A Robust Areal Residual-restrained Variational Mode Decomposition for Filtering on Surface Texture Analysis

Zhuowei Li¹, Yuanping Xu¹,¹*, Chaolong Zhang¹, Chao Kong¹,², Iain Macleod¹,²,³, Tukun Li², Xiangqian Jiang², Benjun Guo¹

¹School of Software Engineering, Chengdu University of Information Technology, Chengdu, 610225, China
²School of Computing and Engineering, University of Huddersfield, Huddersfield, HD1 3DH, UK
³IMA Ltd, 29 Clay Lane, Cheshire, WA15 8PJ, UK

E-mail: E-mail address: ypxu@cuit.edu.cn

Abstract

This study proposes a novel filter, namely areal RrVMD based on Variational Mode Decomposition (VMD), for decomposing surface areal texture into the form, waviness and roughness. VMD is one of the latest signal decomposition techniques and has been introduced into the field of surface metrology recently. The paper develops a residual-restrained method to further improved the VMD algorithm. It consists of three processing steps: firstly, calculating the robust weight function; secondly, decomposing the surface into the corresponding k modes and a residual by using the devised areal residual-restrained VMD; thirdly, identifying different surface topography features by different wavelengths of modes. This study also proposes a robust algorithm to handle outliers and defects on the measured surface. The experimental results demonstrate that the robust areal residual-restrained VMD can precisely separate form, waviness and roughness and eliminate outliers efficiently.

Keywords: surface filter, surface topography evaluation, spline filter, RrVMD, BVME

1. Introduction

Surface texture analysis plays an essential role in monitoring the manufacturing process of a workpiece and predicting its function performance [1,2]. The functional surfaces of workpieces usually contain complex surface texture information, which can be decomposed into form, waviness and roughness according to the wavelength of the component. They are closely related to material adhesion, sealing, friction, and other functional properties [3–5]. Filtration is essential to obtain the functional correlation components on surfaces. Thus, accurate separation of surface texture by filters is of vital importance to surface texture analysis.

Many filters have been proposed and developed [4], such as the 2RC filter, Gaussian filter, R9 filter, Spline filter, Robust Spline filter, Gaussian regression filter, Wavelet filter, and Morphological filter [6–9]. Some of those filters have been standardised in ISO 16610 [10,11]. In these filters, the 2RC filter is the most primitive one, and the Gaussian filter is the mainstream. However, the Gaussian filter has several shortcomings for specific manufacturing applications, e.g., the classical Gaussian filter has a boundary mutation problem and is susceptible to outliers and defects [12]. The effect of outliers and defects can be suppressed by robust estimation algorithms that assign a corresponding weight to each point on an input surface, e.g., Brinkmann et al. [13] proposed a Gaussian regression filter to reduce the impact of end effects by modifying the Gaussian weight function. Gurau et al. [14] applied the M-estimation to extend the Gaussian filter into a robust one to deal with engineering surfaces. In addition, investigations on advanced filters for better separating surface topographies and reflecting surface textures become a research hotspot, e.g., the spline filter is a more practical
approach than the Gaussian filter for processing end effects, and it is filtered by constructing the variation of surface signals, which is similar to the Wiener filter but differs in implementation details, i.e., it can restrain the boundary effect effectively by introducing different boundary conditions. Tong et al. [15] provides the in-depth study on boundary conditions for the potential of a potential.

The corresponding robust spline filter is specified in ISO16610-22 [11]. When the profile spline filter is extended to the areal one, both isotropy and boundary conditions need to be considered. Thus, the areal spline filter is hotly investigated in recent years. Tong et al. [8,9] proposed the high-order areal spline filter by adding high-order derivative terms to overcome the anisotropic feature. However, there are spectrum overlaps between surfaces due to the 50% transmission characteristic [16].

In addition, the wavelet filter also possesses a benign capacity for surface topography restoration by decomposing a functional surface into multiresolution coefficients [17,18]. Gogolewski et al. [19] researched the influence of wavelet parameters on the filtering effect and found that the different mother wavelets had different impacts on edge deformations. Thus, wavelet filters cause weak generalisation. Other multiresolution algorithms have been introduced into the field of surface metrology. In 1998, Huang et al. [20] proposed the Empirical Mode Decomposition (EMD) method that was introduced into the area of surface filtering in 2010 [21], which was further improved by Du et al. [22] in 2018. EMD is a data-driven algorithm instead of using a basis of wavelets or Fourier transforms. It performs signal decomposition by calculating the upper and lower envelope of a signal, which is especially suitable for non-stationary signal analysis. However, the limitation of EMD is boundary effect and mode mixing. Empirical Wavelet Transform (EWT) algorithm [23] can reduce the mode mixing by dividing the frequency domain into contiguous intervals, and then appropriate orthogonal filter banks can be constructed. A surface filter was developed by Shao et al. [24] in 2021. Variational Mode Decomposition (VMD) is one of the latest signal decomposition techniques which was first proposed by Dragomiretskiy et al. in 2014 [25]. Recent research works have shown that the variational mode decomposition (VMD) has good capacity compared to the weak generalisation of wavelet filters and avoids the mode aliasing in EMD. Previous works [26] show that VMD is superior to EWT and EMD in multiresolution signal analysis. VMD is similar to the spline filter. Both are variational methods, but spline filtering is a low-pass filtering method, while VMD is a multiresolution analysis one. The 2D-VMD process is analogous to its 1D predecessor [27]. Therefore, 2D-VMD can be applied in the field of areal surface filter through a particular improvement; however, several modes and parameters need to be manually determined, which decreases the performance of 2D-VMD. In 2022, Li et al. [28] introduced the VMD into surface texture analysis, and two VMD factors were obtained by using optimal combination algorithms. However, these optimisation methods are time-consuming and lack orthogonality strictly of the mode itself.

To tackle these shortcomings, this study explores the Residual-restrained VMD (RrVMD) to decompose the areal surface texture. Compared with the original VMD method, RrVMD is developed to improve the orthogonality between surface modes, and it can also avoid the preset number of modes. It is inspired by the idea of VME (variational mode extraction) [29,30]. Furthermore, the optimized RrVMD has concrete improvements than the traditional filters on reducing fitting errors, mode mixings and spectrum overlaps.

The rest of this paper is organised as follows: Section 2 presents the general formulas of the proposed algorithm that are derived through several objects to be optimised; the adaptive optimisation of parameters and the robust algorithm are presented in Section 3; Section 4 describes in detail of proposed approach; the proposed algorithm is compared with several classical algorithms to verify the advantages in Section 5, and two real cases are performed by the proposed surface areal texture analysis algorithm; finally, Section 6 summarises this work and ends with a discussion of a potential research direction.

2. Approaches

2.1 Brief of Variational Mode Decomposition

VMD aims to minimise the estimated bandwidth of each mode at the central frequency location, and the optimisation is based on the Lagrange method, which is formulated in the frequency domain as the following:

\[ L = \sum | \alpha | \left[ \sum (\nu - \nu_i)^2 \right] F_i(w) + \frac{\tau}{2} \sum (\sum (|w| - S(w))^2 + \sigma(w)) \sum (|w| - S(w))dw \]  

(1)

where \( \alpha \) is the Lagrange factor, \( w \) is frequency, \( F_i \) is the mode, \( S \) is the original signal, \( \tau \) is the penalty coefficient. Among these parameters, the number of mode \( k \), the \( \alpha \) and \( \sigma \) are the essential items, and \( \alpha \) is related to the bandwidth of the mode, i.e., the larger \( \alpha \) refers to the bandwidth being more compact, and visa versa. Therefore, to decompose the components on surface texture via VMD, the values of two parameters must be appropriately set. Moreover, it is essential to separate the abnormal values, outliers and defects on the premise of minimal loss and identify which mode includes them. Therefore, choosing the appropriate number of modes and the parameter \( \alpha \) based on the inherent characteristics of a signal is the key to effectively applying the VMD algorithm to surface topography separation.

2.2 Proposed approach

Although VMD offers a promising direction in the field of multiresolution decomposition, it has some challenges, e.g.,
the two most important parameters are difficult to determine in advance, and the spectrum of each mode is much more overlapping. Moreover, the presence of outliers will carry this effect into every mode, so it is difficult to merge a good benchmark. Take a 1D signal in Fig.1 as an example, and Fig.1b shows the expected results. To this end, a method conducted based on the following steps: Fig.1a and Fig.1b are frequency domain images of Gaussian noise, which are decomposed by the VMD and the Residual-restrain VMD, respectively. The overlap of modes in Fig.1b is less because of the least overlap between them; such narrowband signals can better reflect the optimal subcomponents of the surface.

![Fig.1. The results of VMD and spline filters.](image)

![Fig.2. frequency characteristics of areal VMD filter.](image)

The unoptimized modes and components can be regarded as residuals to reduce overlaps between modes. It also includes irrelevant outliers. Unlike the classical VMD, the object is transformed into k times optimisation to continuously extract effective modes from residuals. Therefore, to analyse surface areal texture, a surface $S_1(x, y)$ can be regarded as the sum of a current mode $f_i(x, y)$, a residual component $r(x, y)$ and other modes, as shown in the following:

$$S_1(x, y) = f_i(x, y) + r(x, y) + \sum f_j(x, y) \quad (2)$$

The mode updating of 2-D VMD[27] in the frequency domain is shown in the following:

$$H^{n+1}_k(w_u, w_v) (\hat{S}_k(w_u, w_v) - \sum_{i=1}^k f_i^n(w_u, w_v)) + \frac{\lambda}{2} \left[ (w_u - w_{u,k})^2 + (w_v - w_{v,k})^2 \right]$$

where the $H^{n+1}_k$ represents the 2D Hilbert mask, $\hat{S}$ is the input signal, $\hat{f}_i$ denotes $i$th mode, $\hat{\lambda}$ is the dual variable, $\alpha$ is the regularisation parameter. The amplitude-frequency characteristic of filtering in 2D-VMD is demonstrated in Fig.2. This study aims to make the modes more compact and separate residuals, so it is advisable to minimise the correlation between the current frequency response of mode, residuals and other modes [30]. Based on this analysis, the frequency response can be expressed as the following[31]:

$$\hat{\lambda}(w_u, w_v) = \frac{1}{\alpha (w_u - w_{uk})^2 + (w_v - w_{vk})^2}$$

where $w_{uk}$ and $w_{vk}$ are the frequencies in the $u$ and $v$ directions; $\alpha$ is the penalty factor that affects the bandwidth of the filter; $w_{uk}$ and $w_{vk}$ are the centre frequencies in the $u$ and $v$ directions. Because the residuals and other modes should be infinitely amplified at the location of centre frequency, and to make the residuals and different modes more orthogonal with the current mode when minimising the correlation between them, the proposed objective optimisation and constraint of areal Residual-restrain VMD (areal ReVMD) can be constructed as Eq.5 and Eq. 6 in the time domain[30]:

$$\min \left[ \sum_{i=k} \left[ \rho^2_k(x, y) + \sum f_{i,k}(x, y) \right] \right]$$

$$s.t. S_1(x, y) = f_i(x, y) + r(x, y) + \sum f_j(x, y) \quad \ldots (5)$$

where $t_1$ is to minimise the correlation between the current mode, residual component and other modes; $t_2$ is to increase the orthogonality between the current mode and other modes; the unfolding view of $S$ is in the following[31]:

$$\nabla f_{i,k}(x, y) = \frac{\partial f_{i,k}}{\partial x} + \frac{\partial f_{i,k}}{\partial y}$$

It is designed to simulate the filtering characteristics of 1D VMD, and it is isotropic in the condition of 2D. In order to address the aforementioned optimal problems, a regularisation
parameter $\alpha$ and the augmented Lagrange method are introduced [27], so the reconstruction constraint problem in equations 5 and 6 can be translated into the following:

\[
\begin{align*}
\min & \left\{ \alpha \sum_{i=1}^{n} \left[ \left| \mathcal{F}_{\Delta}(x, y) \right| e^{\frac{-j\pi r_i u x - j\pi r_i v y}{2}} \right|^2 + \rho_i (x, y) \right\} \right. \\
& + \frac{\tau}{2} \sum_{i=1}^{n} \left( S_i (x, y) - \sum_{k=1}^{p} f_{\Delta,k}(x, y) - r_{\Delta,k}(x, y) \right)^2 \\
& \left. + \left\{ \lambda (x, y), S_i (x, y) - \sum_{k=1}^{p} f_{\Delta,k}(x, y) - r_{\Delta,k}(x, y) \right\} \right. \\
& \left. \left( \lambda (x, y), S_i (x, y) - \sum_{k=1}^{p} f_{\Delta,k}(x, y) - r_{\Delta,k}(x, y) \right) \right\} \right. \\
\end{align*}
\]

where $\tau$ and $\lambda$ are step size and dual variable, respectively. For simplification, the term $\lambda$ can be merged into the quadratic penalty term, then based on the Parseval theorem [27], Eq.8 can convert to the following:

\[
\begin{align*}
\min & \left\{ \alpha \sum_{i=1}^{n} \left[ \left| \mathcal{F}_{\Delta}(w_i, w_i) \right| \right]^2 + \\
& + \frac{\rho_i (w_i, w_i)}{\rho_i (w_i, w_i)} \sum_{i=1}^{n} f_{\Delta,k}(w_i, w_i) \right\} + \\
& + \frac{\tau}{2} \sum_{i=1}^{n} \left( S_i (w_i, w_i) - \sum_{k=1}^{p} f_{\Delta,k}(w_i, w_i) - r_{\Delta,k}(w_i, w_i) \right)^2 \\
& + \frac{\tau}{2} \left( \lambda (w_i, w_i), S_i (w_i, w_i) - \sum_{k=1}^{p} f_{\Delta,k}(w_i, w_i) - r_{\Delta,k}(w_i, w_i) \right) \right. \\
& \left. \left( \lambda (w_i, w_i), S_i (w_i, w_i) - \sum_{k=1}^{p} f_{\Delta,k}(w_i, w_i) - r_{\Delta,k}(w_i, w_i) \right) \right. \\
\end{align*}
\]

where $f_{\Delta,k}(x, y)$ and $r_{\Delta,k}(x, y)$ are 2D analytic signals which are extended from the 1D functions, and the unfold expressions are shown in Eq.10 [27]; $\Omega$ is represented as Eq.11 [31], which mergers two partial derivatives transformed to the spectrum domain.

\[
\begin{align*}
\hat{f}_{\Delta,k}(w_i, w_i) &= [1 + \text{sgn}(ww_i + wv_i)] \hat{f}_k (w_i, w_i), \\
\hat{r}_{\Delta,k}(w_i, w_i) &= [1 + \text{sgn}(ww_i + wv_i)] \hat{r}_k (w_i, w_i) \\
\Omega &= \sqrt{(w_i - w_{\Delta})^2 + (w_i - w_{\Delta})^2} \\
\end{align*}
\]

Then, according to the Alternating Directions Method of Multipliers (ADMM), the entire process only needs to operate in half-plane, so the mode $\hat{f}_k$ can be obtained by optimising the following formula [27]:

\[
\hat{f}_{k}(w_i, w_i) = \min_{\{w_i, w_i\}} \left[ \frac{\lambda (w_i, w_i)}{2} \right] \right. \\
\left. + \frac{1}{2} \alpha [\hat{f}_{k}(w_i, w_i) - \hat{f}_k (w_i, w_i) - \frac{\hat{r}_k (w_i, w_i)}{\lambda (w_i, w_i)}]^2 \\
\end{align*}
\]

Taking the derivative to $\hat{f}_k$ using Eq.12 can get the extreme point, thus $\hat{f}_k$ can be obtained by the following:

\[
\hat{f}_{k}(w_i, w_i) = \frac{\hat{f}_k (w_i, w_i) - \sum_{i=1}^{n} \hat{f}_k (w_i, w_i) - \hat{r}_k (w_i, w_i) + \frac{\hat{r}_k (w_i, w_i)}{2}}{1 + \alpha [\hat{f}_k (w_i, w_i) - \hat{r}_k (w_i, w_i)]^2} \\
\end{align*}
\]

where the acquirement of residual $\hat{r}$ is the same as above; thus, the most optimal result is the following:

\[
\hat{r}_k (w_i, w_i) = \frac{\sum_{i=1}^{n} \hat{f}_k (w_i, w_i) - \sum_{i=1}^{n} \hat{f}_k (w_i, w_i) + \frac{\hat{r}_k (w_i, w_i)}{2}}{1 + \alpha [\hat{f}_k (w_i, w_i) - \hat{r}_k (w_i, w_i)]^2} \\
\end{align*}
\]

In practice, it is not necessary to calculate the residual separately, hence taking Eq.14 to Eq.13 can obtain Eq.15 that achieves the mode optimization. In contrast to the mode equation of VME [29], it can be seen that the new mode is updated independently of other methods. The equation will degrade into the mode equation of VME when $\alpha$ is large.

\[
\hat{f}_k^{(n)}(w_i, w_i) = \frac{\sum_{i=1}^{n} \hat{f}_k^{(n)}(w_i, w_i) - \sum_{i=1}^{n} \hat{f}_k^{(n)}(w_i, w_i) + \frac{\hat{r}_k^{(n)}(w_i, w_i)}{2}}{1 + \alpha [\hat{f}_k^{(n)}(w_i, w_i) - \hat{r}_k^{(n)}(w_i, w_i)]^2} \\
\end{align*}
\]

The centre frequency is also an important parameter which needs to be updated, shown as the following [30]:

\[
\hat{f}_k^{(n)}(w_i, w_i) = \frac{\sum_{i=1}^{n} \hat{f}_k^{(n)}(w_i, w_i) - \sum_{i=1}^{n} \hat{f}_k^{(n)}(w_i, w_i) + \frac{\hat{r}_k^{(n)}(w_i, w_i)}{2}}{1 + \alpha [\hat{f}_k^{(n)}(w_i, w_i) - \hat{r}_k^{(n)}(w_i, w_i)]^2} \\
\end{align*}
\]

where $p$ represents the two orientations of the spectrum domain. When $\alpha$ in the numerator is a significant value, Eq.16 can be converted into Eq.17 [29], which is the same as the 2D-VMD:

\[
\hat{w}_k^{(n)} = \frac{\int_{w_i} w_i \hat{f}_k^{(n)}(w_i, w_i) \, dw_i \, dw_i - \frac{1}{2} \int_{w_i} w_i \hat{f}_k^{(n)}(w_i, w_i) \, dw_i \, dw_i}{\int_{w_i} \hat{f}_k^{(n)}(w_i, w_i) \, dw_i \, dw_i}, \forall p \in \{u,v\} \\
\end{align*}
\]

Finally, the last parameter is updated by the following dual ascending method:
\[ \hat{\lambda}^* (w_j, w_k) = \hat{\lambda}^* (w_j, w_k) + \tau \left[ S(w_j, w_k) - \sum_{i=1}^{j} f_i (w_j, w_k) - \hat{f}(w_j, w_k) \right] \]  

where \( \tau \) is a tolerance of noise, and it is set to zero in general.

3. The parameters optimisation

3.1 The spline filter and VMD

The spline filter is a classical surface profile filter listed in ISO 16610-22[11]. In the discrete case, Eq.19 [11] and Eq.20 [25] are the optimised objects of the spline filter and VMD, respectively. The VMD and spline filter are both variational methods and have been widely applied to the vibration signal domain.

\[
\min \left\{ \left[ s(t) - f(t) \right]^2 + \lambda \left[ \frac{d^2 f(t)}{dt^2} \right]^2 \right\} 
\]  

(19)

\[
\min \left\{ \left[ s(t) - \sum_{i=1}^{j} f_i (t) \right]^2 + \alpha \sum_{i=1}^{j} \left[ \left[ \delta(t) + \frac{1}{\pi t} \right] * u_i (t) \right] e^{-\alpha t^2} \right\} 
\]  

(20)

Through the Parseval theorem and Fourier transform of the derivative theorem, Eq.19 and 20 can be transposed to the frequency domain, as shown in follows. Eq.19 corresponds to Eq.21, while Eq.22 is transposed from Eq.20:

\[
\min \left\{ \left[ S(w) - F(w) \right]^2 + \lambda \left[ w^2 F(w) \right]^2 \right\} 
\]  

(21)

\[
\min \left\{ \left[ S(w) - \sum_{i=1}^{j} F_i (w) \right]^2 + \alpha \left[ (w - w_i) F(w) \right]^2 \right\} 
\]  

(22)

where \( F(w) \) is the Fourier transform of \( f(t); f_i(w) \) and \( S(w) \) are as same as \( F(w) \). By taking the part of \( F(w) \) in both Eq.21 and Eq.22, the final results are obtained, as shown as follows:

\[
F(w) = \frac{1}{1 + \lambda w^2} S(w) 
\]  

(23)

\[
F_i (w) = \frac{1}{1 + \alpha (w - w_i)^2} (S(w) - \sum_{i=1}^{j} F_i (w)) 
\]  

(24)

In the VMD, when there is only one mode, and the central frequency is zero, Eq.24 degrades into:

\[
F(w) = \frac{1}{1 + \alpha w^2} S(w) 
\]  

(25)

It can be seen that the one difference between Eq.23 and 25 is the power of \( w \), and they are both low-pass filters. As reported by Janecki et al. [31], the VMD filter is a spline of a lower order. They all belong to the Butterworth filter of a different order, and the Lagrange coefficient can determine the cut-off frequency. The figure of their amplitude-frequency is as shown in Fig.3; thus, it is defective when \( \alpha \) is a constant in the original VMD.

3.2 The setting of \( \alpha \) in the presented approach

Although the spline filter and VMD have some similarities, VMD is a multiresolution decomposition while the spline filter is a low-pass filter; thus, the \( \alpha \) setting method of the spline filter cannot be directly applied to the proposed algorithm. However, when low-frequency surface signals need to be extracted, the parameters \( \alpha \) should be as large as possible to extract a relatively smooth form. Thus, based on [32], the parameter can be calculated by the following:

\[
\sqrt{\alpha_i} = \frac{f_s}{2} \left( \frac{1}{\kappa_{10^5} (\alpha w)^{n_u} + \alpha w} - 0.5 \right) 
\]  

(26)

where \( u \) and \( v \) represent the two orientations of the spectrum, respectively; \( f_s \) is the sampling frequency, and \( w \) is the angular frequency that is equal to the real frequency multiplied by \( 2\pi \).

3.3 The optimisation of \( k \) and centre frequency

3.3.1 Spectrum analysis

In RrVMD, the initial centre frequency can be calculated directly by spectrum analysis rather than the random initial centre frequency selection in VMD, such that the convergence of algorithm can be accelerated. Spectrum analysis in this study is to select the location of the maximum value from the spectrum. The initial centre frequency influences the result of mode decomposition since it has strong impact on the algorithm convergence. The modes are the components which have higher energy on the surface[33], and the determination of the centre frequency depends on the property of modes. For the initial centre frequency of the \( k^{th} \) mode, the location of the maximum value from the spectrum of the \( k^{th} \) residual will be selected.

3.3.2 Parameter \( k \) avoidance

The VMD relies on the parameter \( k \) that determines the number of modes since selecting a sound number of modes is
fatal for the mode decomposition result. However, the number of modes in 2D VMD is selected in a subjectivity way, which leads to unrefined decompositions if the number is too small, and further obtain unrealistic surface topographies. To tackle the problems, the number of mode k in RrVMD can be obtained through k iterations, i.e., the modes of RrVMD can be obtained iteratively with several mode extraction and input replacement iterations. Theoretically, the modes of the proposed areal RrVMD do not have the problems of over- and under-decompositions, but efficiency does not allow for infinite decomposition. The final residual is relatively small in the overall surface, and some modes are included in the residual when they have not been iterated, so the iteration termination condition of mode number can be set to that the residual no longer changes significantly.

The processing steps of the areal RrVMD are presented as the following:

(1) **Initialisation.** A half-plane of a 2D spectrum $S_i$ is selected for maximum detection. The location of maximum is the initial centre frequency $(\omega_{id}, \nu_{id})$ of the first mode. Calculating the parameter $\alpha$ by Eq.26 and obtaining the optimal first mode $f_i$ by substituting the parameter $\alpha$, centre frequency into Eq.15.

(2) **Repeat.** Updating $S_{i+1}$ through

$$S_{i+1} = f_{i+1}(w_u, w_v)$$

and calculating the centre frequency and $\alpha_{i+1}$ to get the new mode $f_{i+1}$. Stop the above course until it is satisfied the following condition:

$$\left\| r^{i+1} - r^i \right\|_2 / \left\| r^i \right\|_2 < \rho$$

(3) **Output.** The final result is k optimal modes and a residual surface.

The above process effectively avoids determining the number of modes in advance when lacking prior knowledge, and the main modes making up the functional surface can be obtained rapidly.

4. Robust algorithm of RrVMD

There are outliers or defects in an actual measured engineering surface, which will affect the evaluation result. In general, the value of the outlier is more significant than the normal surface value, so areal RrVMD can correctly separate outliers into final residuals. However, some exceptional situations may occur in practice, e.g., if the outlier is abnormally large, it will significantly impact the results. In robust estimation of RrVMD, M-estimation is introduced to further eliminate the influence of these outliers on the decomposition results. However, the approach in this study is carried out in the spectrum domain, while the weight function of M-estimation requires assigning different weights to each point in the time domain. To merge the robust estimation into the mode optimization (see Eq.15), the weight function is introduced into the 2D Fourier transform, as the following:

$$j_i\hat{w}_{ik}(w_u, w_v) = \hat{\beta}_{ik}(w_u, w_v) \frac{1}{2 \pi} \int_{-\alpha}^{\alpha} \int_{-\beta}^{\beta} \left( |r_{ik}(w_u, w_v)|^2 + (r_{ik}(w_u, w_v) + \hat{\lambda}_{ik}(w_u, w_v))^2 \right) \sin(\pi v) \sin(\pi \nu) \delta(|v| < 1)$$

where $\hat{\beta}_{ik}(w_u, w_v)$ is the frequency response calculated by using Eq.4, $\delta$ is the weight function of Andrews estimation [34], calculated by the following:

$$\delta(|v|, y) = \begin{cases} \frac{1}{\pi v}, & |v| < 1 \\ 0, & |v| \geq 1 \end{cases}$$

where $\beta$ calculated by:

$$\beta = \text{med} \left[ \frac{|S(x,y) - f_{ik}(x,y)|}{\text{std} \left| S(x,y) - f_{ik}(x,y) \right|} \right]$$

where med denotes taking the median, std is the standard deviation, $c=1.5 \pi$ is a flexible parameter. The whole algorithm is summarised as Algorithm 1.

5. Surface topography separation

5.1 The surface topography type of the subcomponents

Their wavelength should determine the relationship between residual, modes and surface topography types. A surface $S(x,y)$ can be written as shown in Eq.32, where $F(x,y)$ is a form, $W(x,y)$ represents waviness, $R(x,y)$ denotes the roughness, and benchmark is equal to the sum of $F(x,y)$ and $W(x,y)$. The cut-off wavelength of each surface topography is shown in Fig.4, where the cut-off wavelength between form and waviness is $\lambda_f$ and the cut-off wavelength between roughness and waviness is $\lambda_c$. Here, the conversion for wavelength and frequency is Eq.33, where $f$ is frequency.

$$S(x,y) = F(x,y) + W(x,y) + R(x,y)$$

$$\lambda = \frac{1}{f}$$

Fig.4. The filter of VMD and spline filter.

5.1.1 The disposal of residual

In general, roughness at high frequencies is weak in the spectrum, so the final residual can be directly regarded as roughness. However, when the energy of surface topography
is fragile in the spectrum, there is no mode belonging to the cut-off wavelength range of this surface topography. To tackle this case, the residual can be further decomposed by an areal wavelet filter, and the detailed technology can be found in [17]. This study only introduces the calculation of frequency about each layer of signal decomposed by wavelet transform.

According to the Nyquist Criteria, assuming the sampling frequency is \( f_s \), the frequency of the low- and high-frequency coefficients of each layer are Eq. 34 and 35, respectively, where \( s \) is the starting frequency, \( e \) is the terminal, \( f_0 \) is the frequency of high-frequency coefficients, \( f_1 \) is the frequency of low-frequency coefficients.

\[
\begin{align*}
    f_s &\in \left\{ \frac{f_{s+1} + f_{s+2}}{2} \right\}_e \\
    f_0 &\in \left\{ \frac{f_{s+1} + f_{s+2}}{2} \right\}_e \\
    \end{align*}
\]

**Algorithm 1**

Input surface: \( S_t(x,y) \)

Repeat

\[
\delta = \max \left\{ \frac{1}{\Delta P} \sin (\pi P) \right\} \left| v_p \right| < 1
\]

Repeat

\[
\delta = \max \left\{ \frac{1}{\Delta P} \sin (\pi P) \right\} \left| v_p \right| \geq 1
\]

Create Hilbert mask:

\[
H_{s,k}^n(u,v,w) = \max \{ w_{s,k}^n + w_{s,k}^n \}
\]

Repeat

\[
\alpha = 1 - \left( \frac{1 - \Delta P}{1 + \Delta P} \right) \left( \frac{1 - \Delta P}{1 + \Delta P} \right)
\]

Dual Ascent:

\[
\hat{f}^{n+1}(u,v,w) = \hat{f}^{n+1}(u,v,w) + \alpha \left( \frac{\| \hat{f}^{n+1}(u,v,w) - \hat{f}^{n+1}(u,v,w) \|^2}{\| \hat{f}^{n+1}(u,v,w) \|^2} \right)
\]

Until convergence

\[
\| \delta^{n+1} - \delta^n \|^2 < \eta
\]

Repeat

\[
k = k + 1
\]

Repeat

\[
S_{ks}(w_{s,k}w_{k+1}) = \rho \times \alpha \times (w_{s,k}w_{k+1})
\]

Update \( f_{k+1} \)

Until convergence

\[
\| f_{k+1} - f_{k+1} \|^2 < \rho
\]

Where \( k \) is the number of iteration.

5.1.2 The frequency of modes
To obtain the exact expression of modes, the Riesz transform is used to replace the Hilbert transform when getting the instantaneous frequency because of Hilbert transform is not isotropic in 2D conditions[35]. The processing steps for the specific instantaneous frequencies are as follows:

1) The Riesz transform in the frequency domain is shown in the following:

\[ F_1(\Omega), F_2(\Omega) = \begin{bmatrix} \tilde{w}_w \tilde{F}(\Omega) \tilde{w}_w \tilde{F}(\Omega) \end{bmatrix}, \]

\[ \Omega = \left[ \tilde{w}_w \tilde{w}_w \right] \]

It can be obtained the monogenic signal \( F_m \) in the time domain through the 2D inverse Fourier transform:

\[ F_m(x, y) = \left\{ F(x, y), iF_1(x, y), jF_2(x, y) \right\} \]

2) Then the phase calculation formula can be obtained as Eq.38, where \( k \) is equal to 1 and 2.

\[ \varphi(x, y) = \arctan \left( \frac{F_1(x, y)}{F(x, y)} \right) \]

3) Finally, the instantaneous frequencies are acquired by Eq.39. It is the square root of phase derivatives in different orientations.

\[ f_i = \sqrt{\left( \frac{\partial \varphi}{\partial x} \right)^2 + \left( \frac{\partial \varphi}{\partial y} \right)^2} \]

The above steps merge subcomponents in the same cut-off wavelength range to obtain topographies. Moreover, some recommended cut-off wavelengths are summarized in Table 1 according to ISO 21920 [36].

<table>
<thead>
<tr>
<th>Cut-off wavelength (mm)</th>
<th>(mm)</th>
<th>(mm)</th>
<th>(mm)</th>
<th>(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_t )</td>
<td>0.8</td>
<td>2.5</td>
<td>8</td>
<td>25</td>
</tr>
<tr>
<td>( \lambda_c )</td>
<td>0.08</td>
<td>0.25</td>
<td>0.8</td>
<td>2.5</td>
</tr>
<tr>
<td>( \lambda_s / \lambda_c )</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

5.2 Areal surface parameter calculation

To obtain abundant surface information, the areal surface parameters have been defined in ISO 25178[37]. Assuming the areal surface is \( h(x,y) \), and the area is \( A \), height and hybrid parameters are calculated in Table 2, which are critical for evaluating functional surfaces. In height parameters, \( S_q \) is the root mean square height parameters; \( S_d \) is the quotient of the mean cube value of the ordinate values of \( h(x,y) \) and the cube of \( S_q \); \( S_m \) is the quotient of the quartic value of the ordinate values of \( h(x,y) \) and the fourth power of \( S_q \); \( S_t \) is the sum of \( S_p \) and \( S_q \); \( S_p \) is the maximum peak height; \( S_d \) is the maximum pit height; \( S_u \) is the mean of the absolute of the ordinate values of \( h(x,y) \) [37]. For the hybrid parameters, \( S_o \) is the square root of the mean square of the surface gradient of \( h(x,y) \); \( S_t \) is the ratio of the increment of the interfacial area of \( h(x,y) \). The workflow of areal VMD is illustrated in Fig.5a [28], and the workflow of the proposed robust areal RvVMD is shown in Fig.5b for comparing the differences between original areal VMD and the robust areal RvVMD. The main improvements of RvVMD can be summarized as: 1) it requires fewer parameters than the areal VMD (i.e., don’t need to preset the number of mode \( k \) (Section 3.3); 2) the initial centre frequency can be simply calculated by spectrum analysis rather than the random initial centre frequency selection in VMD, such that the convergency of algorithm can be accelerated (see Section 3.3); 3) the Andrews estimation is merged in the equation (29) to enable better robustness (see Section 4); 4) it has two mode optimisations (see equation (5) and equation (6), so the spectrum overlap between surfaces are suppressed.

6. Design of experimental and simulation methods

6.1 Experimental conditions and simulation methods

This study validates the proposed areal RvVMD by using two different workpieces. The first is a groove workpiece, while the second is a flat metal part. The experimental setup and workpieces are shown in Fig.6. and Fig.7. The validation includes two steps: firstly, the 3D structured light measurement instrument (accuracy is 0.05mm) was employed to collect the point cloud of workpieces; then, two case studies were presented using two different parts, and the algorithm was implemented via Python programming languange.
The principle of this measurement system is to project a sinusoidal stripe onto a workpiece and use the camera to capture the degree of bending modulated by the workpiece. Then the phase can be obtained by demodulating the bending stripe, and it is converted into the height information of the workpiece. Two groups of simulation experiments have been used to analyse areal RrVMD. The simulation experiment uses two different types of surfaces, i.e., one is a surface with outliers, to test whether the proposed algorithm can correctly avoid outlier interference.

To avoid contingencies, all experiments are repeated five times since there is no random parameter using as the input of the proposed areal RrVMD, and it gets the same results five times in the experiments.

6.2 Simulation experiments

6.2.1 The proposed algorithm and classical algorithm

In this section, the advantages and disadvantages of the proposed areal RrVMD algorithm will be analysed by comparing other widely used surface areal filters, including the areal Gaussian filter, areal spline filter, original 2D VMD and wavelet filter. The simulation surface expressions for filtering are as shown in the following:

\[ 0.1(x^2 + y^2) + 0.5\cos(10\pi x + 10\pi y) + 0.5\text{normal}(0, 0.8) \]  

where the terms in Eq.40 correspond to form, waviness, and roughness, respectively. The normal is a noise function, and the sampling area is 1cm×1cm with 400×400 points, and \( \lambda_0=2.5\text{mm} \) and \( \lambda_y=0.25\text{mm} \), for the cut-off wavelength of each surface topography. For qualitatively analysing the performance of surface topography separation, the simulated surface and its corresponding surface components are shown in Fig.8. After eliminating the end effect of the Gaussian filter via boundary extension, the filter results of the areal Gaussian filter are shown in Fig.9. It can be seen that the form is smooth but coarse in waviness and roughness. There is also a little mixing of roughness and waviness. The result of the surface filtered by discrete wavelet transform is depicted in Fig.10. Only roughness is relatively reliable in vision, while the form shows a serious distortion. The wavelet basis is Daubechies wavelet which separates the simulated surface into six layers. The result of the wavelet is not good enough, which also shows that the selection of wavelet basis dramatically influences results.
Like the areal Gaussian filter, the areal spline filter also performs well in form processing, obtaining a better result on form than the Gaussian filter. This kind of low-pass filter has more frequency band mixing in intermediate frequency and high frequency, as shown in Fig. 11. The original areal VMD decomposes the simulated surface into four modes, and the wavelength of each mode is listed in Table 3, and the centre frequency is randomly initialised, $\alpha$ is set to 4780. However, the residuals are ignored. Although the original areal VMD has many residuals in its form, as marked in Fig. 12, it still has a sound reduction ability for waviness and roughness. The corresponding mode decomposition results are shown in Fig. 13. The filter results of the proposed RrVMD method are shown in Fig. 14. Comparing to the original areal VMD, it not only solves the contamination of form by residuals but also improves the separation of waviness and roughness (see Fig. 16). The areal RrVMD gains better results than original areal VMD (see Table 7 and Table 8) due to its spectrum analysis and iteration setup. The wavelength of each mode calculated by Riesz transform is shown in Table 4, and the position of the initial centre frequency in the half-plane spectrum before updating each mode is shown in Fig. 15. In summary, each surface topography filtered by areal RrVMD is more precise than other filters in the qualitative analysis.

To evaluate each surface filter more accurately, the Pearson coefficient (PC)[24] and Root Mean Square Error (RMSE) methods are employed to measure the similarity quantitatively between the filtered surface topography and the simulated surface topography. The formulas are shown as follows:

$$PC(x, y) = \frac{\sum \sum (x_{ij} - \bar{x})(y_{ij} - \bar{y})}{\sqrt{\sum \sum (x_{ij} - \bar{x})^2 \sum \sum (y_{ij} - \bar{y})^2}}$$

$$RMS(x, y) = \sqrt{\frac{\sum \sum (x_{ij} - y_{ij})^2}{mn}}$$

where $x$ and $y$ represent two surfaces, respectively; $\bar{x}$ and $\bar{y}$ are the mean value of two surfaces, respectively. In PC, the higher value refers to the higher correlation, and in RMSE, the lower value corresponds to the lower average difference. The PC and RMSE of each algorithm are listed in Table 7 and Table 8, respectively. PC is also used to verify the degree of spectrum overlap for surface topography in which the cut-off wavelength is adjacent, and the result is listed in Table 9. It is shown that the Gaussian filter and spline filter have relatively spectral overlaps. In contrast, the proposed areal RrVMD has a better effect and can reflect surface texture information more precisely.
Fig. 10. Filtering result by wavelet: (a) form; (b) waviness; (c) roughness.

Fig. 11. Filtering result by areal spline filter: (a) form; (b) waviness; (c) roughness.

Fig. 12. Extracted modes by areal VMD.

Fig. 13. Filtering result by areal VMD (a) form; (b) waviness; (c) roughness.

Fig. 14. Filtering result by areal ReVMD: (a) form; (b) waviness; (c) roughness.
To verify the effectiveness of the areal RrVMD for surface texture analysis, the areal surface parameters listed in Table 2 are applied to evaluate the three types of surface features. According to ISO 25178 [37], the evaluation area should be larger than the corresponding cut-off wavelength. Thus, the evaluation of all filters was carried out within the same 5mm×5mm area on the simulation surfaces. To validate the experimental results, Eq. (43) is applied to calculate the deviations between the surface texture parameters and the actual surface parameters among different filters. The results are listed in Table 10, Table 11 and Table 12 respectively. The flatness tolerance limits the form error, so Table 10 presents the flatness result. The original areal VMD has more than 20% deviation in the flatness, and the effect is the worst among the four filters according to Table 10. Although the Gaussian filter performs well in flatness, it is inferior in dealing with waviness and roughness due to spectrum overlaps between surfaces (see Table 9). Moreover, it can be seen that the areal DWT gains overall benefit over the other three traditional filters except RrVMD in roughness (see Table 12). Based on experimental results illustrated in Table 10, 11 and 12, the proposed areal RrVMD has the smallest deviation over benchmarking data for all form, waviness and roughness.

$$D = \frac{|R - S|}{S}$$  \hspace{1cm} (43)

### 6.2.2 The robustness of the proposed algorithm

To test the validity of robust estimation merge, the simulation surfaces with outliers have been employed to evaluate the capacity to reduce outliers and defects. Typically, the outlier and defect on the functional surface do not belong to modes and must be correctly assigned to the residual. To better evaluate the robustness, the simulated roughness should remove small amounts of low-frequency components to ensure that the roughness ultimately falls on the high-frequency part. Two 5mm outliers are merged to the simulated surface (Fig.17a), and wavelength of each mode is shown in Table 5. The modes and residuals obtained by the areal RrVMD method without robust estimation are shown in Fig.17b. The outliers can be well separated into the residual, and the modes are not affected by the outliers. Nevertheless, when the outlier is raised tenfold, the benchmark in Fig.18a has an apparent bulge at the location of outliers. The PC and RMSE are applied to verify the robustness of RrVMD on the filtered benchmark after adding the Andrews estimation method (see Table 6). According to Fig.18c and Table 6, the bulges on the benchmark can be effectively eliminated by incorporating robust estimation.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Mode 0</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength(mm)</td>
<td>24.35</td>
<td>0.709</td>
<td>0.203</td>
<td>0.116</td>
</tr>
</tbody>
</table>

Table 3. The wavelength of each mode obtained by areal VMD.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Mode 0</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength(mm)</td>
<td>0.741</td>
<td>25.08</td>
<td>0.192</td>
<td>0.180</td>
</tr>
</tbody>
</table>

Table 4. The wavelength of each mode obtained by areal RrVMD.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Mode 0</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength(mm)</td>
<td>0.741</td>
<td>25.08</td>
<td>0.192</td>
<td>0.180</td>
</tr>
</tbody>
</table>

Table 5. The wavelength of each mode for simulated surface in Fig.17.
<table>
<thead>
<tr>
<th>Mode</th>
<th>Mode 0</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength(mm)</td>
<td>25.12</td>
<td>0.752</td>
<td>0.119</td>
<td>0.098</td>
</tr>
</tbody>
</table>

Table 6. The PC and RMSE between benchmark and simulated benchmark for RrVMD and Robust RrVMD respectively.

<table>
<thead>
<tr>
<th>Method</th>
<th>PC</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>RrVMD</td>
<td>0.8910</td>
<td>6.923×10⁻³</td>
</tr>
<tr>
<td>Robust RrVMD</td>
<td>0.9941</td>
<td>6.296×10⁻³</td>
</tr>
</tbody>
</table>

Table 7. The PC of each filter method between simulated topography and filtered topography.

<table>
<thead>
<tr>
<th>Filter</th>
<th>form</th>
<th>Waviness</th>
<th>Roughness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Areal Gaussian filter</td>
<td>0.9898</td>
<td>0.9874</td>
<td>0.9834</td>
</tr>
<tr>
<td>Areal Spline filter</td>
<td>0.9903</td>
<td>0.9601</td>
<td>0.9911</td>
</tr>
<tr>
<td>Areal DWT</td>
<td>0.9812</td>
<td>0.9849</td>
<td>0.9814</td>
</tr>
<tr>
<td>Areal VMD</td>
<td>0.9809</td>
<td>0.9994</td>
<td>0.9904</td>
</tr>
<tr>
<td>Areal RrVMD</td>
<td>0.9913</td>
<td>0.9998</td>
<td>0.9947</td>
</tr>
</tbody>
</table>

Table 8. The RMSE of each filter.

<table>
<thead>
<tr>
<th>Filter</th>
<th>form</th>
<th>Waviness</th>
<th>Roughness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Areal Gaussian filter</td>
<td>6.312×10⁻³</td>
<td>67.82×10⁻³</td>
<td>56.42×10⁻³</td>
</tr>
<tr>
<td>Areal Spline filter</td>
<td>5.911×10⁻³</td>
<td>42.68×10⁻³</td>
<td>41.23×10⁻³</td>
</tr>
<tr>
<td>Areal DWT</td>
<td>12.48×10⁻³</td>
<td>62.51×10⁻³</td>
<td>51.76×10⁻³</td>
</tr>
<tr>
<td>Areal VMD</td>
<td>7.137×10⁻³</td>
<td>9.121×10⁻³</td>
<td>42.47×10⁻³</td>
</tr>
<tr>
<td>Areal RrVMD</td>
<td>6.289×10⁻³</td>
<td>5.784×10⁻³</td>
<td>32.88×10⁻³</td>
</tr>
</tbody>
</table>

Table 9. The PC of each filters.

<table>
<thead>
<tr>
<th>Filter</th>
<th>PC between form and waviness</th>
<th>PC between waviness and roughness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian filter</td>
<td>15.67×10⁻³</td>
<td>31.01×10⁻³</td>
</tr>
<tr>
<td>Spline filter</td>
<td>15.88×10⁻³</td>
<td>75.28×10⁻³</td>
</tr>
<tr>
<td>DWT</td>
<td>9.404×10⁻³</td>
<td>1.138×10⁻³</td>
</tr>
<tr>
<td>RrVMD</td>
<td>0.361×10⁻³</td>
<td>0.349×10⁻³</td>
</tr>
</tbody>
</table>

Table 10. The comparison of the flatness in form. D1 is the deviation of the areal Gaussian filter, D2 is the deviation of areal DWT, D3 is the deviation of areal VMD, and D4 is the deviation of areal RrVMD.

<table>
<thead>
<tr>
<th>Unit(mm)</th>
<th>Simulated</th>
<th>Areal Gaussian filter</th>
<th>Areal DWT</th>
<th>Areal VMD</th>
<th>Areal RrVMD</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flatness</td>
<td>2.586</td>
<td>2.591</td>
<td>2.601</td>
<td>3.217</td>
<td>2.590</td>
<td>0.19%</td>
<td>0.58%</td>
<td>24.4%</td>
<td>0.15%</td>
</tr>
</tbody>
</table>

Table 11. The comparison of the waviness parameters. D1 is the deviation of the areal Gaussian filter, D2 is the deviation of areal DWT, D3 is the deviation of areal VMD, and D4 is the deviation of areal RrVMD.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$S_d$</th>
<th>$S_a$</th>
<th>$S_{dr}$</th>
<th>$S_r$</th>
<th>$S_a$</th>
<th>$S_{dr}$</th>
<th>$S_{dr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated</td>
<td>0.353</td>
<td>7.191</td>
<td>1.499</td>
<td>1.000</td>
<td>0.317</td>
<td>1.565</td>
<td>0.788</td>
</tr>
<tr>
<td>Areal Gaussian filter</td>
<td>0.302</td>
<td>6.001</td>
<td>1.586</td>
<td>1.175</td>
<td>0.260</td>
<td>1.657</td>
<td>0.822</td>
</tr>
<tr>
<td>Areal DWT</td>
<td>0.352</td>
<td>6.901</td>
<td>1.608</td>
<td>1.052</td>
<td>0.314</td>
<td>1.607</td>
<td>0.791</td>
</tr>
<tr>
<td>Areal VMD</td>
<td>0.350</td>
<td>7.699</td>
<td>1.589</td>
<td>1.101</td>
<td>0.315</td>
<td>1.554</td>
<td>0.791</td>
</tr>
<tr>
<td>Areal RrVMD</td>
<td>0.353</td>
<td>7.578</td>
<td>1.500</td>
<td>1.032</td>
<td>0.316</td>
<td>1.562</td>
<td>0.786</td>
</tr>
<tr>
<td>D1</td>
<td>14.44%</td>
<td>16.54%</td>
<td>5.80%</td>
<td>17.50%</td>
<td>17.98%</td>
<td>5.87%</td>
<td>4.31%</td>
</tr>
<tr>
<td>D2</td>
<td>0.28%</td>
<td>4.03%</td>
<td>6.77%</td>
<td>5.20%</td>
<td>0.94%</td>
<td>2.68%</td>
<td>0.38%</td>
</tr>
<tr>
<td>D3</td>
<td>0.84%</td>
<td>7.06%</td>
<td>6.00%</td>
<td>10.10%</td>
<td>0.63%</td>
<td>1.34%</td>
<td>0.38%</td>
</tr>
<tr>
<td>D4</td>
<td>0.00%</td>
<td>5.38%</td>
<td>0.06%</td>
<td>3.20%</td>
<td>0.31%</td>
<td>0.19%</td>
<td>0.25%</td>
</tr>
</tbody>
</table>

Table 12. The comparison of the roughness parameters. D1 is the deviation of the areal Gaussian filter, D2 is the deviation of areal DWT, D3 is the deviation of areal VMD, and D4 is the deviation of areal RrVMD.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$S_d$</th>
<th>$S_a$</th>
<th>$S_{dr}$</th>
<th>$S_r$</th>
<th>$S_a$</th>
<th>$S_{dr}$</th>
<th>$S_{dr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated</td>
<td>0.319</td>
<td>0.016</td>
<td>3.022</td>
<td>2.924</td>
<td>0.254</td>
<td>12.72</td>
<td>10.13</td>
</tr>
<tr>
<td>Areal Gaussian filter</td>
<td>0.330</td>
<td>0.019</td>
<td>3.016</td>
<td>3.831</td>
<td>0.203</td>
<td>11.65</td>
<td>12.54</td>
</tr>
<tr>
<td>Areal DWT</td>
<td>0.321</td>
<td>0.018</td>
<td>3.023</td>
<td>2.895</td>
<td>0.255</td>
<td>12.74</td>
<td>10.15</td>
</tr>
<tr>
<td>Areal VMD</td>
<td>0.336</td>
<td>0.017</td>
<td>3.025</td>
<td>2.998</td>
<td>0.253</td>
<td>12.67</td>
<td>10.12</td>
</tr>
<tr>
<td>Areal RrVMD</td>
<td>0.317</td>
<td>0.017</td>
<td>3.017</td>
<td>2.954</td>
<td>0.253</td>
<td>12.68</td>
<td>10.10</td>
</tr>
<tr>
<td>D1</td>
<td>3.44%</td>
<td>12.50%</td>
<td>0.19%</td>
<td>31.01%</td>
<td>20.07%</td>
<td>8.41%</td>
<td>23.79%</td>
</tr>
<tr>
<td>D2</td>
<td>0.62%</td>
<td>18.75%</td>
<td>0.03%</td>
<td>0.99%</td>
<td>0.39%</td>
<td>0.15%</td>
<td>0.19%</td>
</tr>
<tr>
<td>D3</td>
<td>5.32%</td>
<td>6.25%</td>
<td>0.09%</td>
<td>2.53%</td>
<td>0.39%</td>
<td>0.39%</td>
<td>0.09%</td>
</tr>
<tr>
<td>D4</td>
<td>0.62%</td>
<td>6.25%</td>
<td>0.16%</td>
<td>1.02%</td>
<td>0.39%</td>
<td>0.31%</td>
<td>0.29%</td>
</tr>
</tbody>
</table>
6.3 Case study

6.3.1 The metal workpiece with grooves

As shown in Fig. 19, the 82 mm × 45 mm metal workpiece with grooves is used for the first case study. The areal surface parameters mentioned in section 4 are used to evaluate the surface of the 12 mm × 12 mm area marked by the red box, and the sampling interval is set to 0.002 mm. The specification Ra and Rz of the workpiece are 1.6 µm, 7 µm for roughness, respectively [36], and the sampling area must be larger than the maximum cut-off wavelength, so λ_f is set to 8 mm and λ_c is set to 0.8 mm. The proposed areal RrVMD is applied to decompose it five times in total; hence five modes and a residual are obtained, as shown in Fig. 20 and the wavelength of them are calculated in Table 13. The residual spectrum after extraction of each mode is shown in Fig. 21. The In these modes, the first mode belongs to form, the second to fifth modes are waviness, and the residuals mainly consist of roughness with a small amount of form and waviness. Further, DWT is used to decompose the residual six times, and the mode and wavelet are merged to reconstructed surfaces in the corresponding cut-off wavelength range for obtaining the surface topographies, as shown in Fig. 22. Finally, three evaluation parameters are shown in Table 16.

Table 13. The wavelength of each mode from the experimental surface.

<table>
<thead>
<tr>
<th>Mode i</th>
<th>Mode 0</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength(mm)</td>
<td>22.66</td>
<td>22.10</td>
<td>0.866</td>
<td>0.582</td>
<td>0.091</td>
</tr>
</tbody>
</table>

Table 14. The wavelength of each mode in region a of Fig. 24.

<table>
<thead>
<tr>
<th>Mode i</th>
<th>Mode 0</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Mode 4</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength(mm)</td>
<td>19.26</td>
<td>1.599</td>
<td>1.771</td>
<td>0.574</td>
<td>0.836</td>
<td>0.103</td>
</tr>
</tbody>
</table>

Table 15. The wavelength of each mode in region b of Fig. 24.

<table>
<thead>
<tr>
<th>Mode i</th>
<th>Mode 0</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Mode 4</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength(mm)</td>
<td>9.743</td>
<td>1.468</td>
<td>0.756</td>
<td>0.729</td>
<td>0.425</td>
<td>0.121</td>
</tr>
</tbody>
</table>

6.3.2 The flat metal workpiece

The second case is a 66 mm × 33 mm flat metal plate, as shown in Fig. 23. The two 10 mm × 10 mm areas in two red boxes marked as a and b are selected for surface texture analysis, and the sampling interval is set to 0.005 mm. The specification Ra = 0.1 µm, Rz = 0.5 µm. Thus, the cut-off wavelengths of different topographies are λ_f = 2.5 mm, and λ_c = 0.25 mm, and the areal RrVMD is used to filter the two regions. For region a, a total of five decompositions is performed to obtain five modes and one residual, and the results are as shown in Fig. 24, and the wavelength is calculated in Table 14. It shows that the modes are very smooth and hardly mixed with other modes. Each topography
is obtained by integrating the modes, as shown in Fig. 26, and each spectrum before extracting each mode is shown in Fig. 25.

Region $b$ is analysed in the same way, and each mode is shown in Fig. 27; each spectrum before extracting each mode is shown in Fig. 28, and the wavelength of each mode are displayed in Table 15, the surface topography is shown in Fig. 29, and the evaluation parameters are listed in Table 18.

### Table 17. Surface parameters of the region $a$ on the metal flat workpiece

<table>
<thead>
<tr>
<th>Unit($\mu m$)</th>
<th>$S_q$</th>
<th>$S_d$</th>
<th>$S_{10}$</th>
<th>$S_1$</th>
<th>$S_a$</th>
<th>$S_{dp}$</th>
<th>$S_{dr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>waviness</td>
<td>0.471</td>
<td>1.404</td>
<td>2.386</td>
<td>0.191</td>
<td>0.038</td>
<td>0.125</td>
<td>0.007</td>
</tr>
<tr>
<td>roughness</td>
<td>0.229</td>
<td>-</td>
<td>3.252</td>
<td>0.712</td>
<td>0.056</td>
<td>3.539</td>
<td>2.268</td>
</tr>
</tbody>
</table>

### Table 18. Surface parameters of region $b$ on metal flat workpiece

<table>
<thead>
<tr>
<th>Unit($\mu m$)</th>
<th>$S_q$</th>
<th>$S_d$</th>
<th>$S_{10}$</th>
<th>$S_1$</th>
<th>$S_a$</th>
<th>$S_{dp}$</th>
<th>$S_{dr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>waviness</td>
<td>0.296</td>
<td>0.724</td>
<td>2.315</td>
<td>0.184</td>
<td>0.024</td>
<td>0.104</td>
<td>0.005</td>
</tr>
<tr>
<td>roughness</td>
<td>0.209</td>
<td>-0.0606</td>
<td>3.216</td>
<td>0.715</td>
<td>0.056</td>
<td>3.532</td>
<td>2.264</td>
</tr>
</tbody>
</table>

Fig. 19. The metal workpiece with grooves and the region used for filtering.

Fig. 20. The modes and residual extracted by areal RtVMD.

Fig. 21. The spectrum of $S_q$ before extracting each mode.
Fig. 22. Filtering result by areal RrVMD: (a) form; (b) waviness; (c) roughness.

Fig. 23. The flat metal workpiece: (a) the region a for surface texture analysis; (b) the region b for surface texture analysis.

Fig. 24. The modes and residual extracted by areal RrVMD.

Fig. 25. The surface spectrum of Sk and r before extracting every mode in region a.
Fig. 26. The filtering result of the region a by areal RrVMD: (a) form; (b) waviness; (c) roughness.

Fig. 27. The modes extracted by areal RrVMD in region b.

Fig. 28. The surface spectrum of $S_s$ and $r$ before extracting every mode in region b.

Fig. 29. The filtering result of region b by areal RrVMD: (a) form; (b) waviness; (c) roughness.
7. Conclusions and future work

This study explores a novel robust filter, named as areal Residual-restrained Variational Mode Decomposition (RrVMD), to decompose areal surface texture features. Like other VMD-based filters, RrVMD has sound capability on dealing with non-stationary signal components and capturing these components self-adaptively. The experimental results demonstrate that areal RrVMD gains sound improvements compared with the mainstream filters:

1) The deviations of all surface parameters are below 7%, all the PC between the surfaces filtered by areal RrVMD and simulations are more than 0.99, and all RMSE values are above 0.34. These results gained by RrVMD are better than traditional filters, and they prove that areal RrVMD has notable optimizations on the reduction of fitting error and the suppression of spectrum overlap.

2) Compared with areal VMD, the determination of initial center frequency and the preset parameter k avoidance can significantly reduce the fitting error. The suppression of spectrum overlap of RrVMD can increase PC for form to 0.9913. The robust estimation of areal RrVMD can effectively alleviate the impact of outliers on the benchmark to enhance the robustness (decreasing the PC of benchmark to 0.9941).

The improvements prove that the filtered outputs by areal RrVMD can reflect the surface texture more precisely. The centre frequency and residual play a vital role in mode optimisation. In future work, centre frequency and residual may be applied to explore a new spline filter that can directly decompose a functional surface into three optimal surface topographies according to the cut-off wavelength to form the target formulas.

Acknowledgements

This research is supported by the National Natural Science Foundation of China (NSFC) (61203172); the Sichuan Science and Technology Programs (2023NSFSC0361, 2022002).

References

[18] Gogolewski D 2021 Fractional spline wavelets within the surface texture analysis Measurement 179 109435

Zhang Z, Zhang Y and Zhu Y 2010 A new approach to analysis of surface topography Precis Eng 34 807–10


Gilles J 2013 Empirical Wavelet Transform IEEE TRANSACTIONS ON SIGNAL PROCESSING 61 3999–4010

Shao Y, Du S and Tang H 2021 An extended bi-dimensional empirical wavelet transform based filtering approach for engineering surface separation using high definition metrology Measurement 178 109259

Dragomiretskiy K and Zosso D 2014 Variational Mode Decomposition IEEE TRANSACTIONS ON SIGNAL PROCESSING 62 531–44

Ywa B and Rm B 2016 Filter bank property of variational mode decomposition and its applications Signal Processing 120 509–21


Nazari M and Sakhaei S M 2020 Successive variational mode decomposition Signal Processing 174 107610

Janecki D 2013 A two-dimensional isotropic spline filter Precis Eng 37 948–65


Felsberg M and Sommer G 2001 The monogenic signal IEEE Transactions on Signal Processing 49 3136–44
