Investigation of VMD denoising method based on Monte Carlo simulation: A comparative study between newly introduced autocorrelation-based method and PDF-distance based method

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Abstract: A signal with low signal to noise ratio is always difficult to be analysed by the traditional signal processing methods, especially the vibration and acoustic signals that contain non-linear, non-stationary, modulation phenomenon. Extracting features contaminated in heavy background noise requires an effective denoising tool and hence variational mode decomposition based denoising method has been considered in this paper. An initial investigation has been carried out for a simulation signal with very low signal to noise ratio. Firstly, VMD is introduced to decompose the signal into a number of intrinsic mode functions. The selection of IMFs is very important to get the reconstructed denoised signal. For this purpose, a noble method based on autocorrelation has been proposed along with the frequency domain denoising technique. Use of Monte-Carlo method proves the effectiveness of the proposed Autocorrelation based method and provides a comparative analysis between probability distribution function-based method and the proposed method.

Keywords: Variational mode decomposition; Denoising; PDF-distance; Autocorrelation; Monte-Carlo simulation.

1. Introduction

Acoustic feature extraction from a machine can be a challenging task especially when strong background noises exist in the acoustic signals (Mondal et al., 2019, Vilela et al., 2004). Due to low signal to noise ratio, it's difficult to extract feature parameters for diagnosing the machine faults (Vilela et al., 2004). Similarly, vibration signal obtained from a machine may get contaminated with other uninterested sources and the characteristic components might get submerged in huge background noise (Cheng et al., 2019). Hence a good denoising tool can be very useful to reduce the background noise and enhance the characteristic signatures. Several different denoising techniques have been developed over the years. Wavelet thresholding (Jiang et al., 2020) method is one that is being widely used in recent years. However, the performance of this method is highly dependent on choosing the basis functions that depends on the experience and hence has uncertainty.

To overcome the shortcoming of the above method, Huang et al (Wu and Huang, 2004) developed empirical mode decomposition method, popularly known as EMD method that is adaptive in nature and demonstrated its capability in handling the nonstationary and non-linear signals (Wu et al., 2007, Mondal et al., 2019). In similar with the wavelet denoising method, EMD thresholding technique is also proved to be effective in denoising the signals (Liu and Chen, 2019). In addition, EMD based denoising method is combined with other methods to achieve good denoising effects. For example, EMD along with Hausdorff distance (Komaty et al., 2012) has been proposed. Though these methods have achieved great success, EMD still has some problems such as lack of solid mathematical formulation to support the algorithm, mode mixing etc (Fosso and Molinas, 2018).

Variational mode decomposition (VMD) can be used as a good denoising tool. In most of the cases, VMD combined with other methods serve as a promising tool for denoising a signal instead of VMD itself though a using of Wiener filter makes this method effective to deal with the noisy signal (Lahmiri and Boukadoum, 2014). It overcomes the drawback of the mode-mixing problem and non-optimal solution of the reconstructed signal of EMD. Therefore, VMD is considered as potential denoising tool for this analysis.

In recent years, VMD based hybrid methods are used for denoising purposes. VMD combined with wavelet threshold denoising and singular spectrum analysis was applied to a noisy time domain signal and the denoised output signal was reconstructed by grouping the decomposed components by singular value decomposition (SVD) based on the singular entropy increments (Fu et al., 2020). In another study, the noise level of a noisy ECG signal was obtained by using detrend fluctuation analysis (Liu et al., 2016). Then VMD was applied to decompose the signals and the modes were selected based on the threshold value to reconstruct the filtered signal. Li et al. developed denoising and feature extraction method based on VMD and combination of new kind of permutation entropy (NPE) for ship radiated noise (Li et al., 2017). After VMD decomposition, the NPEs are calculated for the decomposed IMFs; the noises are cancelled out based on the values of NPE, and the denoising process can be achieved by reconstructing the rest of IMFs. Couple of studies show that VMD combined with distance measurement can successfully denoise the signal. One study uses Hausdorff distance (Ma et al., 2017), whereas, another study focuses on calculating Euclidean distance of the PDFs (Ren et al., 2017) obtained from the IMFs and that of the signal.

Denoising preserves the important features of a signal by removing the unwanted noise. The denoising algorithm can be validated by using Monte Carlo simulation. The simulation is assisted with additive noise and the input signal is manipulated to validate the denoising algorithms.

Generally, the vibration and acoustic signals of the machine components are complex in nature. Basically, they are non-stationary, sometimes non-linear, having modulation components due to fluid-mechanical interactions and incorporation of the responses from the other machine components, often submerged in a heavy background noise (Mondal et al., 2019, McInerny and Dai, 2003). Hence in this paper, a non-stationary, multicomponent simulation signal is established and then white Gaussian noise is added using Monte Carlo simulation approach. The algorithm was validated for signals of twelve different SNRs. Firstly, the simulation input signal is decomposed by the VMD method into several band limited intrinsic mode functions (BLIMFs). Then the probability distribution function (PDF) of each BLIMs is obtained using Kernel smoothing (K-S) density method. After that the Euclidean distance (ED) between the PDFs of each BLIMF and that of the input noisy signal are estimated. A lower ED value signifies the probability of having more feature contents in the selected IMF and hence the particular BIMFs with small ED values are selected for reconstruction of the signal. Similarly, correlation-based method has been applied for choosing the BLIMFs. For both cases, there still exists some noises in the reconstructed signals. To eliminate the remaining noise, denoising technique based on Fourier Transform has been applied. For validation of the denoising effects, the root mean-squared error (RMSE) is calculated and plotted against the different SNRs. The advantage of VMD method has also been highlighted by showing relevant comparison results between the denoising effect on the raw signal and that of the reconstructed IMFs by the VMD process.

This paper is structured as follows: the section 2 provides the overview of the theoretical background of the applied methods. Section 3 depicts the proposed algorithms. In section 4, detailed analysis results with in-depth discussion have been presented. The PDF based denoising result is compared with the correlation-based method and best suitable method is selected based on the denoising capability of the low SNR signal. In the process, the performance of PDF based and correlation based denoising algorithms is compared in terms of root mean square error and signal-to-noise ratio. Finally, the conclusion is drawn in section 5.

2. Theoretical Background

2.1. VMD Method

VMD is a novel, completely inherent, data-driven and quasi-orthogonal decomposition technique. The extraction of the modes in non-recursive way is another advantage of this method compared to the other data-driven techniques (Dragomiretskiy and Zosso, 2013). It decides the significant band adaptively and calculate the respective modes appropriately. VMD strategy depends on three primary concepts: using of Wiener filtering, Hilbert transform, and mixing of frequencies. It breaks down a signal into number of band limited intrinsic mode functions (BLIMFs) that have different frequency bands starting from low to high and believed to be compact around the centre pulsation, calculated during the decomposition. The following schemes help to access the modes in frequency domain with a particular bandwidth.

i. Compute Hilbert transform of the analytic signal to get a one-sided frequency spectrum.
ii. Sifting of the mode frequency response to the baseband by using exponential tune mixed with the individual assessed centre frequency.
iii. The estimated bandwidths are obtained by applying Gaussian smoothness on the demodulated signal.

VMD could be a great technique for denoising a noisy signal. This method has an optimal capacity to bargain with the noise in a signal due to its connection with Weiner filter (Dragomiretskiy and Zosso, 2013). Moreover, VMD method overcomes the limitations of the current decomposition techniques, that include missing numerical hypothesis, recursive sifting process which does not permit reverse error calculation, the prerequisite to filter bank boundaries and present of background noise in a signal. VMD is found to be widely used in many applications such as recognition of speech signal, image processing, machinery fault diagnosis, wind speed estimating etc. due to its advantages over other decomposition strategies (Zhang et al., 2017, Lahmiri and Shmuel, 2017, Upadhyay et al., 2017, Zhang et al., 2017, An et al., 2017). The hypothesis of VMD technique can be portrayed as below (Jianwei et al., 2017):

Step 1: Initialise $\{\hat{u}_k^1\}, \{\omega_k^1\}, \hat{\lambda}^1,\ n \leftarrow 0$

Step 2: The values of u_k, ω_k and λ are updated based on the following Equations (1-3):

$$\hat{u}_k^{n+1} \leftarrow \frac{\hat{f}(\omega)-\sum_{i<k}\hat{u}_i^{n+1}(\omega)-\sum_{i>k}\hat{u}_i^{n}(\omega)+\frac{\lambda^n(\omega)}{2}}{1+2\alpha(\omega-\omega_k^n)^2} \tag{1}$$

$$\omega_k^{n+1} \leftarrow \frac{\int_0^{+\infty}\omega|\hat{u}_k^{n+1}(\omega)|^2 dw}{\int_0^{+\infty}|\hat{u}_k^{n+1}(\omega)|^2 dw} \tag{2}$$

$$\hat{\lambda}^{n+1}(\omega) \leftarrow \hat{\lambda}^{n}(\omega) + \tau\left[\hat{f}(\omega) - \sum_k \hat{u}_k^{n+1}(\omega)\right] \tag{3}$$

Step 3: Continue with the same process again from step 2 until the function is converged. The convergence criteria is $\sum_k ||\hat{u}_k^{n+1} - \hat{u}_k^{n}||_2^2/\left||\hat{u}_k^{n}|\right|_2^2 < \epsilon$, (ϵ = required accuracy).

2.2. Euclidean Distance

The Euclidean distance (ED) measured, is the straight-line distance between the coordinates of two vectors. For example, in two-dimensional cartesian coordinate if a = (a_1, a_2, …a_n) and b= (b_1, b_2, …b_n) are two vectors in Euclidean space. The distance between two is given by Equation 4 (Wang et al., 2005).

$$d(a,b) = d(b,a) = \sqrt{\sum_{i=1}^{n}(b_i - a_i)^2} \tag{4}$$

In one dimension the distance (d) between two vectors is the absolute value of their numerical differences of the points. Thus if x_i and y_i are the points on two vectors (i=1, 2, 3, …. n), then the distance between them is given by Equation 5.

$$\sqrt{(x_i - y_i)^2} = |x_i - y_i| \tag{5}$$

The complex vibration and acoustic signals are mostly symmetrical non-Gauss signals. The probability distribution function (PDF) is calculated using the kernel smoothing density function. The PDF can fully reflect the characteristics of a signal based on statistical information. According to Bayes, a PDF can be described as a state of knowledge of a particular signal instead of being a simple frequency representation. ED can serve as an analysis tool for PDF similarity that can be used to identify the BLIMFs containing the predominant features of the original input signal. The Euclidean distance (ED) between the PDFs of each BLIMF and that of the input noisy signal are estimated. A lower ED value signifies the probability of having more feature contents in the selected BIMF and hence the particular BIMFs with small ED values are selected for reconstruction of the signal.

2.3. Autocorrelation Coefficient

The autocorrelation measures the similarity of a signal with a delayed copy of itself. It is a mathematical tool that is used for finding periodic patterns obscured by noise or identifying hidden frequency components of a signal implied by its harmonic responses. If y is a discrete signal $[y_1, y_2, \dots y_t]$, then the correlation between y_t and y_{t+k}, ($k = 0, \dots, k$) can be measured by using autocorrelation.

Therefore, the autocorrelation calculated for lag k is presented as, $a_k = \frac{C_k}{C_0}$ (Zaiontz, 2015).

Where,

$$C_k = \frac{1}{T}\sum_{t=1}^{T-k}(y_t - \bar{y})(y_{t+k} - \bar{y}) \tag{6}$$

$$\bar{y} = \frac{1}{T}\sum_{t=1}^{T} y_t \tag{7}$$

C_0 = sample variance.

The correlation method can be used to choose the effective BLIMFs from the VMD decomposition in order to reconstruct the signal. The BLIMFs with larger correlation value can be chosen as a similarity measure and the denoised signal can be reconstructed.

2.4. Denoising by Fourier Transform

The reconstructed signals obtained by both the methods based on PDF and autocorrelation still have some noise present in that signal. Hence the denoising can be performed by fast Fourier transform (FFT) using an effective algorithm (Vogel, 2017). The algorithm is shown as follows:

Step 1: Calculate the fast Fourier transform (FFT) of the samples.

Step 2: Obtain the mean value and calculate a threshold value.

Step 3: Remove everything below the threshold value because that will be considered as noise.

Step 4: Get the filtered signal by doing inverse Fourier transform.

3. Proposed method

The denoising method comprised of two individual processes. The first one is the decomposition of the noisy observed signal by VMD method. This method is based on a number of Weiner filters and has a very good noise robustness. The band limited intrinsic mode functions produced by VMD need to be chosen to reconstruct the signal and get rid of the noisy elements present in the signal. In order to remove the remaining noise, a meaningful criterion should be set to choose the BLIMFs for denoising.

The second process is the comparison of the proposed autocorrelation-based method with the PDF based approach and their capability have been demonstrated while handling the low SNR signals. The results obtained from the PDF-ED method and correlation-based method still contain some noise. In order to remove the remaining noise FT denoising has been performed for both the cases.

Both the PDF based and proposed autocorrelation-based methods are compared under different evaluation criteria, especially the mean square error estimation and the various output SNRs. Finally, the effective method has been highlighted with better denoising capability. Process flowchart of the VMD based denoising technique has been shown in Figure 1.

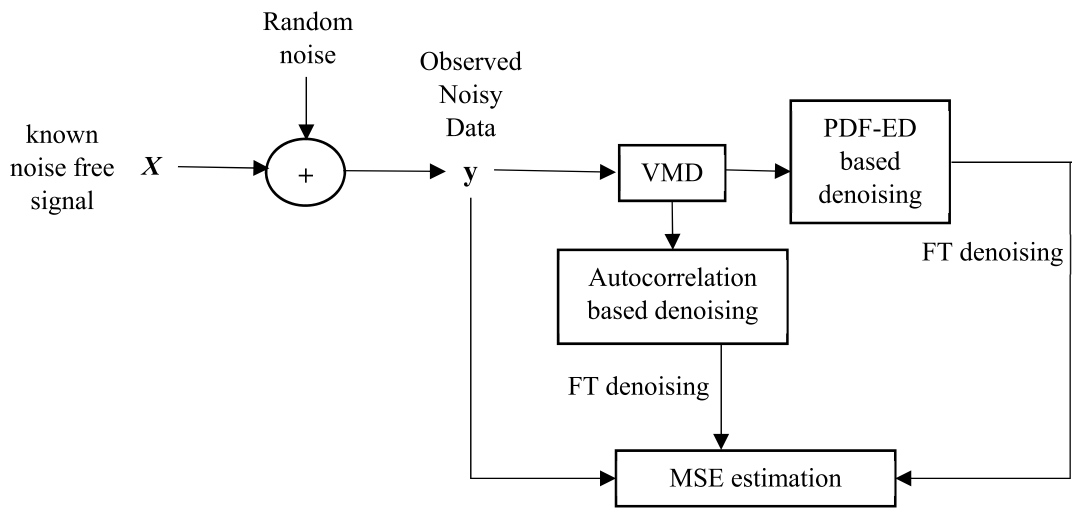

Figure 1: Process flowchart of the VMD based denoising technique.

4. Validation Process

To validate the performance of the proposed denoising method, a non-stationary, non-linear signal is generated and different levels of Gaussian noises are added to it to simulate different levels of SNR. To prove the robustness of the proposed denoising method, Monte-Carlo method is employed and each case is repeated by five times.

Twelve different noise levels have been considered for the study and for each of the cases and five different but of similar characteristic noisy input signals are added to the noise free signal by Monte-Carlo method. This method is effective in optimization of the denoising methods that have been applied later in the process.

5. Simulation Study and Discussion

The acoustic and vibration signals can be contaminated in heavy background noise. The source signal can be modulated by several components and the complex characteristics of the finally measured signal makes it difficult to find the characteristic components. Therefore, a simulation signal (presented in Equation 7-12) that have multicomponent and non-Gauss characteristics can be a good choice to see the effectiveness of the proposed methods.

5.1. Simulation Signal Analysis

The original signal (S-original) has three components among which one is the modulating components. The modulating component has a modulation frequency (f_m) of 50 Hz and a carrier frequency (f_c) of 30 Hz. The other two components have the characteristic frequencies of 150 Hz (f_1) and 250 Hz (f_2).

$$S_1 = 1.6\cos(2\pi f_m t)\sin(2\pi f_c t) \tag{7}$$

$$S_2 = \cos(2\pi f_1 t) \tag{8}$$

$$S_3 = 0.8\cos(2\pi f_2 t) \tag{9}$$

$$S\_original = S_1 + S_2 + S_3 \tag{10}$$

The mixed signal with noise can be found by adding Gauss white noise (S_4) using Monte Carlo approach. S_4 contains five sets of noisy data that can be added for each output SNRs for twelve output SNRs.

$S_4 = \mathrm{randn}(\mathrm{nmonte}, \mathrm{length}(S_1))$; where $\mathrm{nmonte} = 1{:}\,5$.

For each output SNR, the noisy signal input S_noisy can be obtained as:

$$S\_noisy \ = S\_original + \ S_{4x} \tag{11}$$

$$S_{4x} = i * 0.5 * S_4(nmonte, :);\ \text{where } i = increment\ of\ noise = (1,2,3, \ldots, 12) \tag{12}$$

The waveforms of each individual components along with the noise free original signal and the noisy signal of SNR -13.6 dB are shown in Figure 2.

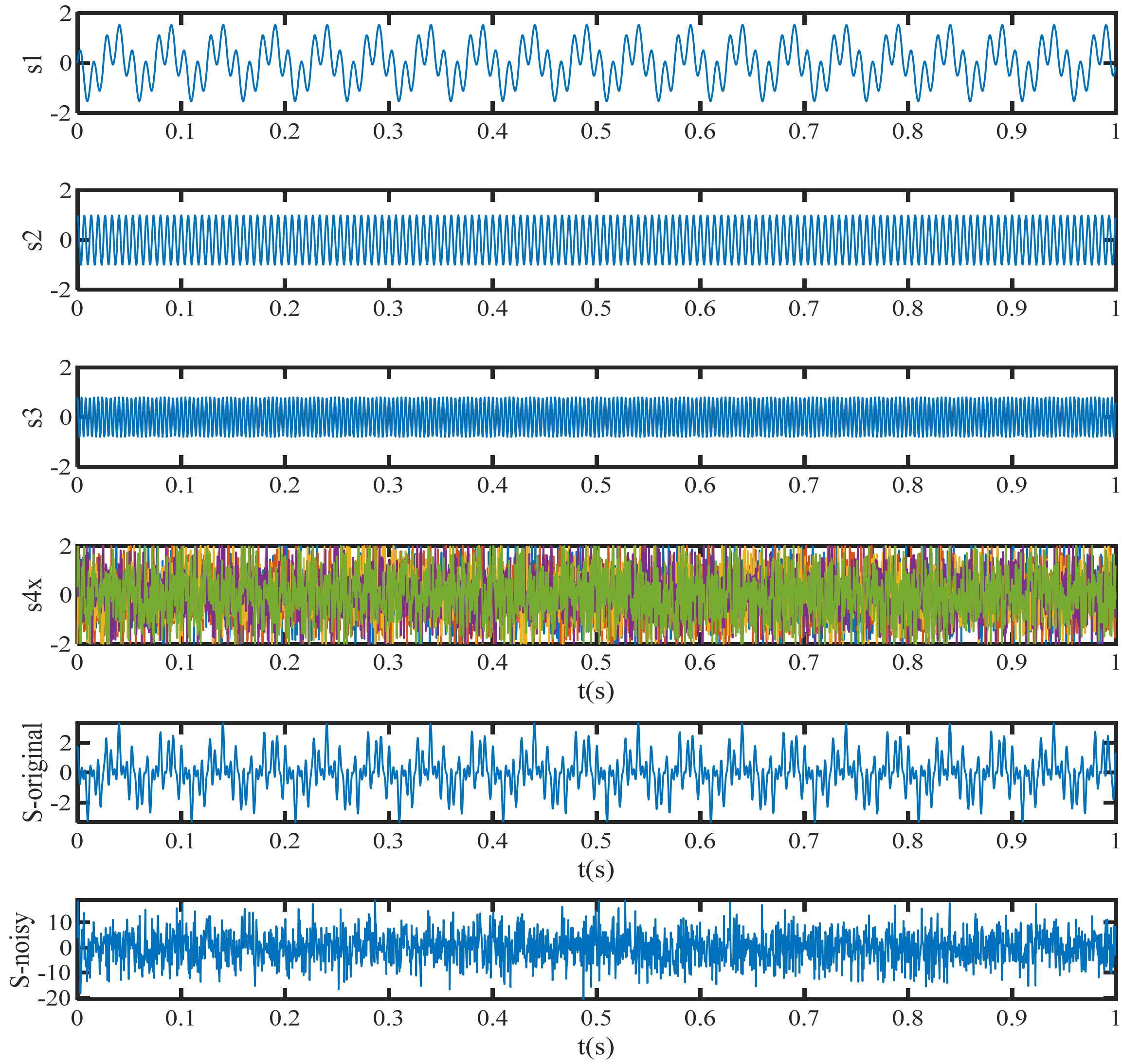

Figure 2: Simulation signal and it's individual components for SNR -13.6 dB.

The number of sampling points that has been considered for the simulation of the signal is 2048 and the sampling frequency is 2048 Hz. From Figure 2, it can be seen that the shock components are clearly contaminated in the signal S_noisy which is not a good indication for extracting fault features. The signal to noise ratio of each input noisy signal S_noisy has been calculated. The signal with SNR of -13.6 dB has been presented in analysis results.

According to the denoising scheme shown in Figure 1, the noisy input signal is then decomposed by VMD and Figure 3 shows the decomposition result where nine modes have been shown.

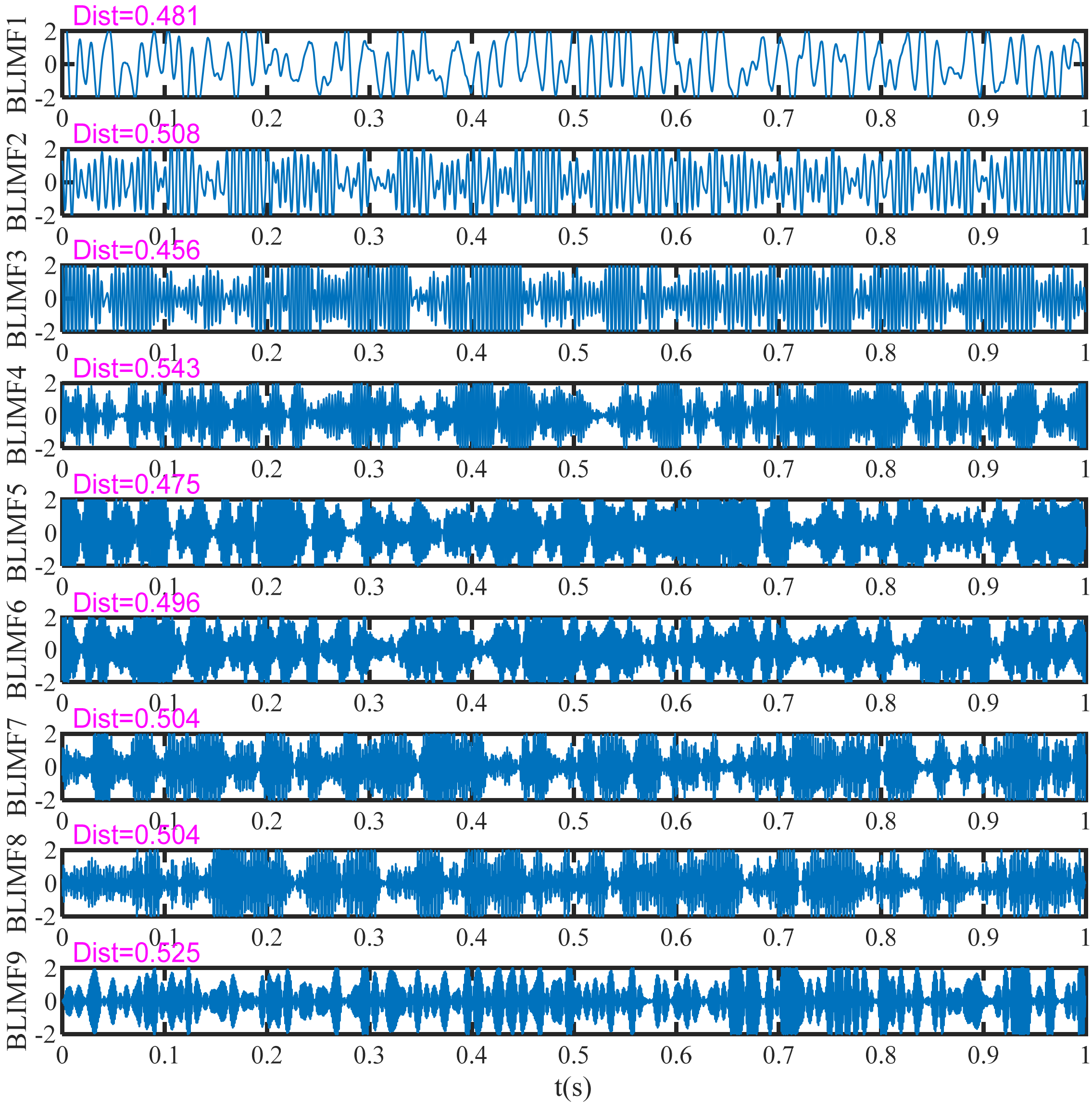

Figure 3: VMD decomposition result and PDF-distance values of the individual IMFs and that of the raw input signal.

From the decomposition result it can be found that among these BLIMFs produced by VMD method, contain pseudo components. To choose the meaningful BLIMFs for the reconstruction of the signal, PDFs of the individual BLIMFs and the input noisy signal are calculated. The PDFs are shown in Figure 4.

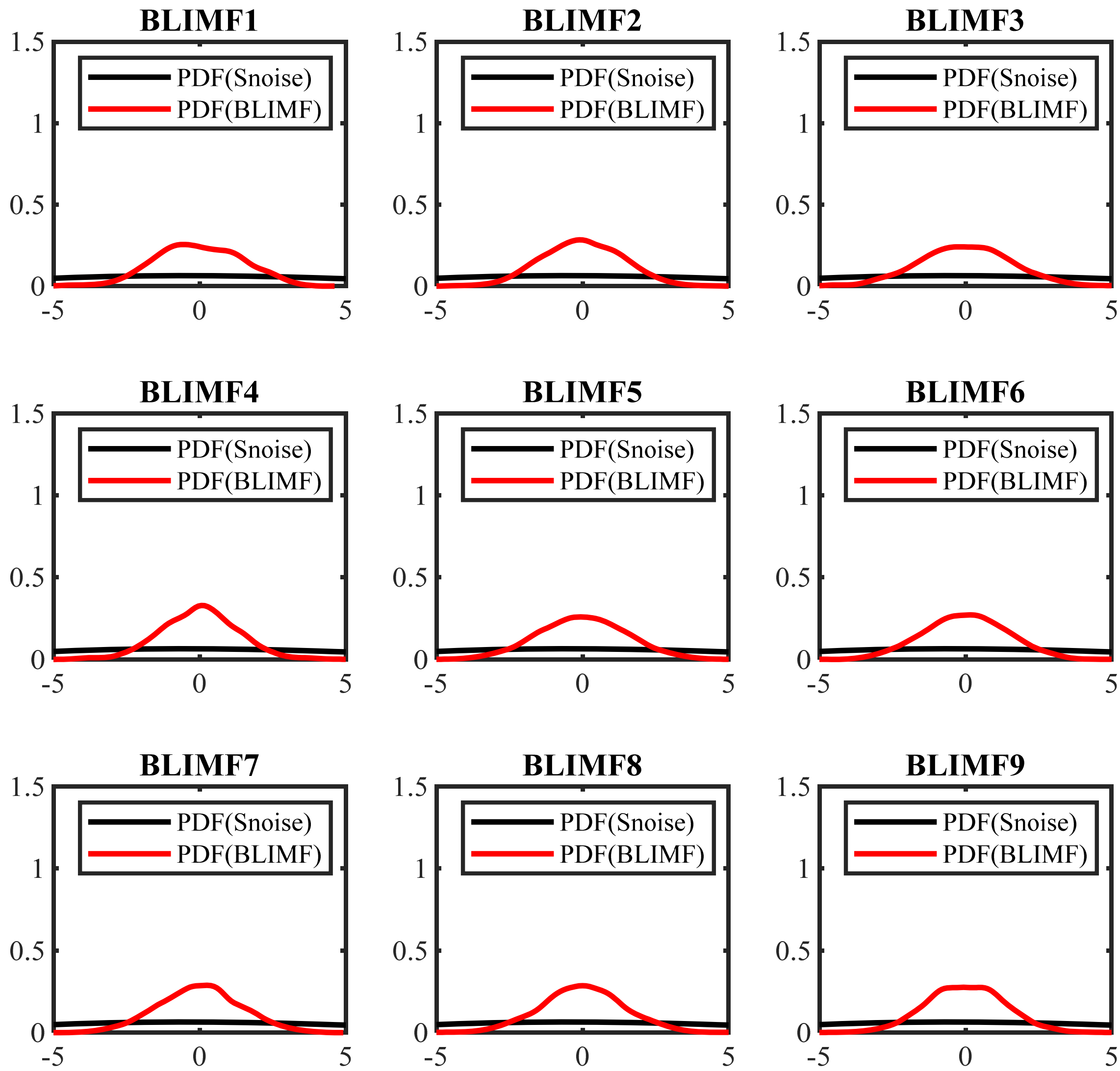

Figure 4: PDFs of the noisy input signal and those of its BLIMFs measured by K-S density method.

From Figure 4, it can be seen that the PDFs of all the BLIMFs are not the same. To choose the correct BLIMFs, a PDF similarity measure as a form of Euclidean distance between the PDF of the noisy input signal and the PDFs of BLIMFs have been incorporated. The Euclidean distance with smaller values will help to identify the BLIMfs that predominantly contain the features of the noisy input signal. The values of the Euclidean distance have been shown in Figure for a noisy input signal of -13.6 dB.

The frequency domain representation of the BLIMFs and that of the noisy input signal have been shown in Figure 5. From the figure it can be seen that each of the BLIMFs contains certain band of frequencies from lower to higher order range.

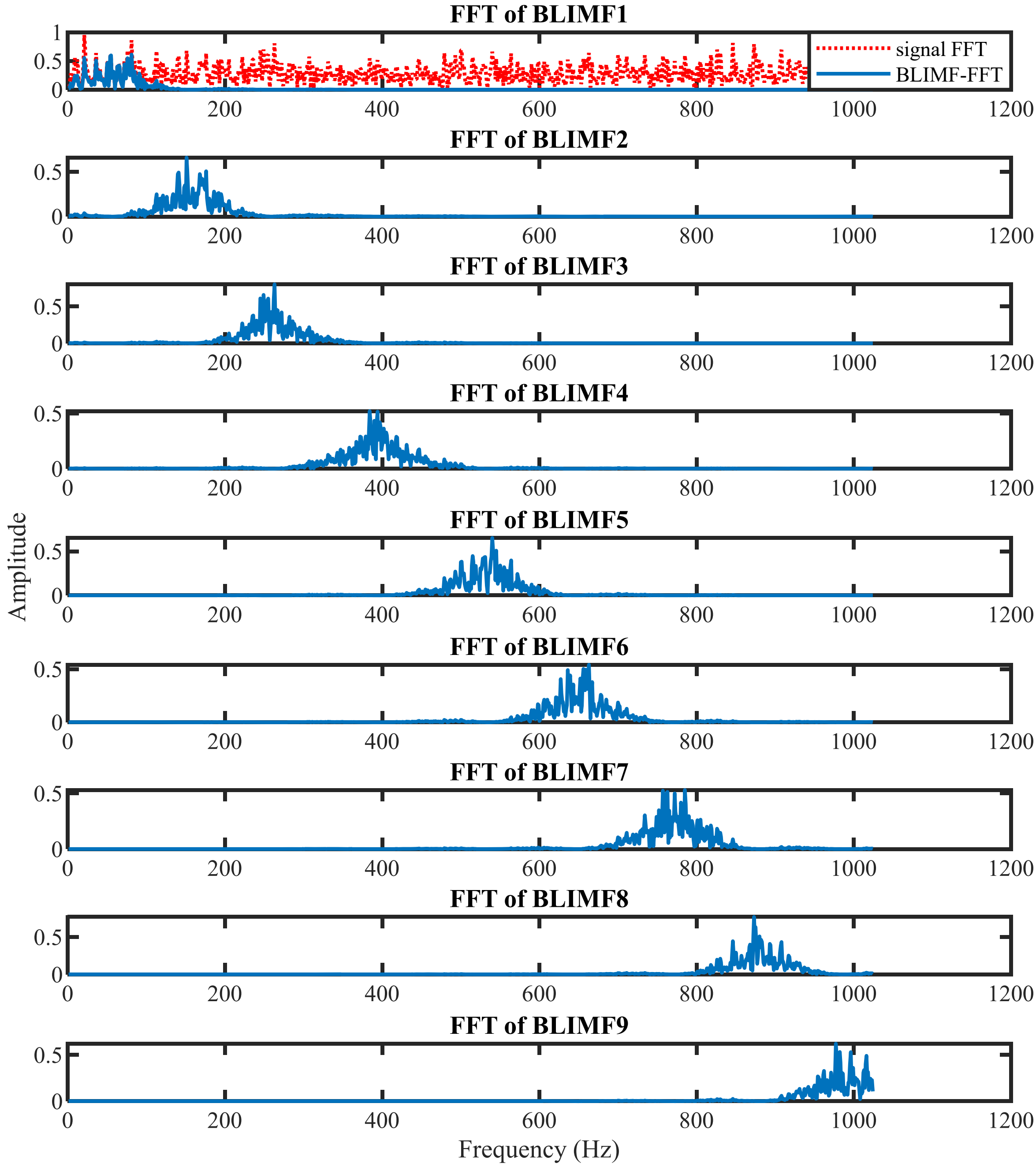

Figure 5: Fast Fourier transform of the BLIMFs and the noisy input signal.

In another method, the autocorrelation functions of each BLIMFs are obtained. Then Hilbert transform of the autocorrelation functions are performed. Then the absolute value of the Hilbert transform is considered for the analysis of the autocorrelation values. The higher values indicate the similar characteristic components and hence those BLIMFs are chosen for reconstruction of the signal. Figure 6 depicts the input noisy signal of -13.6 dB analysed by both PDF-ED method and autocorrelation-based method.

5.2. Performance Evaluation and Discussion

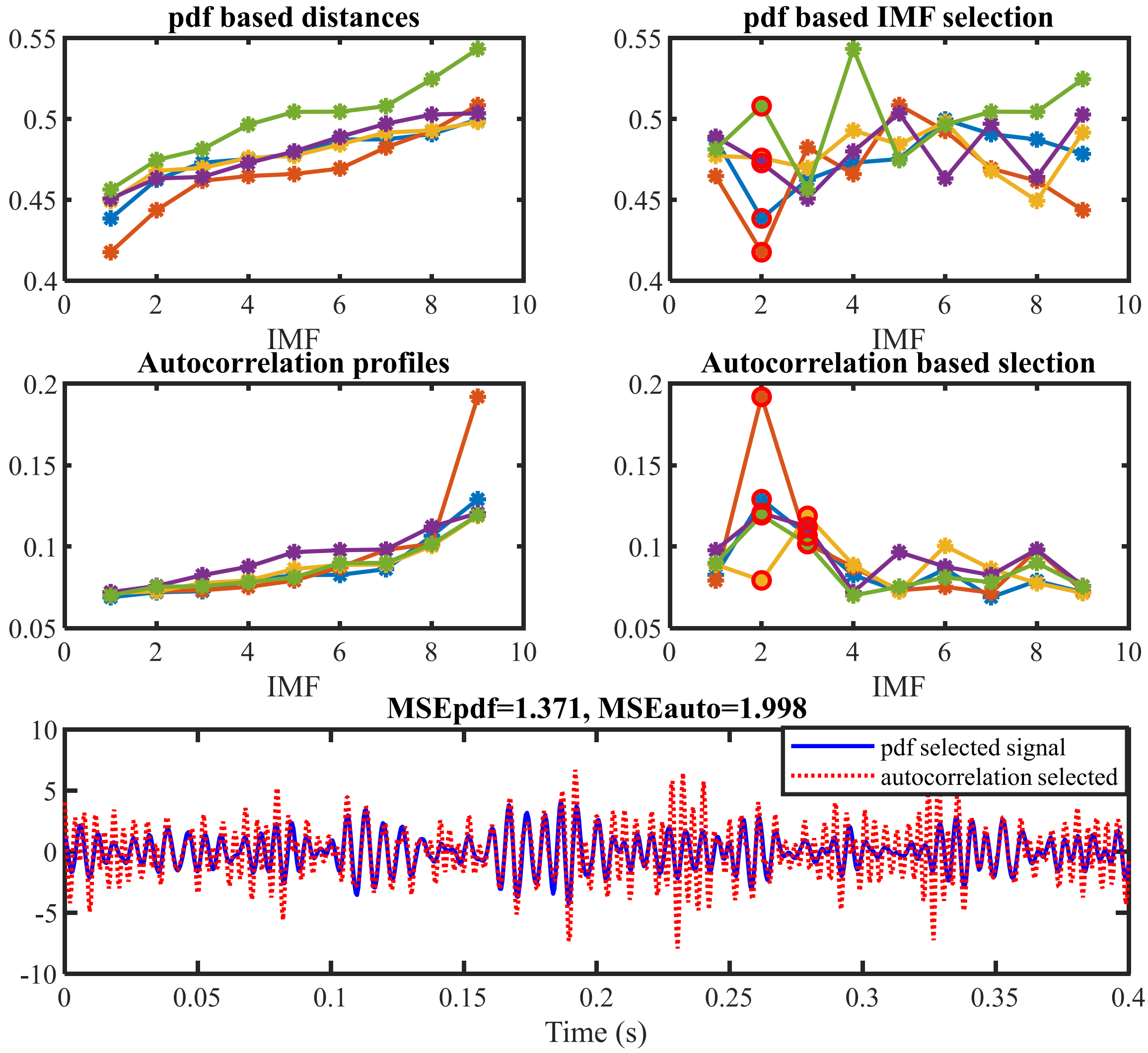

Figure 6: Selection of BLIMFs based on PDF-ED method and correlation-based method.

From the Figure 6 it can be seen, five sets of data obtained from Monte Carlo simulation for a particular SNR have been used to effectively validate the proposed algorithms. Monte Carlo simulation is a tool for validating the denoising methods. The usage of Monte Carlo can help to improve the robustness of the methods.

In case of PDF based method, the smaller values were considered for choosing the IMFs for reconstruction of the signal where a threshold criterion was adopted to observe the sudden jump of the values to the higher magnitudes.

Unlike PDF based method, the autocorrelation-based approach considered the higher values for generating the denoised signal where the threshold criterion was made to observe the sudden drop of the autocorrelation values.

The five sets of noisy data for a particular SNR by Monte Carlo simulation helps to choose the appropriate IMFs for reconstruction of the signal hence, it improves the robustness of the proposed methods. At the end, the root-mean-square-error (RMSE) of the signals constructed by the PDF based method and the autocorrelation-based methods are also calculated.

The denoised signals obtained from both methods still contain some sorts of noise. To make the methods more effective, Fourier transform based denoising technique is applied for both methods. Figure 7 shows the final denoising results as a form of RMSE and the input SNRs. It can be clearly seen that the using of FT denoising enhances the performance of correlation-based method significantly while compared without considering it.

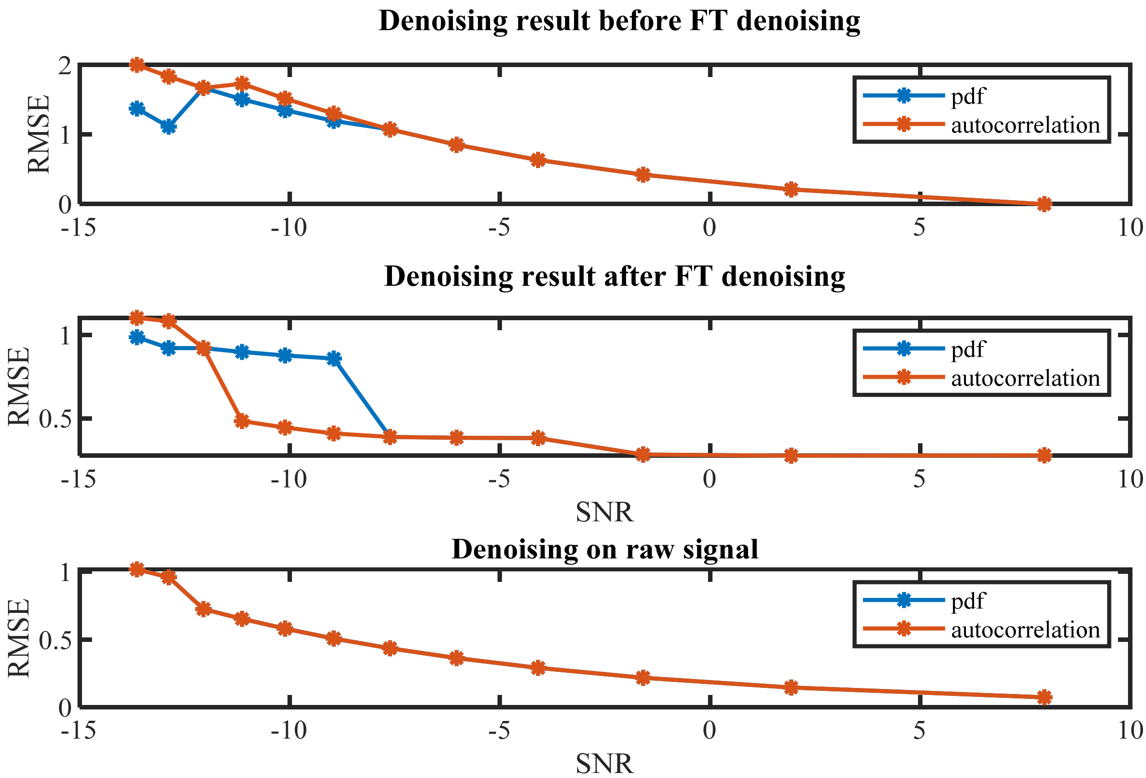

Figure 7: Denoising performance before and after FT denoising.

The result shows the effectiveness of the correlation-based method over the PDF based method for low SNR input signals, whereas, both methods perform the same if applied to the raw noisy input signal. The threshold values of each method with the corresponding SNRs of the raw noisy input signal has been shown in Figure 8.

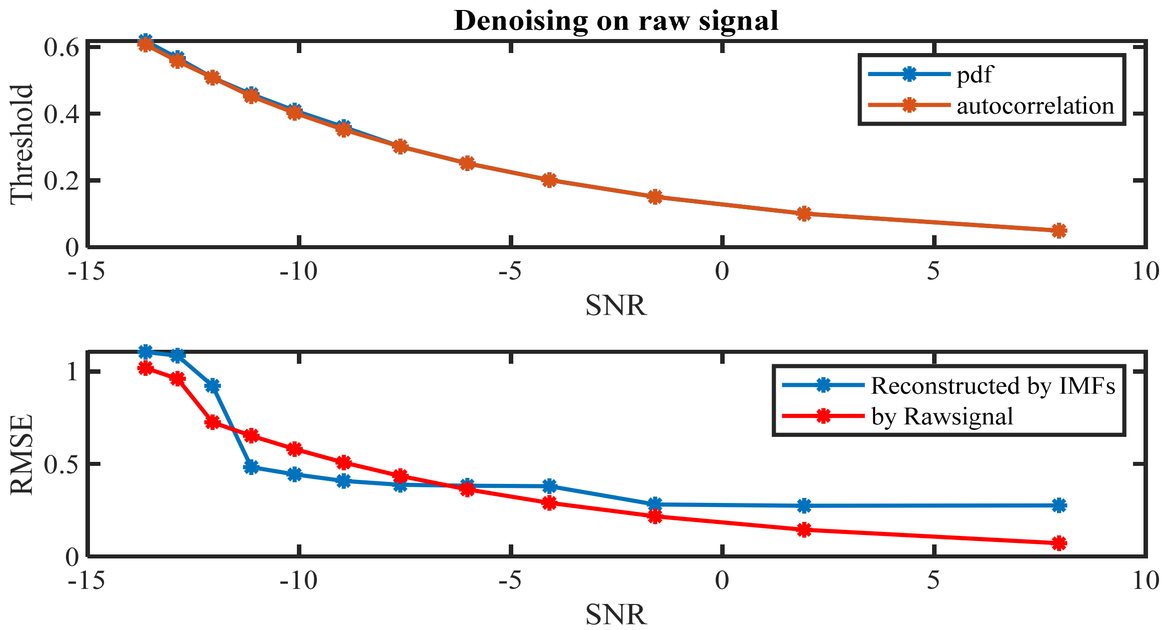

Figure 8: Denoising performance of raw input signal by both methods and the RMSE comparison between the VMD-correlation method and that of the raw signal under different SNRs.

Figure 8 also shows the comparative result of the denoising technique based on the VMD-autocorrelation method and the raw noisy input signal in respect to their RMSE values and corresponding SNRs. Analysis result shows that the proposed autocorrelation-based method is an effective denoising technique for very low SNR signals.

The denoised signal by the proposed autocorrelation and FT denoising based method has been presented in Figure 9.

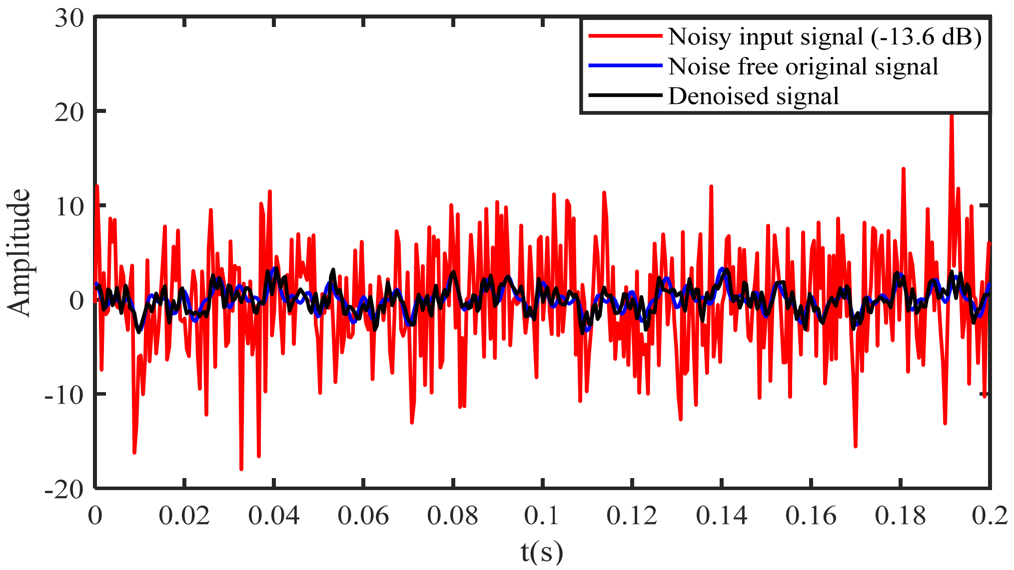

Figure 9: Denoised signal by proposed VMD-Autocorrelation based method.

The noisy input signal of SNR -13.6 dB, the noise free original signal and the denoised signal are compared for a time duration of 0.2 seconds.

Then the proposed method is compared with the VMD based other denoising method from literature (Li et al., 2017). For this purpose, VMD-PE method is considered for the comparison. Where PE is the permutation entropy. The algorithm for the same can be found below:

Step 1: Decompose signal by EMD.

Step 2: Select the decomposition level of VMD according to the decomposition level of EMD.

Step 3: Decompose signal by VMD, IMFs can be obtained.

Step 4: Calculate the PE of each IMF. For calculation of PE, order and time lag are 3 and 1 respectively.

Step 5: Screen out the noise IMFs according to the value of PE. One threshold value has been fixed, depending on that the value greater than 1.5 is regarded as the noise IMF.

Step 6: Reconstruct the useful IMFs with NPE less than 1.5. After the reconstruction, the process of denoising is completed.

The denoised signal has been shown in figure 10 with the original noise free signal and the noisy signal.

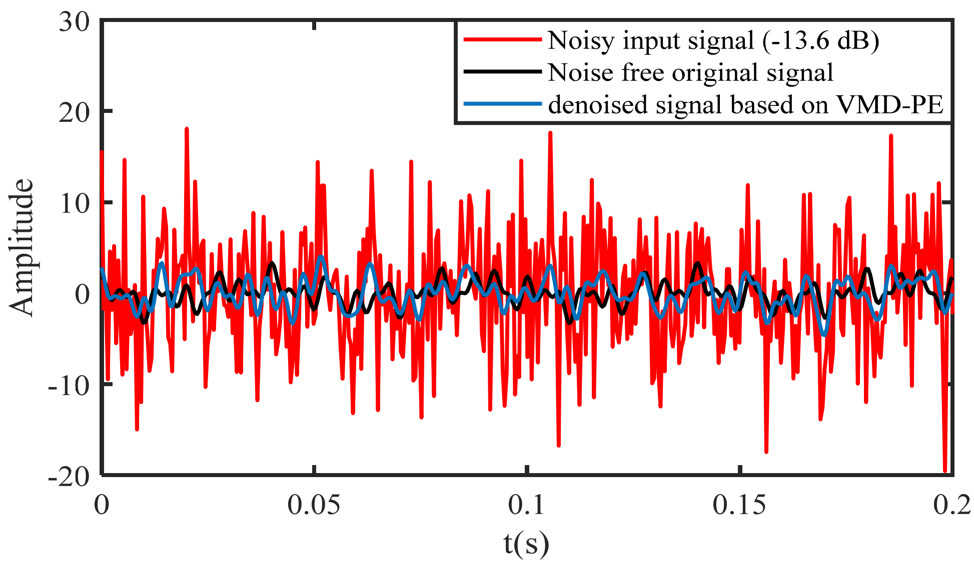

Figure 10: Denoised signal by proposed VMD-PE based method

To summarize, it can be seen that the results of proposed denoising algorithm shown in Fig. 9 has better denoising performance and overcomes the problem of threshold selection of VMD-PE method.

The autocorrelation-based method described in this paper is effective as it can suppress noise in the reconstructed signal and enhance the fault characteristics.

Autocorrelation of a signal is the correlation of a signal with its delayed copy. It is a useful mathematical tool to find out the presence of periodic signal obscured by noise or missing fundamental frequency. If a signal is periodic, then the signal will be perfectly correlated with a version of itself if the time-delay is an integer number of periods. That fact, along with related experiments, has implicated autocorrelation as a potentially important part of sound signal analysis and have a potential application on similar kind of periodic response from the compressor sound signal.

However, Probability density function (PDF) is efficient for random signals and no periodic that we fall to predict in time series analysis. However, spectral analysis, for example by using Fourier Transform, or Time-frequency analysis (wavelet analysis) are easier to understand signal properties. By using PDF, we look at distribution of signal values without caring about time dependence of signal values and indexes. Generally, probability and statistical techniques are used when we cannot describe our signal with deterministic model.

Hence, depending on the nature of the signal and based on the hypothesis we can say that autocorrelation approach is much more suitable to analyse compressor sound signal compared to the PDF based approach as the proposed method considers the periodicity and compressor sound signal has dominant periodic components; thus, suitable for the analysis by VMD-autocorrelation method.

6. Conclusion

This paper focuses on a comparative study between two VMD based denoising methods: VMD-PDF and VMD-autocorrelation. The main contribution of the paper is the introduction of autocorrelation method as a similarity measure of the BLIMF components. Traditionally used cross correlation approach is not reliable for complex signal analysis with unknown noise contents, whereas, the autocorrelation measures the similarity of a signal with a delayed copy of itself, used for finding periodic patterns obscured by noise or identifying hidden frequency components of a signal implied by its harmonic responses. The application of Monte Carlo simulation helps to improve the robustness of the methods. In addition, the paper investigates the effectiveness of using VMD-correlation method for denoising the signals with very low SNR. The application of FT denoising technique also helps to remove the additional noises from the reconstructed denoised signals obtained by the both methods and enhance the capability of VMD-correlation method in denoising of the low SNR signals. The RMSE served as an estimator for comparing the two methods under different SNRs. The analysis result shows that the proposed method can be useful for extracting fault features from a signal heavily contaminated in noise, especially the vibration and acoustic signals with low SNRs. However, the proposed method requires a given number of BLIMFs to decompose the input signal by VMD, which is considered to be a drawback. The future study will focus on the optimisation of the method further. The required modes of the VMD can be easily determined by calculation of the scaling component and introducing mutual information entropy (MIE) thresholding or considering the same number of IMFs obtained by the empirical mode decomposition (EMD) method.

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