideal opportunity for us to assess the effect on operational efficiency of collaboration as well as competition. We control explicitly for the impact of collaborations and the Sevenoaks cartel in the analysis that follows.

Introducing competition into state school sectors has long been advocated as a means of achieving efficient delivery of compulsory education. In England, for example, the 1988 Education Reform Act introduced a quasi-market for schools within which parents could choose which state school to send their child (Levačić 2004). Similarly, between 1988 and 1994 the USA introduced its first school choice programme (Hoxby 2003); New Zealand introduced competition by removing school zoning during the period 1991 to 2000 thereby allowing pupils to attend any public school that would accept them (Harrison and Rouse 2014); and more recently school vouchers were introduced into the Lombardy region of Italy with other regions of Italy subsequently replicating the system. In times of austerity when public funding needs to deliver more for less, competition is still seen as a means to greater efficiency: successive UK governments have further encouraged competition by, for example, the introduction of the free schools programme, permitting the creation of new school places without demonstrating demand, and the encouragement of independence and autonomy for teachers (Morris 2013).<sup>2</sup>

The premise upon which such policies are based is the structure-conduct-performance (SCP) paradigm which predicts that increasing competition (i.e. changing the structure of a market) will impact on the many aspects of the conduct of firms (schools in this case), and that one of these facets will be efficiency. The SCP paradigm then suggests that firms' conduct impacts on their performance. Typically, for firms, performance is measured in terms of profits, but in the schools' context, performance may be in terms of pupils' examination performance, the ease with which the school attracts pupils, or operational efficiency.

It is somewhat contradictory that collaboration amongst schools, which might be viewed as the opposite of competition, is also seen as a way of generating more efficiency. In the UK, for example, the Government has used its academies and multi-academies trusts programme as a means of sharing of good practice. As all state schools are set to become academies (Department for Education 2016), collaboration is likely to become an increasingly important tool for delivering efficiency savings.

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<sup>2</sup> In the UK, free schools are a form of academy school. Academy schools (including free schools) are non-profit-making, state-funded schools where families are not charged fees and which are independent of local government control. Free schools can be set up by like-minded groups, such as groups of parents, teachers, charities or religious groups, for example.

While limited research exists into analysis of the impact of *competition* on operational efficiency in schools (see, for example, Bradley *et al.* 2001), investigation of the effect of *collaboration* on school efficiency appears to be un-researched.<sup>3</sup>

To analyse the impacts of competition, collaboration and a broad range of other factors on school efficiency, we apply a conditional efficiency model to a unique data set of UK independent schools over a 10-year period up to 2012/2013. The data period was selected to cover the years during which fifty leading independent schools were found to be engaged in a school fee fixing cartel (the Sevenoaks cartel), and the period afterwards including the years associated with a major financial crisis and beyond. The focus on economic disruption caused by the financial crisis is particularly relevant given the current economic situation caused by the Covid-19 pandemic.

In the context of education, outputs can be seen to be a function of different inputs involved in the educational production process (Hanushek 1986; 2003). Many previous studies of efficiency in the school context have used Data Envelopment Analysis (DEA) to estimate first-stage efficiencies followed by a Tobit regression in the second stage (based on the premise that the dependent variable is a censored variable) to explore the factors which affect efficiency (Ray 1991; Kirjavainen and Loikkanen 1998; Mancebón and Mar Molinero 2000; Bradley *et al.* 2001; Al-Enezi *et al.* 2010; Agasisti 2013b; Burney *et al.* 2013). Simar and Wilson (2011) have shown that there are no circumstances under which such Tobit regressions would produce consistent estimates in the second stage. Two alternative approaches are instead suggested, the OLS approach of Banker and Natarajan (2008), and the double bootstrap truncated regression procedure of Simar and Wilson (2007)<sup>4</sup>.

Nonetheless, all of these approaches require the restrictive separability condition to hold (Daraio *et al.* (2018), which is a crucial issue that is difficult to expect in empirical frameworks such as education. It implies that the factors which condition the performance of the schools are implicitly assumed to influence only the inefficiency levels and not the shape of the efficient production frontier (i.e., the factors would be considered as fully independent of the input and output space). The conditional approach (Cazals *et al.* 2002; Daraio and Simar 2005; 2007) overcomes this limitation and allows the estimation of efficiency scores to account for the effect of contextual factors in a single step. Moreover, we apply the robust version of those models as well as a dynamic extension of it, in order to avoid the possible drawbacks induced by outliers and extreme data and also to account for the effect of time in the estimation of the efficiencies of independent schools, respectively.

A principal finding of the current analysis is that, contrary to the predictions of the SCP paradigm, whereby greater competition between schools is likely to result in efficiency being

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<sup>3</sup> Although a recent report from the Education Policy Institute highlights that there are not unambiguous gains for multi-academy trusts relative to Local Authorities in terms of school performance (Andrews 2016).

<sup>4</sup> See Wu *et al.* (2020) and Salas-Velasco (2019) for applications of each of these approaches (respectively) in higher education.

boosted, in fact in this case, greater market share (less competition) leads to higher efficiency. There are clear policy implications arising from the conclusion that greater market share is associated with higher school efficiency. However, the analysis below indicates that this relationship between market share and efficiency may also be nonlinear (an inverted U shape): initially positive and then negative at much higher levels of market share. Policy-makers need to take this into account as there might be an optimum level of market share. In addition, evidence suggests that there may be some benefits for efficiency from collaboration, but that cooperation may in some cases have negative effects. Note that the coalitions of schools in the independent sector are arguably similar to the multi-academy trusts which have emerged since the promotion of academies in the state sector. These results relating to the independent school sector can offer insights into the state school sector, and suggest that further work is required to identify which characteristics of coalitions make them likely to be associated with greater school efficiency such that further policy recommendations can be made.

This paper is in 6 sections of which this Introduction is the first. We review the relevant literature in Section 2 and present our empirical strategy in Section 3. Section 4 presents the data and the variables used in our analysis, while Section 5 reports the results of the conditional DEA. Finally, conclusions are drawn in Section 6.

## **2. LITERATURE REVIEW**

There is a growing literature which examines aspects of the (UK) independent school sector, and in particular the senior independent school sector. These studies focus on factors determining demand for an independent school education (Blow *et al.* 2010; Blundell *et al.* 2010; Dearden and Sibieta 2010); the determinants of examination success (Graddy and Stevens 2005); the wage benefits of attending an independent school, (Dolton and Vignoles 2000; Dearden *et al.* 2002; Naylor *et al.* 2002; Green *et al.* 2012); the determinants of school fees (Starkie and Wise 2006; Elliott *et al.* 2016); and details of the Sevenoaks fee-setting cartel (Pesaresi *et al.* 2015; Elliott *et al.* 2016). To date there is no work on determinants of efficiency in the UK independent school sector. However, note that Elliott *et al.* (2016) do consider the impact of collaboration between independent schools on levels of fees set, setting a precedent in the literature for the consideration of possible impacts of school collaborations as well as competition between schools.

In contrast, there is an extensive literature considering the state school sector, both in the UK and overseas. There are two key strands to this literature: the first focuses on factors determining school performance as measured by achievement in examinations or graduation rates at the level of pupils, schools or countries (see, for example, Taylor and Bradley 1998; Woessmann 2003; Chakrabarti 2008; Agasisti and Murtinu 2012; Dolton *et al.* 2014). Some of this literature focuses specifically on the effect of competition on achievement. There is considerable evidence that competition has a positive (and largely significant) effect on

achievement in tests or graduation rates (Dee 1998; Hoxby 2000; Belfield and Levin 2002; Woessmann 2003; Levačić 2004; Millimet and Collier 2008; Agasisti 2011a; Ponzo 2011; Agasisti and Murtinu 2012; Misra *et al.* 2012; Agasisti 2013a; Thapa 2013). Nevertheless, a number of studies find no significant evidence of such a relationship (Geller *et al.* 2006; Zimmer and Buddin 2009; Kamienski 2011; Caldas and Bernier 2012); and Rothstein (2007) disputes the finding by Hoxby (2000; 2007). Of particular interest is the finding that greater independent school activity in a region (measured by enrolments or school numbers) seems to have a positive effect on state school examination outcomes (Couch *et al.* 1993; Dee 1998; Hoxby 2003; Chakrabarti 2008; Agasisti 2011a; 2013a; Thapa 2013). Although again there is some disagreement in the literature as some studies suggest that competition from independent schools has no significant effect on state school outcomes (Jepsen 2002; Geller *et al.* 2006; Caldas and Bernier 2012).

The second strand of this literature considers efficiency levels in the state school sector and is therefore of greater relevance to the research in this paper. School efficiency is distinct from school (examination or graduation) performance in that it looks at the outcomes in relation to the resources used to produce them. Frontier estimation techniques such as DEA, FDH and partial frontiers are typically used to estimate efficiency. Early reviews of studies which use these and other techniques to measure efficiency in the school context can be found in Bradley *et al.* (2001) and Johnes (2004); a more recent review can be found in Johnes (2015), Thanassoulis *et al.* (2016), De Witte and López-Torres (2017) and Emrouznejad and Yang (2018).

Of particular relevance to us in this literature are studies that consider the impact of competition on state school efficiency in the UK and elsewhere (Bradley *et al.* 2001; Grosskopf *et al.* 2001; Bradley and Taylor 2002; Agasisti 2011b; 2013b; Harrison and Rouse 2014). All but one of these studies find unambiguously that the greater the number of schools in close proximity (or in the local region), the higher the schools' technical efficiency. Agasisti (2011b; 2013b) particularly finds that having both public and independent schools in a region is beneficial to state school efficiency. The remaining study (Grosskopf *et al.* 2001) finds no significant relationship between technical efficiency and competition where the latter is measured using (respectively) a 4-firm concentration ratio and a Herfindahl index<sup>5</sup>.

The literature on competition and efficiency even in the state school context is limited, and in the independent school context is non-existent. Much of this literature employs truncated regression or Tobit analysis in the second stage. However, such a two-stage approach is undertaken on the premise that the separability assumption holds which is doubtful.

We therefore build on the existing literature in this area by employing a long panel data set (over 10 years) covering both the pre- and post-financial crisis period, and applying a

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<sup>5</sup> The Herfindahl index for a given market is the sum of the squared enrolment shares for all the public and private school systems in that market. See (Grosskopf *et al.* 2001) for more details.

conditional order- $m$  model (Cazals *et al.* 2002; Daraio and Simar 2005; 2007). This analysis leads us to establish the factors which affect efficiency in independent schools through the interpretation of the ratios among the conditional and the unconditional estimates.

We also test the effect on efficiency of a range of variables, since the methodology allows us to directly incorporate these variables into the efficiency estimation in a single step. We focus particularly on the effect on efficiency of both competition and collaboration amongst schools – the effect of the latter on efficiency has not previously been explored in either the state or independent school sectors.

### **3. METHODOLOGY**

The analysis of efficiency of educational institutions has been explored over the last decades as a result of a growing interest in improving their performance (Liu *et al.* 2013). This stream of literature provides findings that allow for a better understanding of the educational factors that influence students' outcomes. It also provides useful information for decision makers and for policy implementation. In terms of methodology, different empirical approaches can be applied to achieve the goal of measuring efficiency in education, however, frontier methods constitute the most used approach (see Johnes (2015) and De Witte and López-Torres (2017) for a review of the methods for measuring efficiency and productivity). Empirical studies apply this approach in two forms: non-parametric (data envelopment analysis – DEA, free disposal hull – FDH, order- $m$  frontiers) and parametric (stochastic frontier analysis – SFA) methods (see Fried *et al.* (2008) for more details on these approaches)]. Most studies analysing efficiency in education adopt non-parametric frontier techniques because this approach can deal with the efficiency of multiple-input and multiple-output processes (Worthington 2001), and since it does not require a specific functional form for the education production function. This is a considerable advantage when the technology of the production function is not obvious, as in the case of education (Johnes 2006). Based on these advantages, we adopt a fully non-parametric approach to measure efficiency in the UK Independent school sector.

#### ***3.1. Robust conditional model***

We develop a robust conditional model<sup>6</sup> (Cazals *et al.* 2002; Daraio and Simar 2005; 2007) with a major advantage being that it overcomes the potential problems arising from the restrictive separability assumption among the exogenous factors and the inputs and outputs space. This is a critical point in the efficiency estimation which implies that the exogenous factors are impacting on the distribution of the inefficiencies but do not impact on the

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<sup>6</sup> These conditional models have been extensively applied in many empirical papers. See De Witte and López-Torres (2017) and Emrouznejad and Yang (2018) for recent literature reviews.

efficient frontier. Neglecting the possibility of non-separability in the sample could lead to misleading estimates (Daraio *et al.* (2018).

We assume that schools use inputs  $X$  to produce outputs  $Y$ , and their production process is constrained by environmental variables  $Z \in \mathfrak{R}^r$  which are assumed as not independent with respect to  $(X, Y)$ . Thus, following (Cazals *et al.* 2002) the probabilistic joint distribution conditional on  $Z = z$  is<sup>7</sup>:

$$H_{X,Y|Z}(x, y|z) = \text{Prob}(X \leq x, Y \geq y|Z = z)^8 \quad (1)$$

When longitudinal data are available as in this analysis, the model can be adapted by considering the time factor ( $t$ ) as an additional exogenous variable. Accordingly, if we handle the whole sample as a cross-sectional dataset, we can detect effects that are common across different but close periods. Following the extension proposed by Mastromarco and Simar (2015) the probabilistic production function in Equation (1) can be modified to account for that dynamic context as follows, making it possible to assess efficiency over a period of time:

$$H_{X,Y|Z}^t(x, y|z) = \text{Prob}(X \leq x, Y \geq y|Z = z, T = t) \quad (2)$$

This function can be decomposed into two terms: the survival conditional function of outputs  $S_{Y|X,Z}^t(y|x, z)$  and the conditional distribution function of inputs  $F_{X|Z}^t(x|z)$ . Thus, for an output-oriented model, the probabilistic production function is represented as:

$$H_{X,Y|Z}^t(x, y|z) = S_{Y|X,Z}^t(y|x, z)F_{X|Z}^t(x|z) \quad (3)$$

By defining this conditional methodology in these terms, we can assume that environmental factors and time could have a bearing on the shape of the best practice frontier as well as on the distribution of the inefficiencies. This means that the separability condition does not hold for this model since the context in which schools operate has been directly included into the nonparametric educational production function (Bădin *et al.* (2012; 2014).

Under this probabilistic formulation, the conditional efficiency measure can be defined in terms of the Farrell efficiency score:

$$\lambda^t(x, y|z) = \sup\{\lambda > 0 | H_{X,Y|Z}^t(x, \lambda y|z) > 0\} = \sup\{\lambda > 0 | S_{Y|X,Z}^t(\lambda y|x, z) > 0\} \quad (4)$$

Different conditional estimators of the full frontier can be defined from (3) by using a plug-in rule, such as the FDH and DEA. Nonetheless, these traditional estimators are quite sensitive to extreme values and outliers, since the frontier is built by using all the data points in the sample. Moreover, they suffer the well-known curse of the dimensionality problem since the rate of convergence of the efficiency estimates decreases, especially when the number of inputs and outputs is greater than 2 (Simar and Wilson 2015).

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<sup>7</sup> Our presentation of the methodology follows Bădin *et al.* (2010; 2019)

<sup>8</sup> The contextual variables are directly included in the estimation by Equation (1). By doing this, the separability condition does not need to be checked since it is assumed that the  $Z$  factors could have an impact on both, the frontier and on the inefficiencies (Daraio *et al.* (2018).

In this scenario, the best methodological option is to resort to conditional nonparametric partial frontier models that can be considered as the robust version of the full frontier alternatives. The efficient frontier now is built with just a limited number of decision making units (DMUs), so these partial frontier estimators are much less sensitive to outliers than the traditional ones. In particular, we use the robust conditional order- $m$  estimator (Cazals *et al.* 2002; Daraio and Simar 2005; 2007) for which each school is compared only with those that use less or the same input values<sup>9</sup>. The analytical expression of the efficiency measures from the order- $m$  frontier is:

$$\hat{\lambda}_m^t(x, y|z) = \int_0^\infty [1 - (1 - \hat{S}_{Y|X,Z}^t(uy|X \leq x, Z = z))^m] du \quad (5)$$

The size of the comparison subset is determined by the parameter  $m$  which choice is discussed, for example, in (Cazals *et al.* 2002; Daraio and Simar 2005; 2007) and Bădin *et al.* (2012; 2014). In this paper, for practical purposes, we follow the criteria in Tauchmann (2012)<sup>10</sup>. As the frontier will not envelope all observations (only those in that  $X \leq x$ ) there may be *superefficient* units that are below the boundary (with efficiency scores  $< 1$ ).

The estimation of the nonparametric survival function requires smoothing in  $Z$ s to restrict the set of schools for comparison in terms of a bandwidth parameter<sup>11</sup>. These subsamples ensure that each school is only compared with other schools in the same environmental conditions ( $Z = z$  and  $T = t$ ), which makes a fairer comparison. Specifically, this bandwidth allows us to compare units belonging to one and the same year of the period within the observations as a whole. This distinguishes the dynamic extension of the conditional model from other time-analysis techniques.

The computation of conditional efficiency estimators involves the use of smoothing techniques for the exogenous variables in  $z$  and  $t$  (due to the equality constraints  $Z = z$  and  $T = t$ ). This relies on the estimation of a non-parametric kernel function to select the appropriate reference partners and a bandwidth parameter  $h$ . As our dataset includes discrete and continuous environmental factors, we follow the method proposed by Bădin *et al.* (2010) extending the ideas proposed by Hall *et al.* (2004), Racine and Li (2004) and Li and Racine (2006) to smooth all components of  $Z$ <sup>12</sup>. Therefore, the estimator for the conditional survival function can be expressed as:

$$\hat{S}_{Y|X,Z,n}^t(y|x, z) = \frac{\sum_{j=(i,v)} \mathbb{I}(x_j \leq x, y_j \geq y) \cdot \hat{K}_{h_z}(z_j - z) \cdot \hat{K}_{h_t}(v - t)}{\sum_{j=(i,v)} \mathbb{I}(x_j \leq x) \cdot \hat{K}_{h_z}(z_j - z) \cdot \hat{K}_{h_t}(v - t)} \quad (6)$$

<sup>9</sup> Another possibility is to use order- $\alpha$  frontiers (Daouia and Simar 2007). The comparison against the order- $m$  estimator as well as the asymptotic properties can be seen in Daouia and Gijbels (2011). We choose the order- $m$  partial frontiers due to the quantile-based frontiers are less stable to outliers.

<sup>10</sup> The formula that this author suggests is  $\sqrt[3]{n^2}$ .

<sup>11</sup> For interested readers in bandwidth selection methods see, for example, Bădin *et al.* (2010; 2019).

<sup>12</sup> This approach has been applied before in other studies (See, for example, Vershelde and Rogge 2012; De Witte and Kortelainen 2013; Tzeremes 2015; Cordero *et al.* 2017; Cordero *et al.* 2018).

where  $\widehat{K}_h(\cdot)$  represents the multivariate kernel functions for  $Z$ s and  $T$ ,  $\mathbb{I}(\cdot)$  is an indicator function and  $h$  are the appropriate bandwidth parameters for these kernels. Accordingly, this procedure can detect irrelevant factors by providing huge values to the corresponding bandwidths (Bădin and Daraio 2011).

The resulting estimates account for heterogeneity in the educational landscape by including  $Z$  and  $T$  as conditions when estimating schools' efficiency.

### 3.2. Effect of exogenous factors on efficiency

A very interesting advantage that the conditional approach offers is the possibility of exploring the direction of the effect of contextual factors on efficiency. This can be done by following the procedure in Bădin *et al.* (2012; 2014). It can be conducted by analysing the ratios defined as:

$$\widehat{Q}_m^t(x, y|z) = \frac{\widehat{\lambda}_m^t(x, y|z)}{\widehat{\lambda}_m(x, y)} \quad (7)$$

where the  $\widehat{\lambda}_m(x, y)$  are the efficiency scores estimated without including the  $Z$  and  $T$  variables (i.e., the unconditional frontier)<sup>13</sup>.

This ratio can be non-parametrically regressed on a variable in  $Z$  (including  $T$ ) of interest. Graphically, the slope of the smoothed regression line offers an interpretation of the marginal effect of this environmental factor on the attainable set Daraio and Simar (2005; 2007).

In our robust conditional output-oriented model, an increasing shape for the ratios as a function of  $Z$  would correspond to a favourable effect of  $Z$  (higher values of  $Z$  allow us to reach higher outputs,  $Z$  is acting as a freely available input). In other words, the frontier used to evaluate school efficiency with a high value of  $Z$  would be separated (below) from the frontier constructed by the unconditional model. Thus, the larger the  $Z$ , the greater the outcomes the school can achieve. This means that when we compare schools with similar socio-economic and educational environment, the potential output increases.

Analogously, a negative slope indicates a negative effect and the conditional efficiency frontier will be closer to the unconditional frontier ( $Z$  is acting as an undesirable output). Thus, the larger the  $Z$ , the lower the potential outcomes school can achieve. These findings are in line with the literature (for example, Bădin *et al.* 2012; De Witte and Kortelainen 2013; Cordero *et al.* 2018; De Witte and Schiltz 2018). A straight line indicates the absence of an effect. A combination of effects is also possible since the line is smoothed using a nonparametric regression.

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<sup>13</sup> We focus our analysis on the impact of contextual factors on the frontier shifts, whose are derived from ratios estimated by extreme order- $m$  measures. Nonetheless, it is possible to conduct a similar analysis by using ratios from median frontiers, i.e., obtained with small values of  $m$ , with the aim of detecting different effects (see Bădin *et al.* (2012; 2014), for details).

### 3.3. Testing the significance of $Z$

Finally, we can investigate the statistical significance of  $Z$  by applying the nonparametric bootstrap procedure (Racine 1997; Li and Racine 2006; Racine *et al.* 2006) to explain the variations of the ratios. This procedure can be understood as the nonparametric equivalent of standard  $t$ -tests in ordinary least squares regression (De Witte and Kortelainen 2013)<sup>14</sup>. We include as factors in the estimation not only competition and collaboration in the independent school sector, but also an array of other school characteristics which have previously been found to be associated with efficiency (as explained in the next section) as well as the time conditioning factor. Accordingly, it is necessary to control for these other characteristics when examining the association between efficiency and competition and collaboration.

## 4. DATA AND VARIABLES

The data used in this study cover 10 academic years (from 2003/2004 to 2012/2013) relating to the UK independent school sector, with the data period selected to encompass the years during which the Sevenoaks fee setting cartel operated as well as the years of the financial crisis and beyond. The data are drawn from three main sources: The Independent Schools Council (ISC), The Good Schools Guide (GSG) and The Financial Times (FT) Independent Schools Guide. The sample of schools was determined by schools common to all three data sources, as variables from each of the data sources were required. While most data were provided by The ISC, The GSG also provides data on annual day and boarding fees, the numbers of pupils, boarding pupils, any religious affiliation of the school and specialist arts (dance, drama, music) schools where appropriate. Similar data are provided in The ISC and also The FT Independent Schools Guide, so data were compared across these publications to confirm consistency. The FT Guide was required as it contains the most comprehensive data on average pupil General Certificate of Secondary Education (GCSE) and A-Level public examination performance.<sup>15</sup>

Our unit of analysis is the school rather than the student. Although we are aware of the consequences of aggregation, we consider that school level data are more appropriate for our purpose, namely the focus on factors determining school efficiency levels. Our aim is to identify the effect of school level characteristics – with particular emphasis on the competitive environment and collaborative opportunities – on the efficiency of schools.<sup>16</sup> We

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<sup>14</sup> As is noted in Daraio and Simar (2014), this procedure is only acceptable when partial frontiers are used in the estimation, as in this case.

<sup>15</sup> GCSEs are typically taken by pupils at the end of year 11 of full-time school education, around age 16. Pupils are expected to take GCSEs in Mathematics, English Language and Science. Schools will also often advise pupils to take GCSEs in at least one foreign language, a humanities subject and English Literature. A-levels are typically taken by pupils two years later. There are no compulsory A level subjects for pupils.

<sup>16</sup> Individual pupil level data also are not available.

collect data points for 2,792 observations over ten years<sup>17</sup>. However, our panel is unbalanced since not all schools provide information over the 10-year period. The sample ranges from 206 to 319 observations per year, with greater numbers of schools in later years of the dataset.

#### 4.1 Variables in the efficiency analysis

Table 1 contains the definition of inputs and outputs used to estimate efficiency.

**Table 1. Definitions of variables in the efficiency model**

Category	Label	Definition	Source	
Input	$I_1$	<i>Teachers</i>	Number of full time equivalent (FTE) teachers per 100 students.	ISC
	$I_2$	<i>FeeIncome</i>	Weighted average incomes from fees by type of student.	ISC
Output	$O_1$	<i>GCSE</i>	School's average points of pupils in GCSEs.	FT
	$O_2$	<i>A-Levels</i>	School's average points of pupils in A-Levels.	FT

Notes: the sample mean for the average points of pupils in GCSEs is 490.37, with a student receiving 58 points if they obtain the highest A\* grade in a subject. An A grade is worth 52 points, a B grade 46 points etc. The sample mean for the average points of pupils in A-Levels is 350.85, with a top A\* grade in a subject being worth 140 points, an A grade being worth 120 points, a B grade worth 100 points etc.

Source: The authors

Following the literature about efficiency in education (for example, Ray 1991; Agasisti 2011a; 2013b; Brennan *et al.* 2013) we use the results from national level tests that are homogeneous for all students as outputs ( $Y$ ). While test results may not capture the full skills set provided by school education, they are used by policy-makers and parents as key measures of schools' output performance (Hoxby 2000). In fact, most studies of school efficiency use test or examination scores to reflect output; just three exceptions use alternative measures (based on pupil destination after leaving school or attendance rates) in addition to the traditional examination performance (Norman and Stoker 1991; Thanassoulis and Dunstan 1994; Bradley *et al.* 2001). As our unit of analysis is the school, we take at two education levels the arithmetic mean of students' grades (denoted by *GCSE* and *A-levels*).

<sup>17</sup> We are aware that our dataset can suffer from cross-sectional dependency and ignoring that can result in inconsistent estimators and thus, misleading inferences (Baltagi and Pesaran 2007; Pesaran 2015). To check for that, we applied Pesaran's test for cross sectional dependence in panel models (Pesaran 2015). The test's results (Y1:  $z = 7.4533$ ,  $p\text{-value} = 9.1e-14$  and Y2:  $z = 23.327$ ,  $p\text{-value} < 2.2e-16$ ) lead us to reject the null hypothesis of no dependence and accept the existence of cross-sectional dependency. This is an expected result for us, as we are not running a parametric model, neither are we doing inference based on the results. Instead, our model is fully non-parametric, accounts for a multi-input multi-output production function, and incorporates the role of time as a contextual factor ( $Z$ ). Therefore, we are controlling for cross-sectional dependence in our efficiency estimation. Briefly explained, to compute the efficiency score, the conditional model looks for DMUs that are similar to the DMU under evaluation in  $Z$  characteristics but which performs better. Thus, we consider the cross-sectional dependence when computing the bandwidths for our model as we are restricting the scope of comparison.

The levels are GCSE (typically taken at the end of compulsory education) and A-Levels (typically taken at age 18).

In terms of inputs, the vast literature analysing the determinants of educational outcomes distinguishes two main sources. The first focuses primarily on the role of the students' family background, including socioeconomic status (SES), family structure, family resources and parental involvement (Sirin 2005). The second covers factors related to school variables and teaching practices. In this paper, we do not have data on the former characteristics and concentrate instead on school resources that are directly involved with student learning. We include two inputs namely *Teachers* and *FeeIncome*. The variable *Teachers* represent labour input i.e. the number of academic staff working in the school per 100 students. The literature reveals that the number of personnel employed is a good proxy for human capital in education institutions (Haelermans *et al.* 2012; Thieme *et al.* 2012; Brennan *et al.* 2013). On the other hand, *FeeIncome* represents the proxy for capital input into the school. It is defined as the weighted average income from fees by type of student, that is computed as the ratio between the total incomes from day and boarding students divided by the number of students. A higher level of fee income (i.e. resource) is expected to lead to better outcomes (Brennan *et al.* 2013; Agasisti 2014). Fee income is expected to be by far the biggest element of income generation for independent schools. Further, most of these schools have charitable status and as such are not run as profit maximizing entities.

Table 2 reports the main descriptive statistics for the input and output variables included in our model. Figure 1 plots the trend in average values for input and output variables. An examination of Table 2 and Figure 1 indicates that the number of teachers follow a positive trend over time, and the ratio of teachers per 100 students is always higher than 10. In addition, incomes from fees have followed the same trend during the period of analysis, although with a slight decline in 2006. Grades tend to exhibit more stability over the 10-year study period. An examination of Spearman's correlations indicates the expected significantly positive relationship between inputs and outputs, supporting their inclusion in the efficiency model.<sup>18</sup>

A potential weakness of our model is the omission of variables relating to family background and to quality of inputs. The latter is a problem with previous studies of state school efficiency (Bradley *et al.* 2001; Agasisti 2011b; 2013b). We counter this with the argument that it is likely the case that pupils at independent schools represent a more homogeneous set (in terms of family background) than might be the case for the state school sector. Moreover, almost all independent schools have an entrance examination by which they select their pupils, many schools adopting the same common entrance examinations of the Independent Schools Examinations Board for pupils' entrance at 11 or 13 ([www.iseb.co.uk](http://www.iseb.co.uk)).<sup>19</sup>

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<sup>18</sup> Results available on request and withheld only for the sake of brevity.

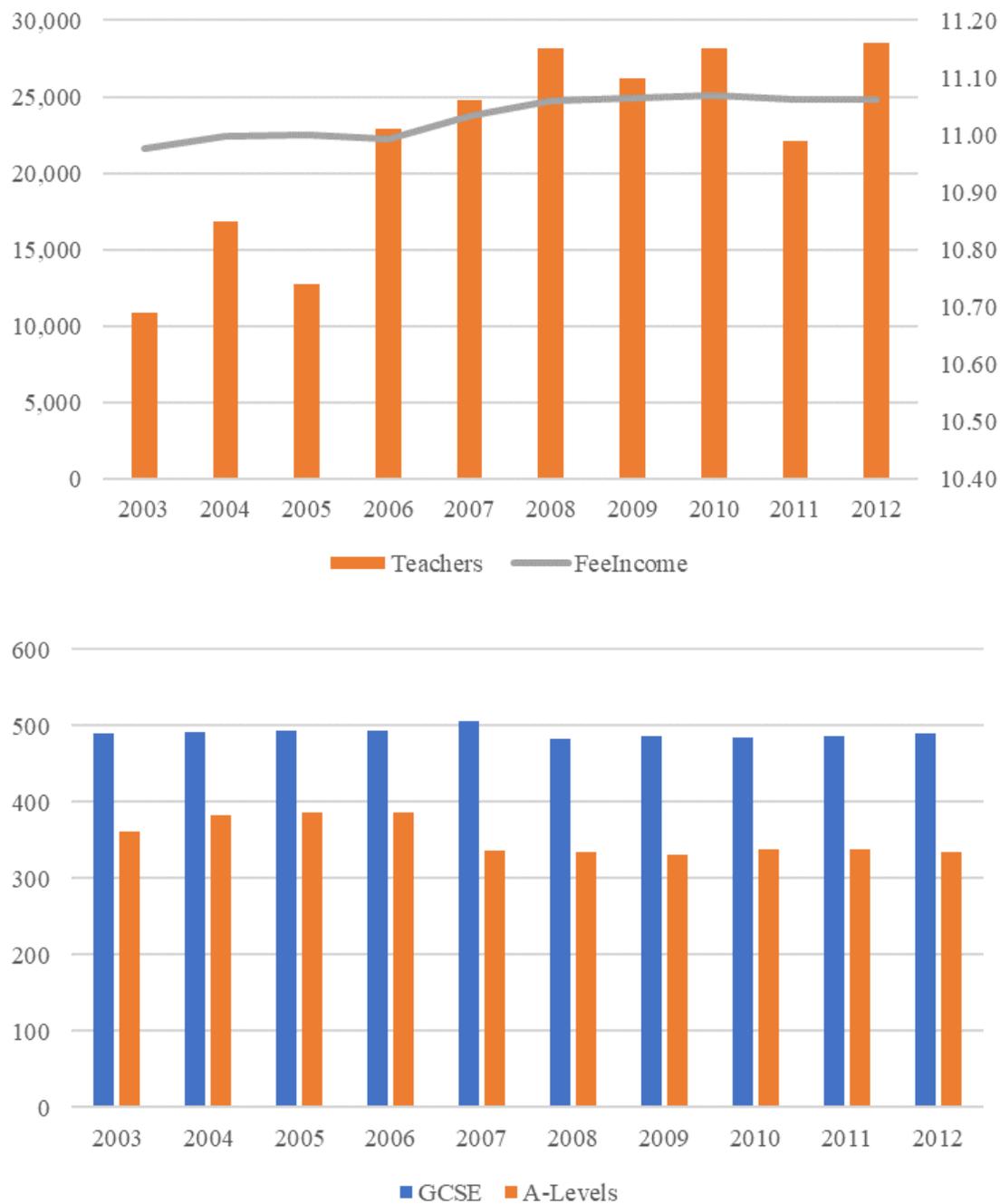
<sup>19</sup> Note that the results of these examinations are not made publicly available so do not provide an additional source of either school or pupil level data.

**Table 2. Descriptive statistics for inputs and outputs**

<b>Variable</b>	<b>2003</b>	<b>2004</b>	<b>2005</b>	<b>2006</b>	<b>2007</b>	<b>2008</b>	<b>2009</b>	<b>2010</b>	<b>2011</b>	<b>2012</b>	
<i><b>Teachers</b></i>	Mean	10.69	10.85	10.74	11.01	11.06	11.15	11.10	11.15	10.99	11.16
	S.D.	2.55	2.77	2.81	3.10	2.88	2.98	3.00	3.00	2.90	3.03
	Min	4.11	4.22	4.56	5.80	5.41	1.53	4.71	4.85	4.35	4.28
	Max	10.60	10.69	10.43	10.66	10.68	10.89	10.81	10.81	10.63	10.88
<i><b>FeeIncome</b></i>	Mean	21,614.89	22,431.68	22,543.49	22,256.42	23,778.49	24,772.25	24,923.91	25,084.87	24,819.50	24,819.50
	S.D.	11,665.01	12,248.36	12,389.66	12,083.07	12,977.87	13,352.37	13,428.91	13,478.91	13,401.85	13,401.85
	Min	6,291.62	6,428.57	6,270.00	6,129.03	6,790.83	6,884.79	6,949.46	6,969.43	6,940.64	6,940.64
	Max	43,377.50	59,020.40	46,410.00	45,366.60	48,871.00	50,087.50	49,472.90	48,013.10	47,240.80	47,240.80
<i><b>GCSE</b></i>	Mean	490.41	491.19	493.67	493.73	506.40	482.73	486.51	484.61	486.54	490.44
	S.D.	34.67	33.82	33.72	33.44	31.89	47.70	47.52	55.22	53.90	55.67
	Min	369.00	357.46	376.70	372.85	353.61	274.00	301.00	48.00	50.00	46.00
	Max	557.63	561.48	561.48	561.48	565.33	584.00	618.00	608.00	603.82	627.00
<i><b>A-Levels</b></i>	Mean	361.26	382.49	385.81	387.28	336.82	335.40	331.36	338.03	337.65	334.06
	S.D.	51.86	57.95	59.49	60.18	51.17	49.62	57.40	57.73	58.56	57.67
	Min	136.42	211.97	154.80	206.87	116.00	147.00	91.50	95.00	107.83	107.83
	Max	491.72	532.56	538.68	567.27	487.63	535.62	595.00	498.00	494.00	493.00
N	206	232	260	260	283	294	306	319	316	316	

Source: The authors.

**Figure 1. Trends of average values of inputs (top panel) and outputs (bottom panel)**



Source: The authors.

#### ***4.2. Factors which might affect efficiency***

We use a variety of environmental factors in our analysis and consider these in various categories below.

### *a) Competitive environment*

We introduce a factor to express the market share of each school (*LnMarketShare*). This measure is calculated by dividing the number of pupils in each school by the number of pupils in the relevant county. This factor includes the total number of pupils in the county from both independent and state schools and is intended to capture possible competition both from other independent schools and also state schools since previous work has found that competition from both state and independent schools affects state school efficiency (Agasisti 2011b; 2013b). It should be noted that the factor is in logarithms as we hypothesise that any possible relationship is likely to be non-linear.

An advantage of using the market share of each school as the measure of competition is that this factor varies at the school level and encapsulates the most information amongst possible measures of competition. A number of alternative variables were considered to capture the extent of competition between schools in the independent schools' context. Two possible Herfindahl indexes were considered, based on the sums of the squares of the student enrolment shares across independent schools in a county or instead across all schools in a county. However, these variables were deemed inappropriate as Herfindahl index measures of competition vary only by region and not by school. Similarly, concentration ratios were ruled out as an appropriate measure of competition because not only do they vary at the regional level but in addition they only capture a limited amount of information relating to the market share of the  $k$  largest independent schools in an area, for example the 4 largest schools. More recently, emerging out of the New Empirical Industrial Organisation literature are measures of competition such as the Panzar-Rosse statistic, the Genesove-Mullin conduct parameter and the Boone indicator (Panzar and Rosse 1987; Genesove and Mullin 1998; Boone 2008). The unavailability of data relating to different elements of labour and capital costs or that would allow estimations of price elasticities of demand unfortunately rule out their use in the present context.

### *b) Affiliation factors*

We include two indicators to reflect information about coalitions or groups of schools. The first reflects membership of the Sevenoaks fee-setting cartel which was uncovered in 2003 (*Sevenoaks*). It includes 50 leading independent schools which the Office of Fair Trading confirmed in 2005 to have been colluding in the setting of school fees. If schools were colluding, they may have been able to share good practice around efficiency, and in fact one of their defences for the collusion was that they colluded in an effort to keep fee increases low (Taylor *et al.* 2005). An additional factor (*Coalition*) is included to reflect other school groups where schools are under common ownership such as Cognita, GEMS, Woodard and UCST, as well as school affiliations such as the Eton Group, the Rugby Group, the YBSG, and the GDST.

c) *Other factors*

Geographical location has been found to be an important determinant of efficiency in previous studies relating to different countries (Agasisti 2011b; 2013b; Burney *et al.* 2013). Given the relatively sparse distribution of independent schools in Scotland and Wales, and contrastingly, the greater concentration of independent schools around London, we include the indicator *Location* to control for these features in the geographical distribution of schools (denoted by *Scotland*, *Wales*, *Inner London* and *Outer London*).

We also include a vector of factors relating to characteristics of the schools. These factors include starting age of pupils at a school (*Starting age*) and schools' religious affiliations (Church of England, Roman Catholic, Quaker, Jewish, Methodist, United Reformed, Christian and non-denomination with the base group being all faiths) (*Religious affiliation*). There is little previous research on the effect of these factors on school efficiency with the exception of religious affiliation where Church of England affiliation has been found to be positively related to efficiency in primary schools (Mancebón and Mar Molinero 2000).

Finally, we include an indicator to refer to *Time*. This might reflect technological changes in the production frontier or changes in market conditions. We know that the financial crisis led to a particularly difficult environment for independent schools in the UK and this might lead to greater efficiency following 2007. Definitions of these variables are fully explained in Table 3.

**Table 3. Definitions of factors hypothesised to affect efficiency**

Category	Label	Definition	Source
<i>Competitive Environment (C')</i>	$Z_1$ <i>LnMarketShare</i>	Logged market share per school: number of pupils in each school divided by the number of pupils in the county (from state and independent schools).	ISC, UK national statistics & own calculation
	$Z_2$ <i>Sevenoaks</i>	Dummy variable to indicate whether a school was part of the Sevenoaks fee-setting cartel.	Own calculation
<i>Affiliation factors</i>	$Z_3$ <i>Coalition</i>	Dummy variables indicating whether a school belongs to any school group coalition (Eton, Rugby, Cognita, Yorkshire Boarding Schools Group (YBSG), GEMS, Girls Day School Trust (GDST), United Church Schools Trust (UCST), Woodard*.	Own calculation
	<i>School characteristics</i>	$Z_4$ <i>Religious affiliation</i>	Classification of schools' religion (Church of England, Roman Catholic, Quaker, Jewish, Methodist, United Reformed, Christian, all faiths and non-denomination).
$Z_5$ <i>Starting age</i>		Variable indicating the starting age of pupils.	ISC
<i>Other factors</i>	$Z_6$ <i>Location</i>	Scotland, Wales, Inner and Outer London.	Own calculation
	$Z_7$ <i>Time</i>	Schools' academic year.	Own calculation

Note: \* Cognita and GEMS are not considered in the final analysis due to lack of observations.  
Source: The authors.

Descriptive statistics for the variables selected to explain efficiency are detailed in Table 4. As can be seen the evolution of competition variable is relatively stable over time. However, it is worth noting that, on average, *LnMarketShare* tends to grow from 2007 onwards, an indication of increased market power of independent schools across the school sector (independent and state schools).

**Table 4. Descriptive statistics for factors hypothesised to affect efficiency**

<b>Factor</b>		<b>2003</b>	<b>2004</b>	<b>2005</b>	<b>2006</b>	<b>2007</b>	<b>2008</b>	<b>2009</b>	<b>2010</b>	<b>2011</b>	<b>2012</b>
<i>LnMarketShare</i>	Mean	-5.235	-5.248	-5.232	-5.233	-5.153	-5.163	-5.104	-5.114	-5.103	-5.110
	S.D.	1.058	1.044	1.025	0.992	1.040	1.017	1.034	1.043	1.037	1.038
<i>Sevenoaks</i>	Mean	0.189	0.172	0.150	0.154	0.145	0.156	0.157	0.150	0.155	0.155
	S.D.	0.393	0.379	0.358	0.361	0.353	0.364	0.364	0.358	0.363	0.363
<i>Coalition* (GDST)</i>	Mean	0.073	0.073	0.077	0.077	0.067	0.065	0.062	0.060	0.063	0.063
	S.D.	0.260	0.261	0.267	0.267	0.251	0.246	0.242	0.237	0.244	0.244
<i>Religious affiliation* (Church of England)</i>	Mean	0.553	0.526	0.500	0.500	0.481	0.483	0.471	0.473	0.484	0.484
	S.D.	0.498	0.500	0.501	0.501	0.501	0.501	0.500	0.500	0.501	0.501
<i>Starting age</i>	Mean	8.092	7.927	7.942	7.904	7.774	7.844	7.650	7.408	7.405	7.263
	S.D.	3.929	3.940	3.947	4.023	4.013	4.042	4.169	4.177	4.204	4.237
<i>Location* (Inner London)</i>	Mean	0.029	0.030	0.027	0.031	0.028	0.034	0.033	0.031	0.028	0.028
	S.D.	0.169	0.171	0.162	0.173	0.166	0.182	0.178	0.175	0.167	0.167
	N	206	232	260	260	283	294	306	319	316	316

Note: \* indicates one example of values for an illustrative dummy variable in a set of nominal variables.

Source: The authors.

## 5. RESULTS AND DISCUSSION

To properly run the robust conditional order- $m$  model and later investigate the role of contextual factors ( $Z$ ), we estimate four models, that is, the unconditional model (UM) and three conditional models considering different specifications, which are summarised in Table 5<sup>20</sup>. Model C0 checks the effect of competition on efficiency over time taking into account schools' location. With model C1 we include the effect of belonging to a coalition. Finally,

<sup>20</sup> We have selected the non-convex estimator for the order- $m$  frontier in all the estimated models after performing a test for convexity (see Kneip *et al.* (2016) for details). The statistic yields a value  $\hat{\tau} \cong -2.8310$  and the corresponding p-value after 1,000 bootstrap replications is 0.0023, thus the convexity assumption can be rejected.

Model C2 includes the specification of C1 and also controls for the remaining characteristics of schools, so this will be the most complete model<sup>21</sup>. The exogenous factor representing time is included in all the three conditional models (C0, C1 and C2).

**Table 5. Models' specifications**

<b>Model</b>	<b>Variables</b>
<b>Unconditional model (UM)</b>	<i>I1, I2, O1, O2</i>
<b>Baseline conditional model (C0)</b>	<i>UM + Z1, Z6, Z7</i>
<b>Conditional model 1 (C1)</b>	<i>C0 + Z2, Z3</i>
<b>Conditional model 2 (C2)</b>	<i>C1 + Z4, Z5</i>

Source: The authors.

Table 6 summarises the main descriptive statistics (mean, median, standard deviation, maximum and minimum) of estimates for conditional models compared to the values of the unconditional model. In addition, we detail the number of efficient and superefficient DMUs and the percentage with respect to the total number of units.

**Table 6. Descriptive statistics of conditional efficiency models**

	<b>UM</b>	<b>C0</b>	<b>C1</b>	<b>C2</b>
<b>Mean</b>	1.2106	1.1517	1.1275	1.0641
<b>S.D.</b>	0.1564	0.1493	0.1456	0.1353
<b>Min</b>	0.9400	0.8967	0.8778	0.8290
<b>Median</b>	1.1921	1.1336	1.1097	1.0454
<b>Max</b>	5.1060	4.8507	4.7486	4.5768
<b># Efficient + Superefficient</b>	65 (2.33%)	221 (7.92%)	336 (12.03%)	867 (31.05%)

Source: The authors.

As expected, when the analysis includes information about contextual factors, the overall mean inefficiency for conditional models decreases (1.2106 for UM compared to 1.1517 for C0, 1.1275 for C1 and 1.0641 for C2) and the number of efficient DMUs increases. As the decision-making units (DMUs) of reference are restricted to those having a similar environment to the unit of analysis, the inefficiency scores are lower, on average. Considering C2, the average school results in the tests considered as outputs could be increased by at least 6.41%, approximately, if schools operated efficiently.

The implementation of a useful non-parametric test based on Li *et al.* (2009) allows us to verify if the observed differences between efficiency values of the designed models are significant or not. This tool is based on a consistent integrated squared difference nonparametric test for equality of densities/probabilities. After applying the test for the four

<sup>21</sup> It is worth noting that we consider a number of environmental factors that allow the model to have discriminatory power.

specified models we reject the null hypothesis of equality of distribution for all of them. The significance among UM-C0, UM-C1 and UM-C2 models is 0.1% level. In practical terms, this means that introducing environmental variables into the analysis of school performance significantly affects the distributions of the efficiency values. Specifically, the greater differences appear when religious affiliation and geographical location variables are incorporated into the model.

We can see the trend and the (in)efficiency score in each school year as illustrated in Table 7 and Figure 2. The standard deviation included in brackets in Table 7 allows us to see that the higher the number of Z introduced in the model, the lower the dispersion in the scores.

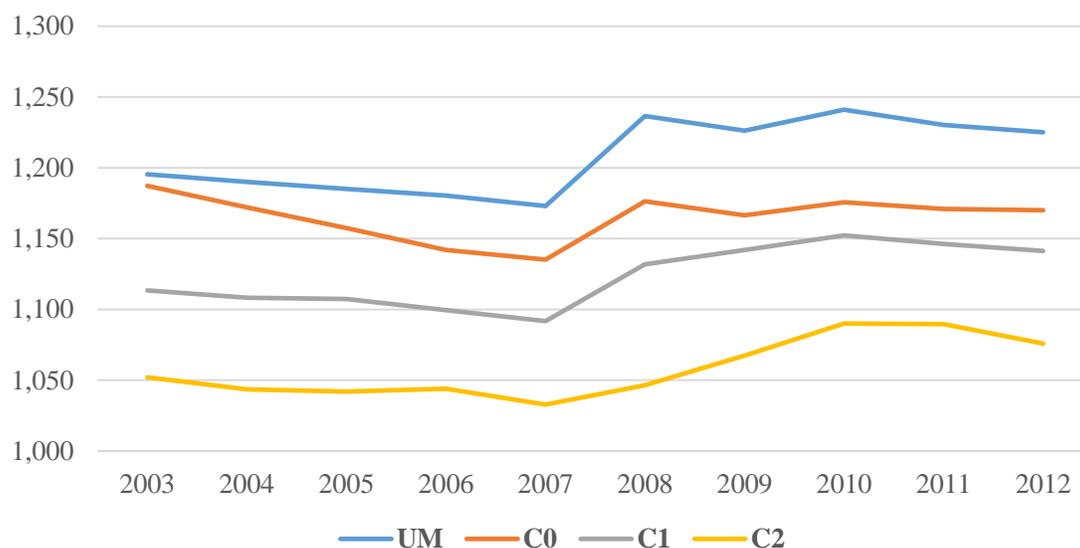
**Table 7. Average inefficiency score by year**

<b>YEAR</b>	<b>UM</b>	<b>C0</b>	<b>C1</b>	<b>C2</b>
<b>2003</b>	1.1954 (0.1149)	1.1873 (0.1091)	1.1134 (0.1067)	1.0521 (0.0959)
<b>2004</b>	1.1902 (0.1118)	1.1720 (0.1063)	1.1084 (0.1042)	1.0436 (0.0927)
<b>2005</b>	1.1852 (0.1055)	1.1575 (0.1004)	1.1074 (0.0983)	1.0420 (0.0878)
<b>2006</b>	1.1804 (0.1038)	1.1421 (0.0987)	1.0996 (0.0967)	1.0441 (0.0859)
<b>2007</b>	1.1730 (0.0897)	1.1352 (0.0854)	1.0918 (0.0838)	1.0328 (0.0778)
<b>2008</b>	1.2365 (0.1374)	1.1763 (0.1309)	1.1318 (0.1283)	1.0465 (0.1173)
<b>2009</b>	1.2263 (0.1342)	1.1664 (0.1278)	1.1419 (0.1252)	1.0675 (0.1140)
<b>2010</b>	1.2411 (0.2560)	1.1756 (0.2432)	1.1523 (0.2381)	1.0901 (0.2275)
<b>2011</b>	1.2303 (0.1657)	1.1711 (0.1601)	1.1462 (0.1543)	1.0896 (0.1431)
<b>2012</b>	1.2252 (0.1990)	1.1701 (0.1915)	1.1412 (0.1854)	1.0759 (0.1736)

Source: The authors.

While schools exhibit inefficiency across each year in the study period, it is worth noting some patterns across the period – and these are observed regardless of model. From 2003 to 2007 independent schools in the UK appeared to improve their efficiency. However, in the years of crisis (2008-2010) inefficiency increased considerably, a sign that schools did not manage their resources well in the context of maximising students' academic results. Since the crisis, efficiency has improved, as educational spending is better adjusted, and more emphasis is placed on improving academic outcomes. Nevertheless, inefficiency continues being higher than in the pre-crisis period (2003-2007).

**Figure 2. Average inefficiency score over the period of analysis**



Source: The authors.

Closer inspection of our conditional model provides a possible explanation of this apparently puzzling result. We have used incomes from fees as a measure of capital input. We know that while fees have risen over the period, the size of the increase has become smaller over time (Independent Schools Council 2019). In addition, we know that schools have increased their bursaries in the period since the financial crisis (Independent Schools Council 2019), and schools which were part of the Sevenoaks cartel have had to provide more generous bursary schemes from 2007 onwards. The ticket price (fee) is therefore likely to be higher than the price received, in which case our measure of fee income will be overestimating the resources actually available to schools, and increasingly so since the crisis. This means that, in calculating efficiency, schools seem to have higher resources (in terms of fee income) than is really the case, and so it is not surprising if output has dropped relative to the measured inputs in this period.

We regress the ratio between the conditional and the unconditional efficiency scores on  $Z$  in order to examine the influence of environmental factors on efficiency. As we have three conditional models, we estimated a nonparametric regression for each one. Table 8 shows the  $p$ -values of the significance tests proposed by Racine and Li (2004) and Li and Racine (2006). Since the significance test is based on a non-parametric regression, the coefficients obtained cannot themselves be interpreted –only their significance.

**Table 8. Non-parametric significance test: Influence of different environmental factors on schools' efficiency**

Environmental factors		C0	C1	C2
<i>LnMarket share</i>	Z1	0.13	0.01**	0.00***
<i>Sevenoaks</i>	Z2		0.00***	0.00***
<i>Coalition</i>	Z3		0.00***	0.00***
<i>Religion</i>	Z4			0.00***
<i>Starting age</i>	Z5			0.00***
<i>Location</i>	Z6	0.00***	0.00***	0.05*
<i>Time</i>	Z7	1.00	1.00	0.02**

\*\*\*, \*\*, and \*: below the 0.1%, 1% and 5% statistical significance thresholds, respectively.

Source: The authors.

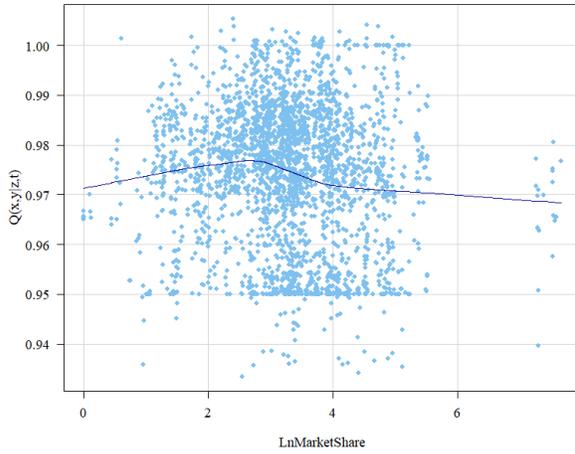
As can be seen, our main environmental factor (*Lnmarketshare*) is significant only when including the effect of belonging to a coalition in the model C1, as this variable is defined at school level. The effect is reinforced with model C2, which also considers schools characteristics. We can conclude by saying that all the environmental factors (*Z*) considered in the model significantly impact on the efficiency frontier (at the 0.1% level), except for location that is significant only at the 5% level in C2. It appears that the effect of religious affiliation and other school features absorb to some extent the effect of location on efficiency when those factors are considered together (C2 model).

In order to continue with the analysis, we investigate how the ratios  $\hat{Q}$  shown in Equation (7) can provide information about the potential role of contextual factors on inefficiencies. This analysis is particularly rich in that we can see the direction of the effect of each *Z* for the different levels of the (in)efficiency distribution. Note that, as explained in Section 3.2, this analysis identifies the effect of the environmental factor on efficiency (rather than inefficiency). We focus the analysis to visualize the effects on the frontier shifts, obtained from robust estimations of the full frontiers<sup>22</sup>. Figure 3 shows the marginal effects of contextual factors on the frontier in panel (a) together with the effect of time in panel (b) for each environmental factor. The effect of time is perceived as favourable in all the graphs (panel (b)), an aspect that is hardly appreciated individually in the marginal plots of panel (a).

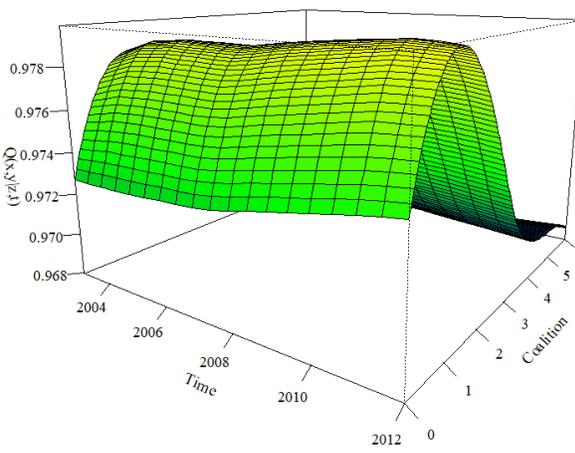
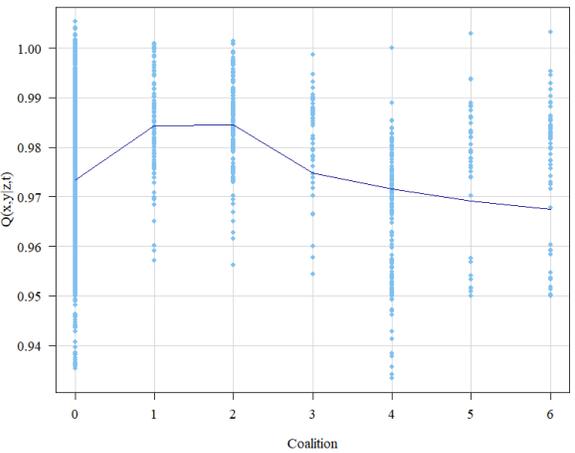
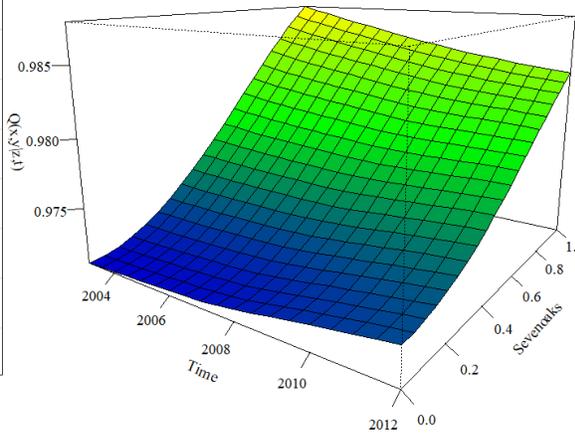
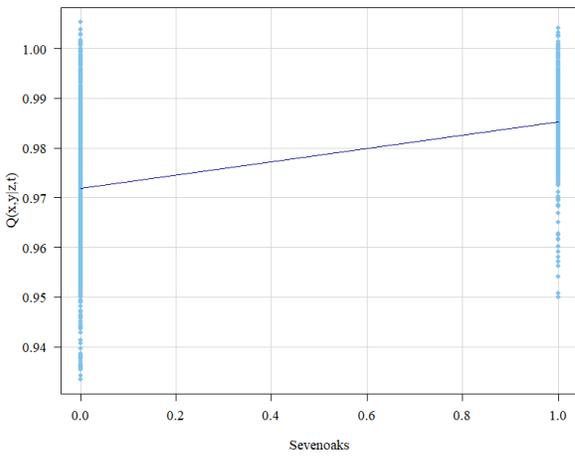
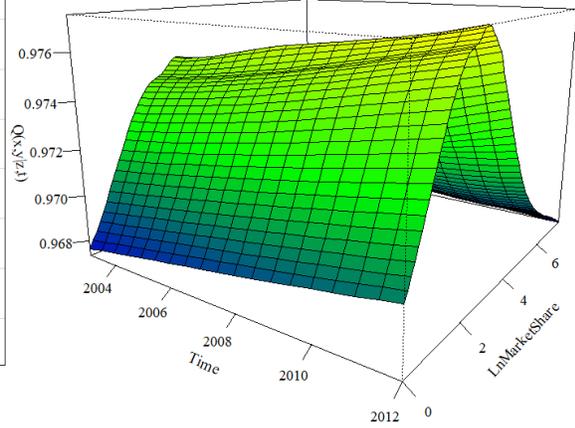
<sup>22</sup> We discard showing here the same analysis for partial frontiers ( $m=1$ ) since we did not find substantial differences among these two trends. Thus, we can assume that we have a shift of the frontier while keeping the same distribution of the efficiencies when the conditioning variables *Z* and *T* change (Bădin *et al.* 2012; 2014; Mastromarco and Simar 2015).

**Figure 3. Plots of ratios  $Q(x, y|z, t)$  versus  $Z (1/2)$**

**(a) Marginal effect**

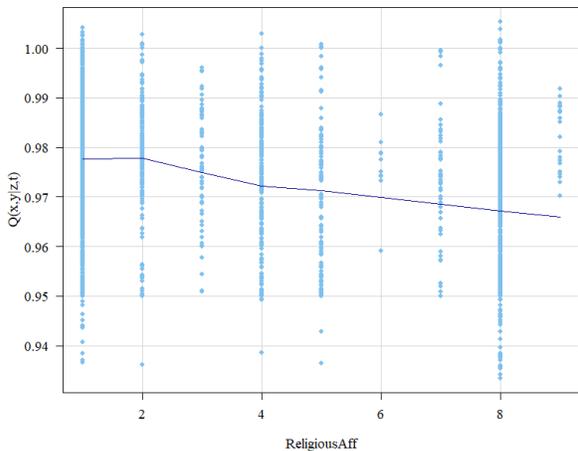


**b) Effect of Z and time on efficiency**

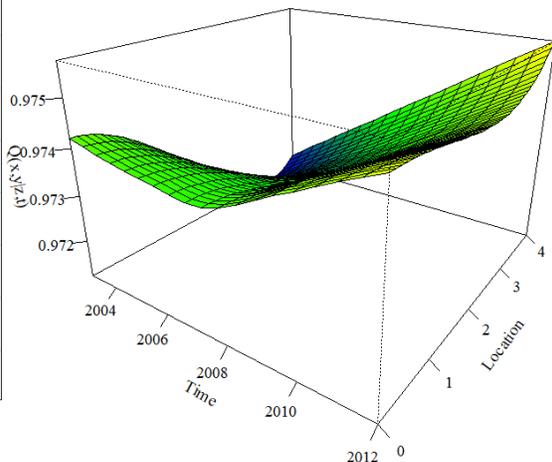
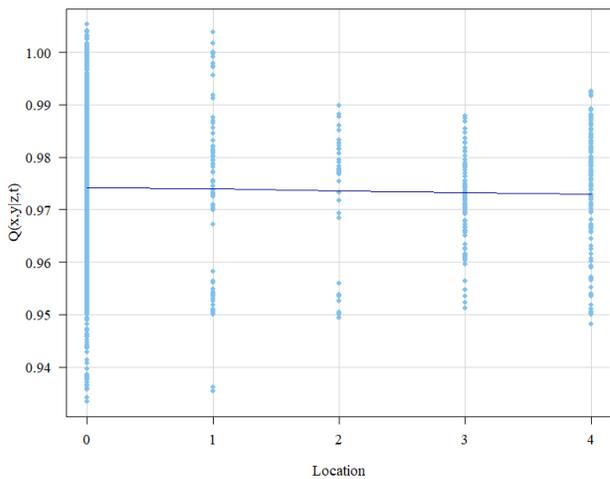
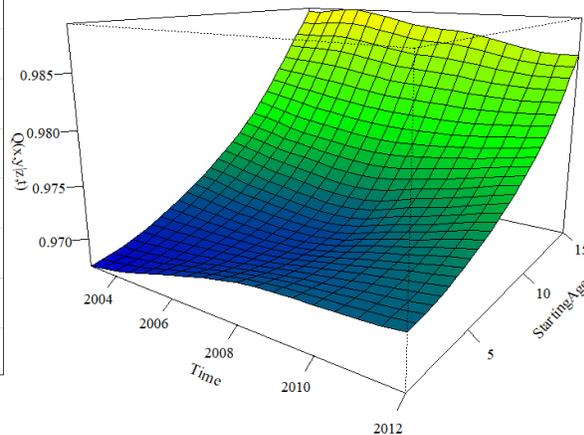
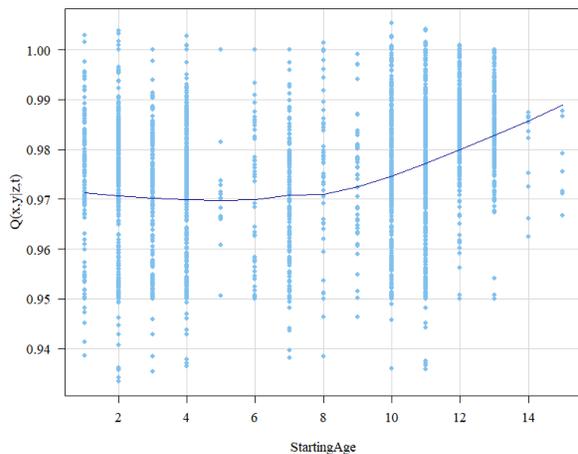
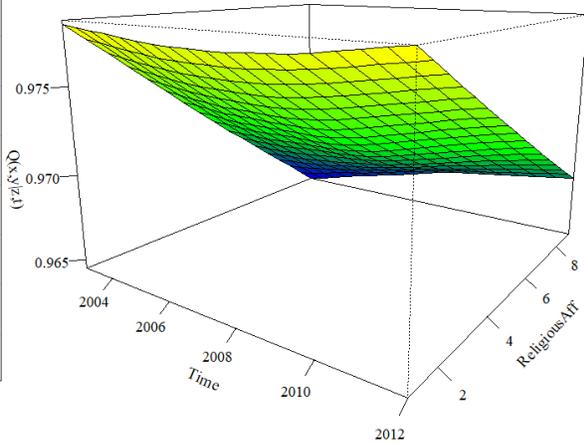


**Figure 3. Plots of ratios  $Q(x, y|z, t)$  versus  $Z$  (2/2)**

**(a) Marginal effect**



**b) Effect of  $Z$  and time on efficiency**



Starting with  $ZI$  – our main environmental factor<sup>23</sup> – as can be seen, higher market shares tend to slightly favour efficiency, but only up to a level of market share (around 20%). Note that this is the range within which schools in our data set are typically operating. From this level onwards, the effect becomes negative, and very high market shares are detrimental to the level of efficiency (note the outliers at the right of the scatterplot). This could be partially justified by the life-cycle hypothesis (Ando and Modigliani 1963) as operating in isolation is negatively related to efficiency and savings. This result of an inverted U shaped relationship between market share and efficiency is striking as previous empirical papers for state schools have suggested a positive relationship between competition and efficiency (Bradley *et al.* 2001; Agasisti 2011b; 2013b)<sup>24</sup>. It should be noted, however, that in previous papers competition was measured by number of schools in radii of varying sizes. In the context of the current analysis such measures of competition are not ideal as the maximum number of competing independent schools in, for example, a 1-mile radius across the sample is 4, with a mean below 1. In addition, the number does not vary much over the period of analysis. However, our result is highly consistent with some findings for Italian universities that find a U-shaped quadratic relationship between inefficiency and market share for the North-Western region of Italy with a turning point at a market share of around 0.29 (not dissimilar to our own findings), and it is also the case that most universities are on the downward sloping part of the curve (Agasisti *et al.* 2016).

It is interesting that the negative relationship between competition and efficiency is found in two studies where competition is measured by market share, and the relationship occurs only up to a specific value for market share. We speculate that greater market share might be an advantage because it might allow schools to enjoy economies of scale of production which, at least to begin with, outweigh any drawbacks from reduced competition. Beyond a certain market share, though, these scale advantages start to be outweighed by the negative impact on efficiency arising from lack of competition.

An interesting finding is that our measure of market share is based on all schools (state and independent). This suggests that when considering competition in the independent school sector, it is appropriate to consider competition from both independent and state schools. Given that in the Introduction we highlighted that over 27% of new independent school pupils in 2019 came from the state sector then this result may not be unexpected. This also mirrors results from the state sector where it is competition from both sectors which is important in affecting efficiency (Agasisti 2011b; 2013b).

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<sup>23</sup> Note that market share has been transformed to non-negative values to fulfil the monotonicity condition.

<sup>24</sup> In order to validate the non-linear relationship found between efficiency and market share, we also employed a semi-parametric fixed effect model following Simar and Wilson (2007), the double bootstrap truncated regression procedure, and the findings are robust. However, this approach requires that the separability condition of environmental factors holds (check Daraio *et al.* (2018) for more details). The conditional approach, our choice, overcomes this limitation and allow us to incorporate the role of contextual factors in a single step.

With regards to affiliation variables –  $Z2$  and  $Z3$  – we observe that, on the one hand, belonging to the Sevenoaks cartel ( $Z2$ ) benefits efficiency. This suggests that, whilst engaging in anti-competitive practices in terms of fee-setting, members of the Sevenoaks cartel became sufficiently close to be able to engage in activities which improved their efficiency – we speculate that these might, for example, relate to sharing good practice or collaborating on purchasing resources. The ETON and RUGBY coalitions also enjoy improved efficiency, whereas GDST, UCST and Woodard coalitions do not. Therefore, the effect of collaboration on efficiency is certainly not clear-cut, and some coalitions may even have a negative effect on efficient school operation. This presumably relates to the nature of the collaboration taking place in any coalition. Policy-makers in England should take note in light of the encouragement of networks of academies in the state school sector, as not all collaboration is beneficial, and further work should be undertaken to identify those practices most likely to positively impact efficiency.

Moving to schools' characteristics –  $Z4$ - $Z6$  – we find noteworthy results. First, mixed effects are found with regards to religion affiliation. Schools with Roman Catholic and Church of England affiliation are more efficient, and those with all-faith affiliation are less efficient. Second, we observe practically no effect of starting age on efficiency in schools whose starting age ranges from 1 to 8. However, schools with a pupils' starting age higher than 8 experience higher efficiency. We hypothesise that schools with a higher starting age have a more homogeneous set of pupils with similar motivations and needs which may lead to higher efficiency. For example, students older than 8 years may start in a new school with more motivation to improve their academic achievement, and their parents may even have switched their school with this goal (Booker *et al.* 2007). Finally, regarding schools' location, we find almost no effect on efficiency. However, time plays an essential role to explain schools' efficiency depending on where they are located. It seems that the popularity of a given area varies over time.

## 6. CONCLUSIONS

This paper offers an early attempt to identify the factors determining UK independent schools' efficiency levels. As such the paper contributes to the growing literature considering factors determining efficiency in different parts of the education sector, including in schools, further and higher education. We also believe that our methodological approach, implementing the non-parametric robust conditional order- $m$  model in the private education sector in the UK, is highly original.

Of particular policy note is the result that schools with a greater market share enjoy higher efficiency at least over much of the observed range of market share. This result has policy implications beyond the independent school sector. In recent years governments around the world (including the UK) have introduced measures to increase competition between schools in the state education sector. Results in this paper suggest that the relationship between

market share and efficiency is an inverted U shape: positive and then negative as market share increases. Further research could test whether this result is replicated using data from state sector schools. While previously research has found that greater competition increases rather than decreases efficiency in the state school sector, measures of competition used have typically not included the market shares of schools as in the current analysis.

Results suggest that collaboration does not generally lead to greater efficiency, and indeed it may lead to lower efficiency. Again, this has policy implications beyond the independent school sector as successive UK governments have encouraged greater cooperation between state schools in recent years. Hence, future research on factors determining technical efficiency levels in state sector schools should be extended to test the impact of school coalitions, and the characteristics of different coalitions that give rise to their differential impacts on school efficiency levels. Finally, the results suggest that for schools in the independent school sector, when considering factors that impact upon a school's operational efficiency, the relevant measure of competition encompasses both independent and state schools. As such independent schools should take care to ensure that they remain vigilant of developments in their local state education sector.

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