

Investigation of the design of creep damage constitutive equations for low stress level

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Abstract

This paper reports the latest attempt to model the creep deformation and rupture for high Cr (P91) alloy under low stress range. While accurately predicting the creep deformation and damage and the ultimate rupture is paramount important, both for the academic research and industrial applications, the current situation is unsatisfactory, particularly for the lower stress range. This paper systematically investigates the formulation as well as tentatively the underline scientific reasons, to improve and accurately predict the creep deformation, creep damage and the ultimate rupture of P91 in the stress range of 80 to 125 MPa. This paper contributes to the knowledge for how to accurately model the creep deformation and damage for P91. Furthermore, it contributes to the methodology through the development of a generic model.

Key words: creep damage constitutive equations, chromium steels, P91, low stress, creep damage mechanics

Introduction

Creep damage is significant for structural parts working at high temperature, which may lead to fracture failure. Therefore, creep damage plays a significant role in all fields involving high-temperature work, especially for components operate in some dangerous working environments such as high-temperature steam pipes. Creep damage may bring high economic losses and human hazards. For example, Kansas City Power's steam pipe had a creep failure that resulted in a catastrophic accident in 1998 (Rodgers, 2007). Therefore, for the safe operation in high temperature environment, accurate

prediction of creep damage is essential.

At present, P91 steel has been widely used in thermal power field all over the world, mainly used in superheater and main steam piping system (Keller et al., 2010). These structures work for a long time under high temperature and certain stress loading, so it is essential to predict their fracture life. With the development of society and the improvement of people's standard of living, the requirements for electricity consumption have been increasing. In order to provide energy and improve efficiency of generating units, many countries have been building supercritical thermal power units in recent years (Vaillant et al., 2008). P91 (9Cr-1Mo-V-Nb) is a high chromium (Cr) alloy steel with an excellent creep and oxidation resistance at high temperatures (Orlova et al., 1998) that is more suitable for high temperature steam pipe.

The traditional common pipeline life prediction of thermal power units generally uses isothermal extrapolation method, which is practical and straightforward, so it is still widely used (Chen et al., 2011). Then, a more accurate time-temperature extrapolation method based on the Arrhenius equation appeared, but it was still not accurate enough and easily overestimated the service life of the structure (Tu et al., 2009).

In order to accurately predict the creep life, it is necessary to understand the creep mechanism and microstructure evolution correctly in the process of materials service according to the physical mechanism of creep, and to use the correct prediction method.

With constant stress loading, there are three different creep deformation stages, these are as follows:

- (1) Primary creep stage. In this stage the initial strain rate is high but decreases with time, it is also called decelerating creep stage. The creep deformation will enter next stage during the strain rate decreases to a value.
- (2) Steady state creep stage. In this stage the creep strain increases slowly. Strain rate, which is a function of stress, increases as the stress increases. It can be described by power-law relation.
- (3) Accelerating creep stage. With strain and time increases, the creep strain increases sharply and causes the material failure and rupture.

Creep damage is accumulating over time and damage occurs within the material. On a macroscopic level, the damages caused during creep are due to particle coarsening and

the growth of voids. As the stress level is reduced from high to low levels, the time to creep failure becomes extended.

Continuum damage mechanics (CDM) model is a popular method of material life simulation and is widely used to develop the creep constitutive equation of P91 steel. CDM based methods were originally proposed by Kachanov (1958), developed from continuum mechanics. In order to describe each damage variable, an equation for each variable was developed and added into a CDM model.

The characteristics of CDM based methods are:

- a. It is not necessary to know the damage mechanism of materials in detail from the micro-level. The degree of creep damage can be directly evaluated from the value of each damage parameter. (Segle, 2002).
- b. The creep constitutive equation based on CDM can be used to analyze the creep damage behavior simply by using the finite element program, which can greatly reduce the research time and cost (Maharaj et al., 2009).
- c. By introducing different damage parameters, the effects of different damage on creep behavior can be conveniently studied. (Dyson, 2000).

A critical review on the development of creep constitutive equations can be found in (Xu et al, 2017a, Xu and Lu, 2020) where the questions about how to model the creep cavitation damage, the coupling of the creep deformation and creep damage, and multi-axial generalization et al were raised. Detailed work on some of these aspects can be found at (Xu, 2000, 2001, 2004, Xu and Barrans, 2003, Xu and Hayhurst, 2003, Xu et al., 2017b, Yang et al., 2013a, Yang, 2018 and Wang et al., 2020).

More specifically, the current creep constitutive equations are unsatisfactory under a wide stress range. Due to its high cost and long duration, the long-term creep tests were rarely carried out, the accelerated creep test under higher stress are usually used hoping to extrapolate to the lower stress level. Therefore, the creep damage equation developed based on the creep test at high stress may not be applicable for low stress levels.

Recently, Yin and Faulkner (2006) proposed a cavitation damage calculation method for 9Cr alloy steel based on McLean and Dyson's (2000) CDM framework. On this basis, Chen (2011) developed a set of creep constitutive equations suitable for medium-high stress range, added some damage parameter such as solute depletion, cavity nucleation and growth and particle coarsening. Yang et al.(2013b) tried to apply it to

the low stress range and found that it did not work, but he did not carry out further research. At present, the research of low stress level and long-term creep behavior is unsatisfactory, the common prediction methods still have high percentage of error.

The main research objectives of this paper are as follows:

1. To research the applicability of the constitutive equation developed by Chen (2011) to assess creep behavior of P91 steel at 600°C in the low stress range of 125MPa~80MPa. The calculated results were compared with the collected experimental data, and analyze the reasons why the constitutive equation could not apply to the low stress range. In order to improve the understanding of the changes in creep mechanisms under different stress loading, this study will discuss from the creep mechanism perspective.
2. To find a method which can improve the applicability of the equation, then apply it on the Chen's equation (Chen et al., 2011) by means of comparison with experimental data to verify whether the proposed method is applicable.
3. To find a modified constitutive equation in low stress, based on the Chen's equation (Chen et al., 2011).

Continuum damage mechanics modeling

1. Parameters in creep damage mechanics

In order to accurately describe the creep damage behavior, it is necessary to add the microscopic damage parameters of the materials in the creep process into the CDM model. During the long-term operation at high temperature, the microstructure and structure of P91 steel will gradually change, the damage parameters related solid solution depletion, particle coarsening and cavitation are defined as D_s , D_p , D_n respectively.

1.1. Solid solution depletion

The decrease of creep resistance is caused by Molybdenum (Mo) depletion in sub grain matrix under the action of long-term high temperature and stress. (Basirat et al., 2012). D_s can be calculated by the equation below (Yin & Faulkner, 2006):

$$D_s = 1 - \frac{\bar{c}_t}{c_o} \quad (1.1)$$

c_o and \bar{c}_t respectively are the initial concentration of solid solution and the average concentration at time t. In Dyson's approach, the rate of D_s can

be calculated by Wert-Zener equation (Wert et al.,1950):

$$\dot{D}_s = K_s D_s^{1/3} (1 - D_s) \quad (1.2)$$

Material constant K_s can calculate by the below equation:

$$K_s = [48\pi^2 (C_0 - \frac{C_e}{C_\beta})^{1/3} n^{2/3} D] \quad (1.3)$$

D is the diffusion coefficient of Mo in the matrix. After measuring C_0 and C_e , it can be concluded that $C_0=0.56\text{mol}\%$ and $C_e=0.33\text{mol}\%$ (Semba et al., 2008).

1.2. Particle coarsening

The coarsening of precipitates in P91 steel has an important influence on the creep resistance of materials (Basirat et al., 2012). The Chen's creep damage constitutive equation (Chen et al.,2011) mainly considers the laves phase coarsening. According to Dyson's approach (Yin & Faulkner, 2006), D_p can be defined as:

$$D_p = 1 - \frac{P_0}{P_t} \quad (2.1)$$

Where P_0 is the particle diameter at the initial creep stage of material, P_t is the particle size at time t .

It is supposed that the coarsening of the particles can be calculated by Livshitz-Wagner equation (Yin & Faulkner, 2006).

$$r^3 - r_0^3 = Kt \quad (2.2)$$

Where K is a constant related to interface energy, equilibrium solute concentration and diffusion coefficient, r and r_0 is the radii of the particle.

In addition, the formula for calculating D_p change rate is defined as:

$$\dot{D}_p = \frac{K_p}{3} (1 - D_p)^4 \quad (2.3)$$

Where K_p is a constant of rate. The value of K_p is the third power of the initial particle size.

1.3. Cavity nucleation and crack formation

The creep behavior of P91 steel is controlled by different mechanisms. One of which is the most important one is the void nucleation and cavity formation (Basirat et al.,2012).

The equation of the rate of D_n is shown below (Yin & Faulkner, 2006):

$$\dot{D}_n = A' \dot{\epsilon} \epsilon^{B'} \quad (3.1)$$

The value of A and B depends on temperature (Yin & Faulkner, 2006). $A'=AB$, $B'=B-1$. The value of D_n might over one when A' and B' at large strain and high strain rates. It will lead to a divergence in the result of calculating. So, the value of D_n should larger than 0 and less than 1.

1.4. Dimensionless parameter of strain hardening (H)

The primary creep model is suggested by Ion et al (1986). H is a dimensionless parameter defined as:

$$H = \frac{\sigma_i}{\sigma} \quad (4.1)$$

The rate of H is:

$$\dot{H} = \frac{h'}{\sigma} \left[1 - \frac{H}{H^*}\right] \dot{\epsilon} \quad (4.2)$$

The value of H is larger than 0 and less than H^* , H^* is the maximum value of Harding in creep damage behavior. Constant $h'=E\Phi$. Φ is the volume fraction, E is the Young's modulus.

2. Introduction of creep damage constitutive equations.

2.1. Kachanov-Rabotnov equation

Kachanov was developed the prototype of the Kachanov-Rabotnov equation in 1958. Then it was improved by Rabotnov (1969), added the mathematical description of tertiary creep.

Kachanov-Rabotnov equation is a nonlinear first order differential equation, used a power-law creep rate function which defined as:

$$\dot{\epsilon} = \epsilon'_0 \left[\frac{\sigma}{\sigma_0(1-D)} \right]^n \quad (5.1)$$

$$\dot{D} = \dot{D}_0 \left[\frac{\sigma}{\sigma_0(1-D)} \right]^v \quad (5.2)$$

The $\dot{\epsilon}$ is the creep rate when creep is occurring, σ is the loaded stress on the materials during the creep behavior. ϵ'_0 and \dot{D}_0 is the initial state. n and v are material constants called stress exponent.

Advantages:

1. Kachanov-Rabotnov equation had a mathematical description of tertiary creep (Gorash, 2008).
2. According to change the value of n, this equation can use on

different stress and temperatures easily (Hall & Hayhurst, 1991).

Disadvantages:

1. Kachanov-Rabotnov equation was not consider hardening, particle coarsening and other creep damage mechanisms (Dyson & McLean, 1983).

2.2. Dyson's equation

Kachanov-Rabotnov equation use D to describe many microdefects like precipitate coarsening, solid solution. It is too simple to describe creep damage behavior accuracy. Dyson (2000) improved the CDM model, combined some damage parameter into CDM model and use sine-law replaced the power law. The Dyson's creep damage constitutive equation defined as:

$$\dot{\varepsilon} = \dot{\varepsilon}_0 \frac{1}{(1-D_s)(1-D_d)} \times \sinh \left[\frac{\sigma(1-H)}{\sigma_0(1-D_p)(1-D_n)(1-D_{cor})(1-D_{ox})} \right] \quad (6.1)$$

$$\dot{H} = \frac{h'}{\sigma} \left(1 - \frac{H}{H^*}\right) \dot{\varepsilon} \quad (6.2)$$

$$\dot{D}_s = K_s D_s^{1/3} (1-D_s) \quad (6.3)$$

$$\dot{D}_p = \frac{K_p}{3} (1 - D_p)^4 \quad (6.4)$$

$$\dot{D}_n = A' \dot{\varepsilon} \varepsilon^{B'} \quad (6.5)$$

2.3. Chen Yunxiang and Yang Ke's equation

Based on Kachanov-Rabotnov's CDM model, in order to improve the accuracy of description of creep damage behavior, some damage parameters were added into CDM model by Chen Yunxiang and Yang Ke (2011). Only considered the effect of the coarsening of Laves phase, the new equation defined as:

$$\dot{\varepsilon} = \dot{\varepsilon}_0 \frac{1}{(1-D_s)} \left[\frac{\sigma(1-H)}{\sigma_0(1-D_p)(1-D_n)} \right]^n \quad (7.1)$$

$$\dot{H} = \frac{h'}{\sigma} \left(1 - \frac{H}{H^*}\right) \dot{\varepsilon} \quad (7.2)$$

$$\dot{D}_s = K_s D_s^{1/3} (1-D_s) \quad (7.3)$$

$$\dot{D}_p = \frac{K_p}{3} (1 - D_p)^4 \quad (7.4)$$

$$\dot{D}_n = A' \dot{\epsilon} \epsilon^{B'} \quad (7.5)$$

The advantage of Chen's (2011) equation:

By analyzing the microstructure changes of materials in the creep process and adding damage parameters in a targeted way, the simulation of creep life is more accurate after experimental verification (Chen et al., 2011).

Chen's equation was based on the initial CDM model, and various damage parameters were added to facilitate the analysis of the effect of different damage parameters on the creep behavior.

The disadvantages of Chen's equation:

Due to Chen's equation is a power-law function, it may lead an imprecise prediction in the primary and steady state creep stage at low stress level, the creep strain is usually lower than experimental data.

3. The current situation of application to wide stress level

At present, there are no creep model can be used to predict creep life accurately at all temperature and stress applied. With the change of the working environment of material, the parameter in creep model may also change.

Yang et al. (2013b) tried to use the constitutive equation by Chen (2011) to predict creep life at 600°C and low stress level, the result is not work. But he did not propose improve method or assumption. Basirat et al. (2012) developed a creep model which can be used at 80MPa~200MPa. As his research, the value of damage parameter A will be change with applied stress. Basirat tried to establish a mathematical relationship between the constant A and the stress level, but he only optimized A value in terms of macroscopic phenomena, there is no clear trend in the constant A when the stress level changing from low to high. Therefore, it is difficult to use it in prediction with confidence. Yin and Faulkner (2006) stated the constant A is a function related to temperature, which is used to describe the void nucleation and crack formation in different stress and temperature, no relationship with applied stress.

By the conclusions of Chen (2011) and Rouse et al. (2013), in the power-law system, with the change of the stress range, the value of n is not fixed and needs to be

changed with the change of the stress level.

Christopher (2019) improved a creep model based on Dyson and McLean, developed a model of MX precipitates and formation and the growth of Z-phase with time. The simulated result in his research has a good agreement with experiment data at 50MPa~100MPa. In his research the value of damage parameter K_s and K_p within a specific range, the specific values were calculated by numerical optimization. The relationship between damage parameters and stress levels needs to be further investigated. Basirat et al. (2012) was also state the value of parameter K_s and K_p are not fixed while applied stress change.

The method of calculation

From the constitutive equation above, it can be observed that almost all equations are ordinary differential equations of the first order. As generally mathematic differential knowledge, Runge-Kutta method and Euler method are usually used to solve the ordinary differential equations of the first order. The results by Runge-Kutta method are more accurate than Euler method.

In this study, it is focus on the usability of the improvement of constitutive equation. The accuracy of creep life simulation is not key point. Therefore, in order to simplify calculating, the Euler method has been chosen to use.

Methodology

1. The creep test parameters of multiple groups of P91 steel at 600°C for a long time with low stress were collected. The total stress range was 125~80MPa, and the creep life range was 4000~10000 hours. Then collect all the parameters which were used in the creep modeling by Chen (2011).
2. Based on the equation by Chen (2011), calculate the creep life and compare with the collected experiment data. Try to adjust A to improve the applicability of equation at low stress level, if it does not work, then adjusting n.
3. Compare the modified results and experimental data again, draw the curves of creep damage parameters and creep life curves on the same figure, analyze the reason from the perspective of creep mechanism, consider that the accuracy of K_s and K_p could affect on simulated results. Adjusting A, n, K_s , K_p .

4. According to the final result, to assess the applicability of this improve method for creep constitutive equation at low stress level.
- 5.

	A	n	Ks	Kp
Step 1	N	N	N	N
Step 2	Y	N	N	N
Step 3	Y	Y	N	N
Step 4	Y	Y	Y	Y

Table1: The constant that was adjusted in each step

Results

1. Calculation method and verification

All the parameters used in the equation are calculated by Chen (2011) according to the data he collected in different literatures, the material constants and damage parameters used in calculation are shown in Table 2.

Parameter	Value	Unit
K_s	5E-08	S^{-1}
K_p	1.5E-07	S^{-1}
H^*	0.269	
h'	10000	
A'	10.186	
B'	0.95	
N	10.186	
σ_0	200	MPa
ε'_0	0.0000057	S^{-1}

Table 2: Material constant and parameter

Excel software can plot out the curve of relationship between creep strain and time or the creep rate and time when calculation done. The result shown below:

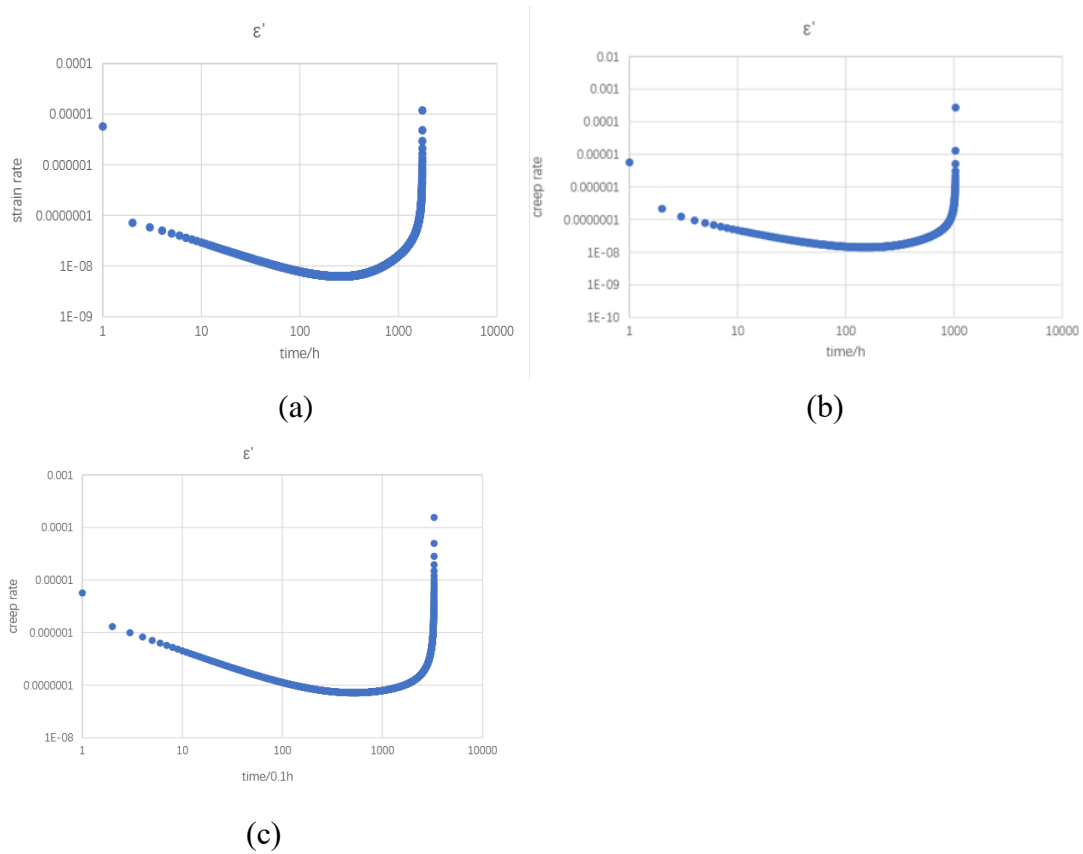


Fig. 1: The curve of strain rate and time. (a) 600°C,130MPa, (b) 600°C,145MPa, (c) 600°C,175MPa

By comparing the calculation results with those posted in the literature by Chen (2011), the two results are consistent, which can prove that there is no error in the calculation, and this calculation method can be used in the next research.

2. Material and experiment

Due to the long time and high cost of low-stress creep test, there are relatively less experimental data can collect. In this study, four groups of low-stress creep experimental data of P91 steel for a long time were used. The total stress range was 125~80MPa, and the creep life range was 4000~10000 hours. The stress decreases gradually in the four sets of data, which is helpful to analyze the trend of each damage parameters in different stress.

Experimental data 1: XCrNiCuNb18-9-3 alloy (P91) was used for constant tension creep test under 600°C and 125MPa loading, and the creep life reached over 4000 hours (Duda et al., 2016).

Experimental data 2: P91 alloy was used for creep test under 600°C and 120MPa loading, use helicoid spring specimen technique, creep life over 15,000h (Sklenička et al.,2003).

Experimental data 3: P91 alloy was used for creep test under 600°C, 100MPa loaded, creep life over 60000h (Lo Conte et al.,2018).

Experimental data 4: P91 alloy as pipe was used for creep test under 600°C, 80MPa loaded, creep lifetime over 100000h (Panait et al.,2010).

The collected experimental data are shown as Figure 2.

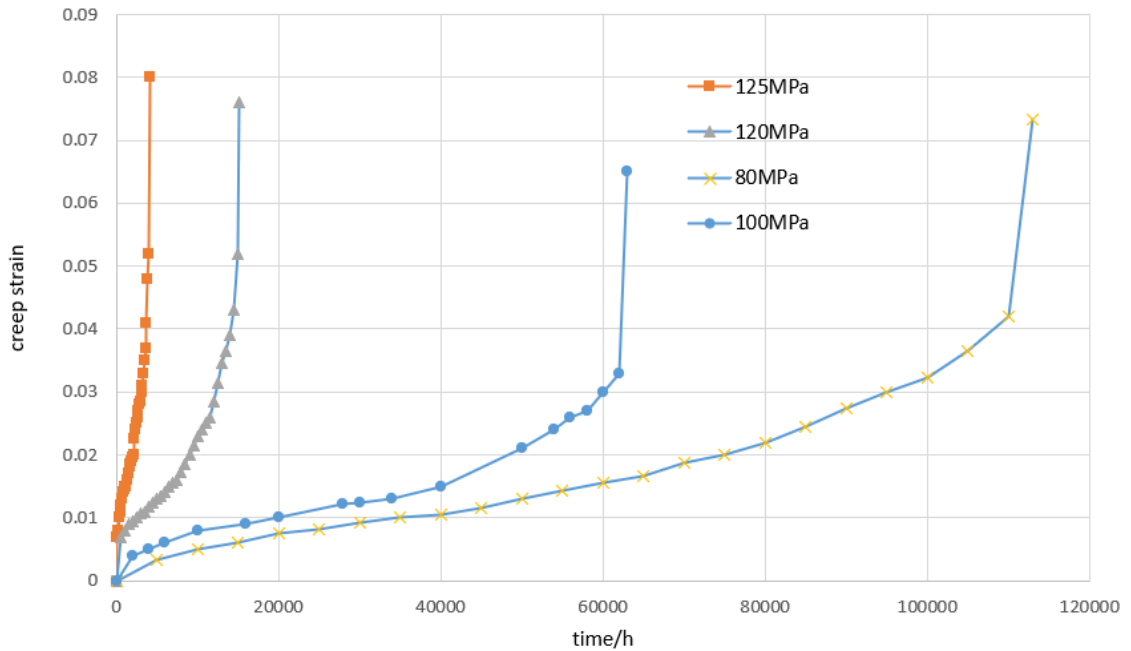


Fig. 2: Collected creep curves under different stress

3. Compare with the experimental data

3.1. Adjusting the value of A

According to Yin's assumption, coefficient A does not change with the stress level.

$A'=AB$, $B'=B-1$, as equation 7.1 and 7.5:

$$\dot{\epsilon} = \dot{\epsilon}_0 \frac{1}{(1-D_s)} \left[\frac{\sigma(1-H)}{\sigma_0(1-D_p)(1-D_n)} \right]^n$$

$$\dot{D}_n = A' \dot{\epsilon} \epsilon^{B'}$$

Firstly, Chen's equation was used for low stress level according to this assumption, and the results are as follows:

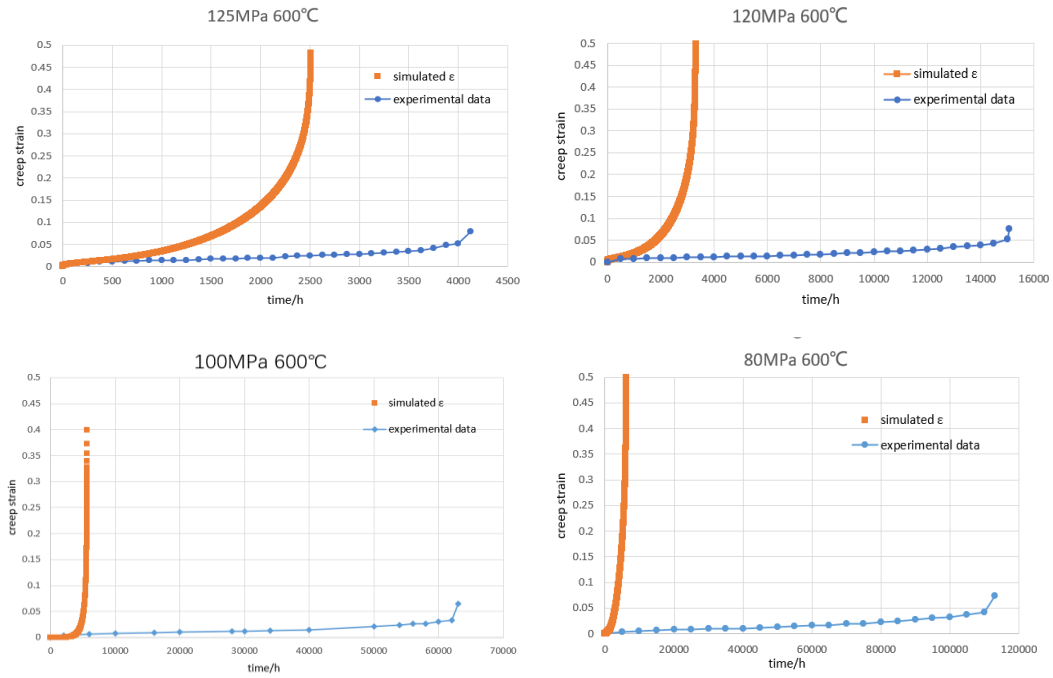


Fig. 3 : Comparison between the modeled creep life and experimental data

Compared with the simulation results and experimental data, there was a big difference between them. Chen's equation cannot be directly used in low stress, this conclusion is consistent with Yang et al. (2013b).

Through data comparison, the results simulated by Chen's equation have a short life and the strain variable increases sharply, which cannot be consistent with the experimental data.

According to the creep damage mechanism, the sharp increase of strain variable is directly related to the cavity damage, and the coefficient A is directly related to the cavity damage D_n . And according to Basirat's results, when the stress is small, the coefficient A decreases with the decrease of stress. Optimization of A -values is beneficial for understanding creep behavior at low stress level. Therefore, trial was conducted by adjusting the value of coefficient A while the other parameters remained unchanged. The results are shown below:

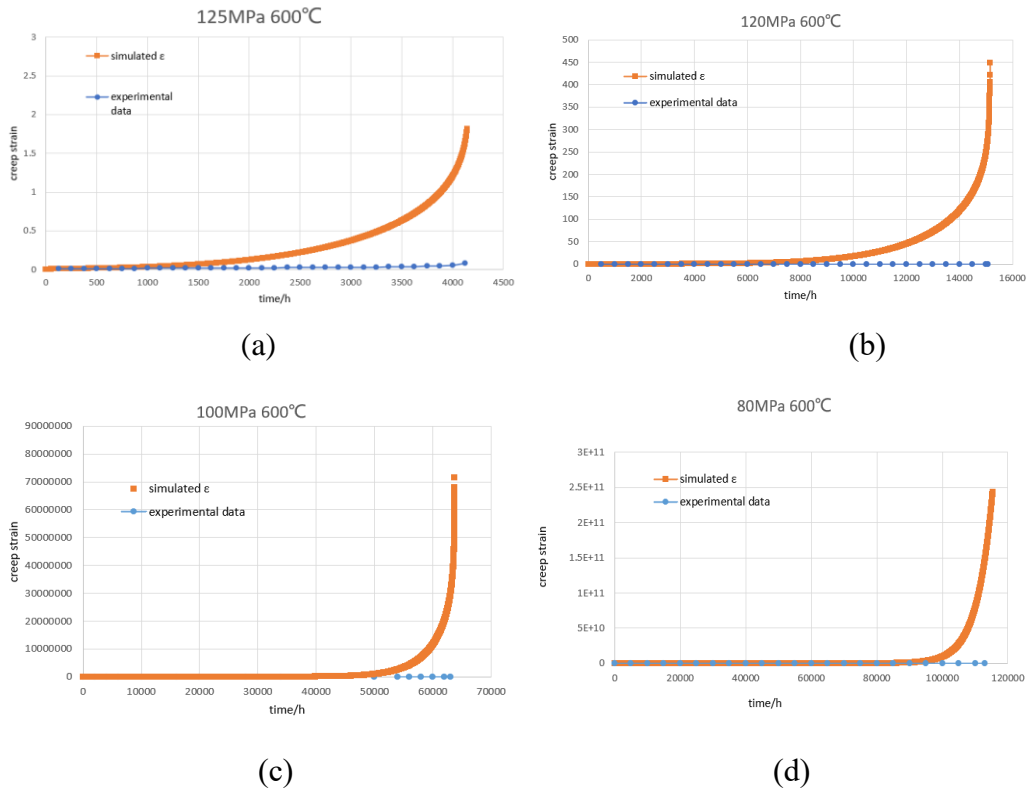


Fig. 4: Comparison between the modeled creep life and experimental data

(a): $A=0.07$, (b): $A=0.25 \cdot 10^{-6}$,

(c): $A=1 \cdot 10^{-16}$, (d): $A=1 \cdot 10^{-50}$

After adjusting the value of A , the simulated creep life has a positive agreement with the experimental data, but the strain is still not consistent. When the simulated results reach the life, the strain is extremely large and has no engineering significance.

3.2. Adjusting the value of A and n

According to the conclusions of Chen (2011) and Rouse et al. (2013), the value of n can be changed with the change of the stress level, so it would be recommended to continue to adjust the parameters A and n at the same time, then observe the comparison results.

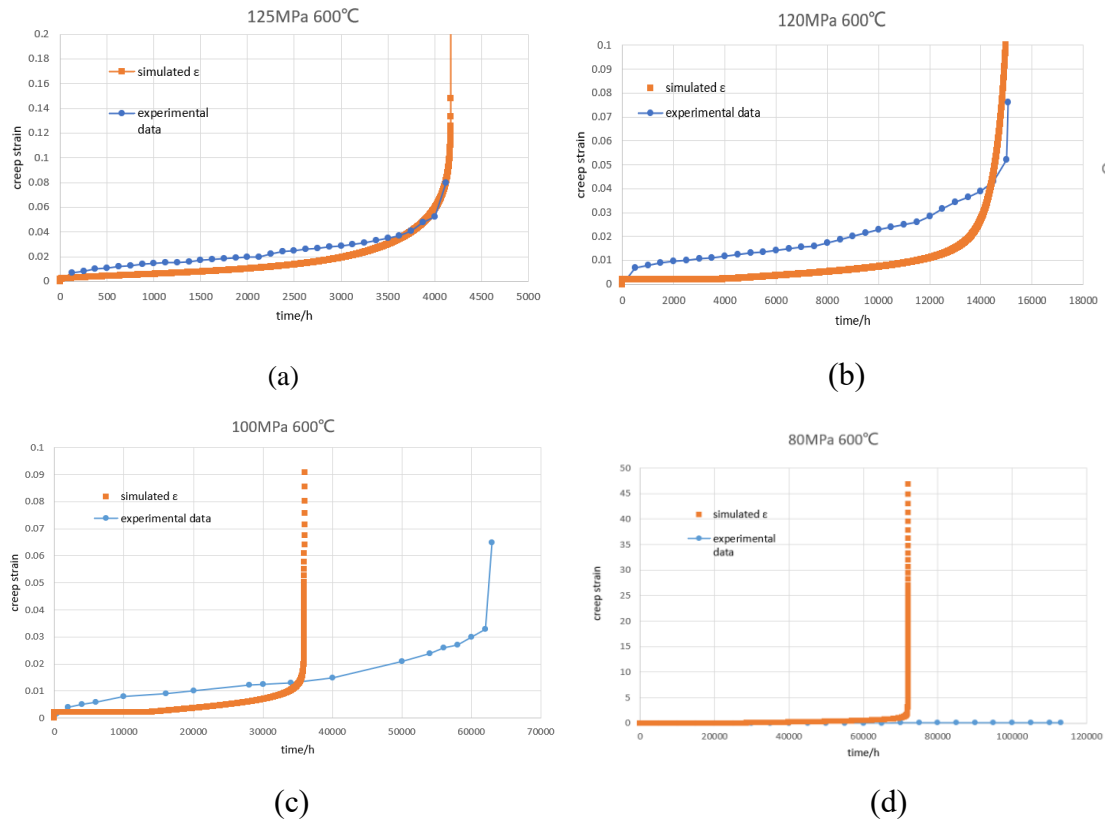


Fig. 5 : Comparison between the modeled creep life and experimental data

(a): $A=15$ and $n=18$, (b): $A=0.5$ and $n=80$,

(c): $A=1 \cdot 10^{-16}$ and $n=10000$, (d): $A=1 \cdot 10^{-30}$ and $n=10000$

Compare the result, the simulated result could have good agreement with experimental data by adjusting the parameter A and n at 600°C, 125MPa, 120MPa.

However, at 100MPa and 80MPa, the creep life over 60000 and 100,000 hours respectively. The simulated result and experimental data cannot consistent although the A is adjusted to $1E-30$, n is adjusted to 10000. And through calculation, it can be concluded that when n at large value and A at small value, continuously increasing n value and decreasing A value will not bring significant increase in creep life, as shown in the following Table:

A=1E-24	
n	creep life/h
100	50430
1000	69320
5000	71590
10000	71900

n=20000	
A	creep life/h
10	71480
1	71950
0.1	72050
0.01	72050

Table 3: Influence of the values of A and n on creep life

According to Chen' equations (Chen et al., 2011) 7.1 and 7.5:

$$\dot{\epsilon} = \dot{\epsilon}_0 \frac{1}{(1 - D_s)} \left[\frac{\sigma(1 - H)}{\sigma_0(1 - D_p)(1 - D_n)} \right]^n$$

$$\dot{D}_n = A' \dot{\epsilon} \epsilon^{B'}$$

When the value of n is tending to extremely large, and A is extremely small, the value of damage D_n will also become extremely small, resulting in no physical meaning. At this time, the damage parameters that dominate the creep behavior may change. So the damage parameters and creep strain are plotted out together for observation.

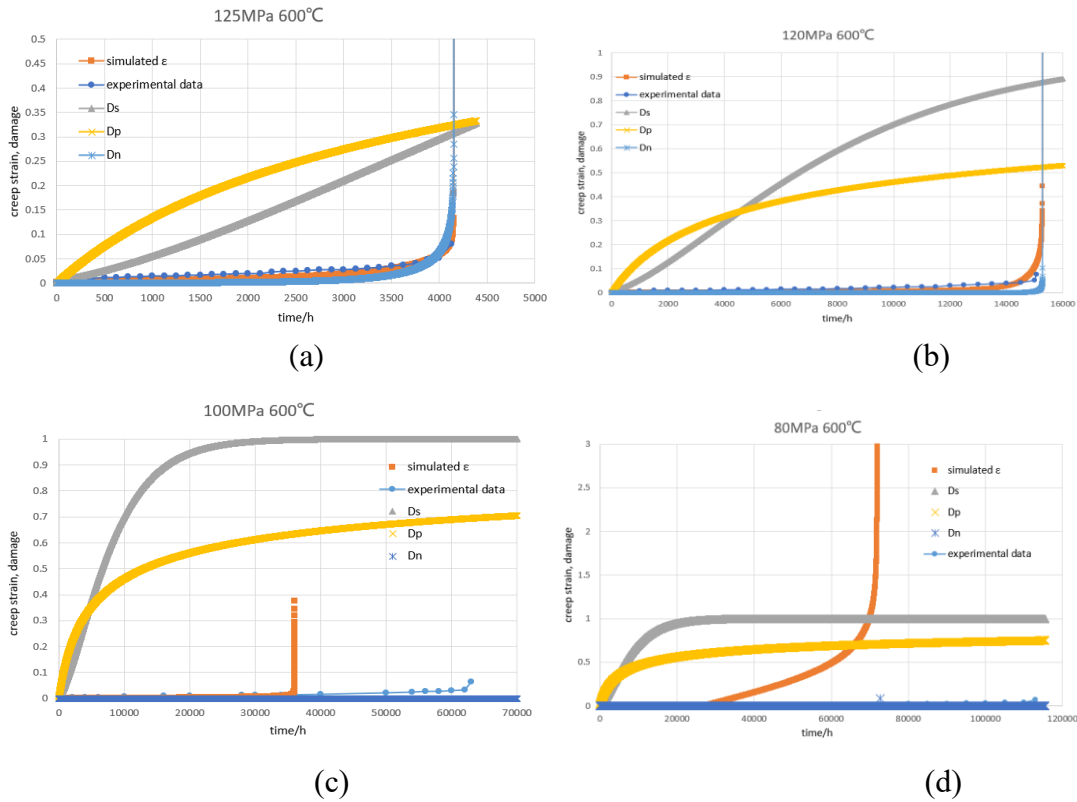


Fig. 6: Comparison between the modeled creep life and experimental data

(a): $A=15$ and $n=18$, (b): $A=0.5$ and $n=80$,

(c): $A=1 \cdot 10^{-16}$ and $n=1000$, (d): $A=1 \cdot 10^{-30}$ and $n=10000$

Comparing the curves drawn with 125MPa, 120MPa in 600°C. It can be observed that by adjusting the value of A and n to fit the modelling results with experimental data, during a significant increase in creep strain. The major damage parameter is

D_n while D_s and D_p is still increasing.

Under 100MPa and 80MPa, 600°C. The value of D_n is small until the failure of material occurs. The D_s and D_p are reaching saturation before the failure of material. It appears that D_s and D_p is dominating the creep strain in modelling result, but by research, the cavity damage D_n should has a jumping when the material failure (Yang et al., 2013b).

3.3. Adjusting the value of A , n , K_s , K_p

According to Basirat et al. (2012) and Christopher (2019). The material constants K_s and K_p should be in a certain range. The value of K_s and K_p calculated by Chen are not fitting well in low-stress range. In this paper will attempt to adjust the value of K_s and K_p to observe the modelling result after lessening the D_s and D_p .

By adjusting the value of K_s and K_p . The modelling result is reaching a better agreement with experimental data, as shown in figure 28 and 29. Therefore, the value of K_s and K_p have a considerable effect to simulate result. Further investigation is required.

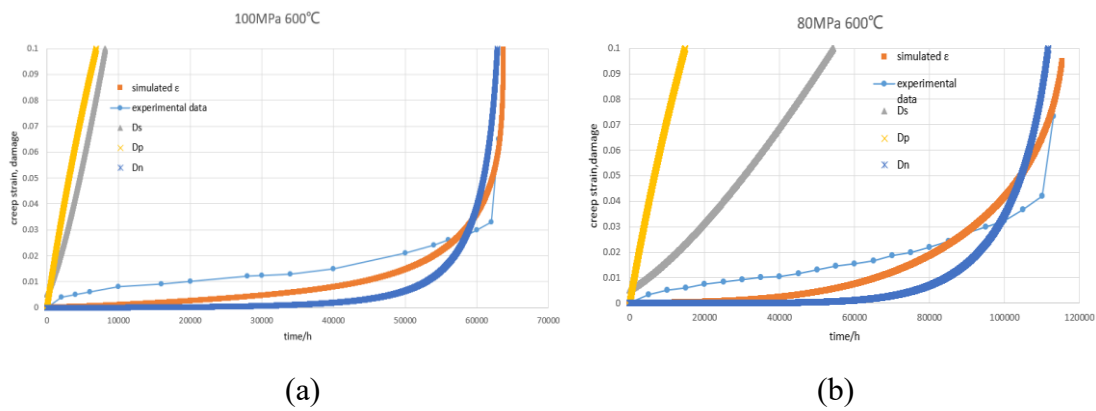


Fig. 7: Comparison between the modeled creep life and experimental data

(a): $A=17$, $n=20$, $K_s = 1 \cdot 10^{-8}$, $K_p = 1.5 \cdot 10^{-8}$,

(b): $A=20$, $n=19$, $K_s = 1.5 \cdot 10^{-9}$, $K_p = 0.7 \cdot 10^{-8}$

3.4. Continue to optimize the model

According to the simulated curves in Fig 7, the creep life has been consistently agreed with experimental data. However, the simulation results are always lower than the experimental data at the primary creep stage and steady state creep stage. Due to a defect in the power function, it is unable to describe more exactly the creep damage in the early stages at long-term low stress level.

To improve this situation, changing the Chen's equation to a hyperbolic sine law with the damage variable held constant. The modified equation shows blow, simulated result shows as Fig 8. A relatively good match was achieved between the simulation results and the experimental data. Meanwhile, as the stress decreases from 100 MPa to 80 MPa, the values of K_s and K_p show a decreasing trend.

$$\dot{\epsilon} = \dot{\epsilon}_0 \frac{1}{(1-D_s)} \times \sinh \left[\frac{\sigma(1-H)}{\sigma_0(1-D_p)(1-D_n)} \right] \quad (8.1)$$

$$\dot{H} = \frac{h'}{\sigma} \left(1 - \frac{H}{H^*}\right) \dot{\epsilon} \quad (8.2)$$

$$\dot{D}_s = K_s D_s^{1/3} (1-D_s) \quad (8.3)$$

$$\dot{D}_p = \frac{K_p}{3} (1 - D_p)^4 \quad (8.4)$$

$$\dot{D}_n = A' \dot{\epsilon} \epsilon^{B'} \quad (8.5)$$

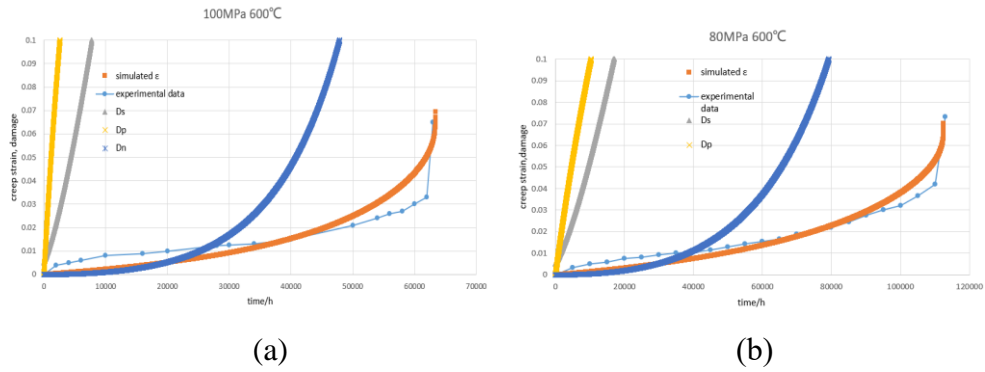


Fig. 8: The simulated result by hyperbolic sine law

(a): $A=200$, $K_s=1.05 \cdot 10^{-8}$, $K_p=4 \cdot 10^{-8}$

(b): $A=200$, $K_s=4.8 \cdot 10^{-9}$, $K_p=1 \cdot 10^{-8}$

Discussion

At present, most of the common creep damage constitutive equations are only applicable to a specific stress range. When the stress level decrease from high stress to low stress, the damage mechanism and creep deformation mechanism may change due to the different stress level. Therefore, the parameters of the creep constitutive equation may also change.

Based on the data used in this study, no obvious rule was found for the value of A and n , which means it is not robust. The adjustment of A value alone has no engineering significance at the long-term of thousands to tens of thousands of hours of creep behavior, the stress exponent n needs to be adjusted simultaneously to make the simulation results have good agreement with the experimental data.

Through a simulation, it is found that in a certain range of low stress level, a simple adjustment of coefficients A and n can still make the simulation results to have a good agreement with experimental data. However, when the creep life reaches tens of thousands or even hundreds of thousands of hours, the adjustment of coefficients A and n has a small effect.

Therefore, this paper proposes a hypothesis from the micro-level, which may be because the creep damage mechanism under low stress is not well understood at present, and the values of K_s and K_p of damaged D_s and D_p are not accurate enough. In the long-term creep behavior, D_s and D_p , which are not dominant in the short-term creep behavior, might have the chance to reach saturation. It is resulting in the predicted life being smaller than the experimental data. If the values of K_s and K_p are accurate, the simulation results may have a good agreement with experimental data.

By adjusting the constants K_s and K_p , the simulation creep life was successfully in agreement with the experimental data. Therefore, this hypothesis is applicable in the long-term low-stress creep behavior. It was shown that phenomenology theory has limitations when applied to the prediction of low-stress and long-term creep. An accurate description of materials at the micro level may be a more effective method to predict the creep behavior of low-stress and long-term. The value of the constant K_s , K_p decreases as the stress is reduced. K_s , K_p could be a function of stress loading.

Hyperbolic sine-law function showed better reliability, but still did not provide a complete description of primary creep stage in this simulation, which might be due to the inaccuracy of some damage variables.

The future work will see the implementation of the improved method to the large stress range and of different creep constitutive equation to verified reliability.

Conclusion

1. Reported in this paper is a proposed method to improve the accuracy of predicted at low-stress and long-term creep behavior for P91 steel at 600°C, the following results were obtained:
 - 1.1. Adjusting the value of A has no obvious advantage on increase the applicability of creep model at low stress level.
 - 1.2. Adjusting the value of A and n can make the simulated result has good agreement with experiment data in some case, but it is still not work at tens of thousands of hours creep behavior.
 - 1.3. Adjusting the value of A , n , K_s , K_p has significant advantage on increase the applicability of creep model at low stress level.
 - 1.4. By the results, adjust damage rate constant is an effective method to improve the applicability of constitutive equation.
 - 1.5. There is a trend in the damage constants K_s and K_p with the stress levels.

The results show that this method is applicable. This improves method links mechanical analysis with materials science.

2. The above results showed that the constitutive equation based on phenomenology theory has its limitations in describing the creep behavior of low stress in a long-term. It is necessary to describe the creep mechanisms more accurately from the perspective of microstructures. Each damage constant has a significant influence on the creep life. It is necessary to calibrate them more accurately in the low stress level, and to establish the relationship between the constants of different variables and temperature or stress levels. For example, clarify the relationship between the constant K_s , K_p and stress level.
3. The current power law minimum creep strain rate and stress relationship is not suitable to be extended to lower stress level. Compared to the power-law function, the hyperbolic sine-law function provides a more accurate description of damage in the primary and steady phase of creep. The modified sine law should be considered in future work.

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