Agency and the intended curriculum:
Teacher enactment of a Department for Education approved primary maths curriculum package.

Ed Southall

In partial fulfilment of a Doctorate degree

April 2024
Abstract

In 2017 the Department for Education endorsed and partially subsidised the adoption of a maths curriculum package for primary schools called ‘Maths, No Problem!’ which claimed to teach ‘for mastery’. This research investigates the curriculum intent of Maths, No Problem! and the role of teacher agency in the enactment of it by enthusiastic adopters in schools. A detailed analysis of the website, associated blog posts and a sample of workbooks, textbooks and lesson plans written by Maths, No Problem! was conducted to determine design principles that were then used as a basis for comparison in four case studies when interviewing and observing seven primary school teachers, each teaching three sequential lessons using the program. The analysis determined 8 lesson design principles and 5 core competencies that were upheld by teachers when both delivering the package and deviating from it with their own improvisations and structural lesson changes. However, teachers across each case study tended to create an imbalance in teaching for relational and instrumental knowledge development by reducing or removing lesson phases focused on instrumental knowledge. This research concludes that a prescribed curriculum package designed to be teacher-enabling rather than teacher-proof can result in enactment closely aligning with intent, sometimes resulting in teacher agency enhancing their own general pedagogy and the package itself. These findings offer further insights into the role of textbooks and curriculum packages in the development of teachers through the lens of enthusiastic adopters, and the potential effects of teacher agency on the enactment of the intended curriculum.
For my children, who look a lot older now.
Copyright Statement

i. The author of this thesis (including any appendices and/or schedules to this thesis) owns any copyright in it (the “Copyright”) and s/he has given The University of Huddersfield the right to use such Copyright for any administrative, promotional, educational and/or teaching.

ii. Copies of this thesis, either in full or in extracts, may be made only in accordance with the regulations of the University Library. Details of these regulations may be obtained from the Librarian. This page must form part of any such copies made.

iii. The ownership of any patents, designs, trademarks and any and all other intellectual property rights except for the Copyright (the “Intellectual Property Rights”) and any reproductions of copyright works, for example graphs and tables (“Reproductions”), which may be described in this thesis, may not be owned by the author and may be owned by third parties. Such Intellectual Property Rights and Reproductions cannot and must not be made available for use without permission of the owner(s) of the relevant Intellectual Property Rights and/or Reproductions.
Table of Contents

Agency and the intended curriculum: 1
Abstract 2
Copyright Statement 4
Chapter 1 Background and Context 6
   A Brief History 6
   The Rise of PISA Testing as an International Benchmarking Tool 7
   Maths Hubs Initiative 9
   The Mastery Agenda 10
      DfE Approved Textbooks: Maths, No Problem! 11
      Summary and Focus 12
Chapter 2: Research Aims and Questions 16
   Research Aims: 16
Chapter 3: Literature Review 20
   Introduction 20
      The Intended Curriculum: Textbooks as drivers of pedagogical change 23
      Trust in textbook integrity 26
      Teacher understanding and interpretation 27
      Perception of Students 37
      Political Landscape 39
      Social Landscape 41
      Closing comments 43
Chapter 4: Methodology 45
   Epistemological Framework: The Interpretative Paradigm 45
   Researcher Bias and Positioning within the Study 46
   The intended and enacted curriculum 49
   The Intended Curriculum 51
      Website analysis 52
      Textbook analysis 52
      Berisha and Bytyqi’s (2020) Textbook Analysis Model 55
   The Enacted Curriculum 61
      Defining the boundaries of each case study 65
      Sampling 67
      Type of School 68
      Geographical Locations 70
      Demographics 70
      Number of Observations 71
      Observation Structure 72
      Interviews 73
      Year Groups 76
      Gaining Access 76
      Ethics 78
      Participant Schools 79
      Documentation 81
Chapter 1 Background and Context

This research is positioned within mathematics education in England during a time of recent significant change towards a particular pedagogical approach and set of principles known as ‘teaching for mastery’. What follows in this chapter is an outline of the educational landscape that has led to the point at which the research was conducted to contextualise the relevance and importance of this study.

A Brief History

Mathematics education in primary schools has seen a number of significant changes since the introduction of the 1944 education act, at which point a clear separation between primary (5-11) and secondary (post 11) was established and the subsequent use of what was described previously as ‘elementary education’ began to disappear (Karp & Schubring, 2014).

Despite there being no national curriculum at the time, preferred mathematical pedagogy was heavily influenced by the Mathematical Association (MA), and later the Association for Teachers of Mathematics, (ATM) (Karp & Schubring, 2014). In 1964 the Nuffield Primary Mathematics Project was established, and between the three organisations, teaching advice and guidance became more freely available to teachers. However as there was no firm curriculum in place, courses were left to schools to design, which often led to courses being closely aligned to textbook content. In the late 1960s, two university chairs in mathematics education were appointed, highlighting the increased interest in developing the pedagogy of mathematics teaching in England (Karp & Schubring, 2014).

This momentum continued into the seventies, when local education authorities began creating mathematics advisor posts with the explicit aims to aid mathematical pedagogy in schools, however these roles began to disappear in the late eighties as funding was increasingly cut (Karp & Schubring, 2014).

In 1977 the Parliamentary Expenditure Committee made a recommendation to set up an enquiry into the teaching of mathematics, citing dissatisfaction amongst employers about
the standards of numeracy in education and too many examination syllabi causing inconsistencies. This resulted in the publication of The Cockroft Report (1982) which outlined a set of pedagogical recommendations and proposals that in turn informed several curriculum schemes including the School Mathematics Project materials which were eventually adopted by more than 70% of schools (Askew, 1996), and expert teachers funded by the Department for Education and Science (DES) appointed into Local Education Authorities colloquially referred to as ‘Cockroft Missionaries’ (Johnson & Millett, 1996). These initiatives were arguably some of the earliest strong and direct state-level interventions to influence the ways in which mathematics was taught in schools.

In 1988, the National Curriculum was established under the 1988 Education Act, and the introduction of a new mathematics GCSE, which 85% of pupils would now sit, following previous qualifications the General Certificate in Education (GCE) O-Level (20%) and the Certificate of Secondary Education (CSE) for a further 40%. This new qualification ensured greater mathematics curriculum standardisation and uniformity across England. Indeed much of the content and structure of the new GCSE was perceived to be an attempt to directly implement many of the suggestions made in the Cockroft Report (Askew, 1996).

The National Numeracy Strategy

In 1996 the National Numeracy Strategy (later known as the Framework for Teaching Mathematics) was published by the Department for Education (DfE) which included both a collection of teaching resources, and teacher guidance notes for students from reception to year six. Arguably, this marked the beginning of the state officially taking a central more direct approach to preferred specific pedagogical strategies for the teaching of mathematics at primary level, and incorporated a large-scale national training programme encouraging teachers to emphasise mental calculation and prescription of pedagogy and lesson structure (Millett, Brown & Askew, 2007).

This type of control, or endorsement of ways of teaching, is one of the key themes of this research. Millett, Brown and Askew (2007) argue that the National Numeracy Strategy impacted upon pupils in a number of ways, finding in specific case studies that the NNS improved pupil attainment, confidence, participation and understanding in mathematics,
with particular benefit to ‘middle learners’, however it appeared to show less impact on low and higher attainers due in part to a promoted ‘whole class teaching’ approach. Johnson and Millett (1996) found that the NNS, whilst based on sound principles, varied in particular with regards to how research-informed it was. While the NNS aimed to standardise teaching and learning outcomes, in practice, earlier models of differentiated entitlement based on perceived differences in pupils overlaid newer ones, leading to mixed messages and tensions for teachers. This resulted in challenges in balancing the goal of achieving a standard level of attainment for all pupils with the need to differentiate the curriculum to meet individual needs (Askew & Brown, 2001).

In 2003 the Numeracy Strategy was absorbed into the broader Primary National Strategy. Teaching strategies were updated in 2006, but the Primary Numeracy Strategy ceased operation in 2011. (University of York, n.d.) leading to a decade of substantial governmental changes and control over primary mathematics teaching, including curriculum changes and what would become known as the Mastery Agenda (Boylan et al., 2018). Here we will briefly outline the factors that influenced the DfE’s decision and subsequent strategy to overhaul mathematics education in the 2010’s, leading to the announcement of a DfE approved curriculum package called ‘Maths, No Problem!’ to be partially funded for schools that adopted the program. The curriculum package ‘Maths, No Problem!’ (MNP) is the focus of this study.

The Rise of PISA Testing as an International Benchmarking Tool

The Programme for International Student Assessment (PISA) is a global testing model developed by the Organisation for Economic Co-operation and Development (OECD) and is utilised by an increasing number of nations to test pupil performance at 15 years of age in mathematics, science and reading. At the time of writing, seventy-nine government education departments were members.

The introduction of the PISA testing model in 2000 has had a profound influence on educational policies around the globe (Baird et al., 2016). The test is carried out every three years on a growing range of participant countries across all continents and has become a comparative tool to contrast the achievements of young students in schools across a range
of subjects, including mathematics (Auld & Morris, 2016). Countries with comparatively low scores have often made drastic changes to their educational policies as a direct result of the tests, in a bid to boost their rankings on the world stage (Grek, 2009) such as Germany (Ertl, 2016), and Sweden (Ringarp, 2016). In Germany, PISA standings in 2000 undermined the country’s self-perception of its education system and triggered high ranking politicians to re-evaluate education in the country resulting in wide ranging reforms of the framework for teaching, assessment and curriculum development and the introduction of national educational standards (Ertl, 2016). Similarly, the 2003 PISA results indicated what Ringarp describes as a drastic loss of knowledge among Swedish students, which resulted in the introduction of a new School Act in 2010 and included a new teacher certificate (Ringarp, 2016). Indeed, the National Numeracy Strategy itself was launched in part due to pressures from Ofsted citing international comparison data as evidence of a need to improve standards (Askew & Brown, 2001).

Countries that perform well in the tests are often the source of emulation strategies for poorer performing countries - for example, in the early years of PISA tests, Finland was the subject of many research projects and teacher excursions from the UK (and indeed Sweden following their 2003 results) to determine how they had succeeded where other countries had perceived themselves to have failed (Chung, 2010).

In 2015, the UK PISA Mathematics scores placed them twenty-seventh out of seventy three, just two points above the average (and one place lower than in 2012). Around this time, the Department for Education began sweeping reforms of the mathematics curriculum and funded several initiatives to train teachers of mathematics to use teaching strategies collectively referred to as ‘mastery teaching’ or ‘teaching for mastery’ (Boylan et al., 2018). Specifically, in November 2013, following a curriculum review in 2011, GCSE reforms across English and Mathematics were announced, including new content, examination-only end of course assessment (replacing coursework, and modular examination structures) and ‘tougher’ exams (DfE, 2013a).

The DfE stated at the time that the intent of the reforms was to match school standards to those of the strongest performing education systems in the world such as Hong Kong and Shanghai. (Pells, 2017). Furthermore, the curriculum reforms were not only triggered by a
desire to match high performing countries, but also the ideas within the reformational measures themselves were influenced by the world’s most successful school systems such as Hong Kong, Massachusetts and Singapore (DfE, 2013b). Whilst Massachusetts may seem like an outsider alongside the frequently mentioned South Asian regions, the US state performed higher than any other on the PISA tests, when analysis is broken down further (NCES 2012). In summary, the PISA tests themselves have been particularly influential in England both in terms of being a motivational trigger for reform, and in providing the reform solutions the Government put forward and developed, leaning into the pedagogical style and structure of mathematics teaching in top ranking countries.

During the last decade the DfE, and in particular schools minister Nick Gibb, have often cited Singapore, Shanghai and Japan (the highest ranking PISA performers) as having outstanding and desirable teaching styles (DfE, 2016; Gibb, 2016; Gibb, 2015a). In 2016 for example, Nick Gibb, then the Minister of State for the Department of Education, said in a speech:

> A large body of evidence from cognitive scientists demonstrates that knowledge and understanding in mathematics proceed in tandem, and should be taught together. This insight is well understood by mathematics teachers in the Far East. I have been most impressed by the focus teachers from Singapore and Shanghai place, not just on basic skills, but also on developing clear conceptual understanding.
>
> (Gibb, 2016)

**Maths Hubs Initiative**

On the fourth of December 2013, Education Minister Elizabeth Truss announced an eleven-million-pound Maths Hub Programme across England, initiated, according to their official press release, because international test results showed England’s performance had stagnated (DfE, 2013c). The press release specifically cited England’s performance in the 2012 PISA tests and associated OECD report as the motivation to create such hubs, and went
on to single out a number of South Asian Jurisdictions as ‘table dominating’ - namely Shanghai, Singapore, Hong Kong, Taipei, Korea, Macao and Japan (DfE 2013a). Truss again named several of these in a July 2014 announcement regarding which schools were becoming hubs across England, stating “there's no reason why children in England cannot reach the same level as those in Japan, Singapore and China.” (DfE 2014)

The maths hub project intended for thirty strategic maths hubs to be created across the country (thirty two were named two years later), led by teaching schools and specialising in: “teacher recruitment, initial training of maths teachers and converting existing teachers to mathematics, coordinating and delivering a range of maths CPD and school-to-school support, ensuring maths leadership is developed, and helping maths enrichment programmes to reach a large number of pupils from primary schools upwards” (DfE 2013a).

In 2014, it was announced that the hubs would develop the maths improvement programme with academics from Shanghai Normal University and England’s National Centre for Excellence in the Teaching of Maths (NCETM). At the same time, fifty teachers from Shanghai were announced to be embedded in the hubs to teach pupils and run masterclasses for other teachers as part of a Shanghai Teacher exchange which at the time of writing had been extended to 2020 (DfE, 2014).

It is worth noting here that the NCETM is directly funded by the DfE, and in turn is responsible for the strategic direction of the Maths Hubs. Maths Hubs are the centres that implement training for teachers in their locality, and thus the NCETM’s publications and general materials to support teachers to teach for mastery are considered here to be directly approved and in line with the DfE’s vision of ensuring teachers in England adopt the South Asian mastery approach (DfE 2016).

The Mastery Agenda

On the twelfth of July 2016, the Schools Minister Nick Gibb announced a four year forty-one-million-pound funded plan to support eight thousand primary schools across England - which is roughly half of all primary schools in the country. The announcement
declared that a “South Asian mastery approach to teaching maths is set to become a standard fixture in England’s primary schools” (DfE 2016). The announcement explicitly used international PISA testing to justify the decision, citing that fifteen-year-olds in Asian ‘mastery teaching’ areas, such as Shanghai, Singapore and Hong Kong (all specifically mentioned) were “more than ten percentage points” more functionally numerate than in England.

Mastery as a specifically cultured and identifiably named style and philosophical approach of teaching is generally considered to originate primarily from Benjamin Bloom’s 1968 article “Mastery Learning” (McCourt, 2019; Slavin, 1987; NCETM, 2016b; Drury, 2017). Mastery Learning popularised ideas and principles that borrowed and built from earlier works such as Carleton Washburne’s model for schooling (referred to as the Winnetka Plan), Michael Scriven’s ideas on formative evaluation, and Fred S. Keller’s Keller Plan (Keller, 1967) - often also referred to as the Personalised System of Instruction (PSI). It was Bloom though, who brought these ideas together to form the backbone of his seminal work Learning for Mastery (1968) which is recognised here as the forefather of the ‘mastery approach’ referred to by the DfE. Bloom is in fact directly referenced by NCETM as one of the originators of the term ‘mastery’ and the idea, in their mastery training materials (NCETM, 2020). Bloom (1968) sets out his case for breaking from what he considered normal expectations - whereby a third of a class were expected to fail, and a further third would not make good progress. Bloom’s objective was to enhance teaching and learning, as well as teacher expectations, so that this self-fulfilling prophecy would be broken and instead replaced with a more optimal and optimistic conjecture of 90 percent of students mastering what is taught to them. In modern day mathematics mastery, the very underpinning philosophy is often cited as “keep up rather than catch up” (NCETM, 2020) - a phrase intended to help classroom culture break away from the notion that a teacher’s expectations of a student are informed by a perception of ability.

DfE Approved Textbooks: Maths, No Problem!

During the mathematics educational reforms in this period, Nick Gibb paid particular attention to the quality of textbooks in Asia. In 2015, Gibb gave a speech titled ‘How to get
more high-quality textbooks into the classroom’ (Gibb, 2015b) which coincided with NCETM producing a list of quality assessment criteria for mathematics textbooks (NCETM, 2015). Both Gibb’s speech and the NCETM’s textbook guidance specifically mentioned and advocated the mastery approach used in textbooks in high performing countries like Singapore including specific commentary regarding their high quality and focus. By 2017 the Government had set up an expert panel to assess the suitability of several new primary mathematics textbooks for use in English classrooms using criteria set out by NCETM (NCETM, 2017), with the intent of announcing DfE endorsed textbooks for primary schools, with financial support of up to £2000 to help schools with adopting them. Despite a range of publishers submitting their textbooks for consideration including established brands Oxford University Press, Pearson, Scholastic, and Collins Learning (Cowdrey, 2017), only a single text was approved, titled ‘Maths - No Problem!’ (Herein referred to as MNP). This was considered an unexpected result (Schoolsweek, 2017) and surprised even the publishers of MNP themselves, with Kate Moore, head of communications at MNP quoted as saying she was “so shocked I had to call the DfE to double check it was correct” (Schoolsweek, 2017). At the time of the announcement, members of the teaching profession warned that the MNP textbooks were prohibitively expensive with resources for a single class estimated at costing around £1015, required annual maintenance costs (replacement workbooks) and that the DfE contribution of up to £2000 would be insufficient for most schools to implement the program (SchoolsWeek, 2017). Further controversy surrounded the expert panel that made the decision, which was not disclosed by the DfE until the Times Educational Supplement applied for details through the Freedom of Information Act. The panel included just three members: Debbie Morgan, director of primary maths at NCETM, Bruno Reddy, founder of Times Tables Rockstars, and Tim Oates, group director of assessment research and development at Cambridge Assessment, and author of the influential 2014 paper ‘Why Textbooks Count’ (Oates, 2014), cited by Gibb in several of his speeches in this period (TES, 2018; Gibb, 2015a; Gibb, 2017; Merttens, 2015a).

Summary and Focus

As we have seen, this research takes place during a time of substantial educational change in mathematics in England, which brought in both a ‘Mastery agenda’, and the first
state-sponsored maths textbooks in primary education in England. The textbooks singularly passed the DfE commissioned expert panel approval process for aligning with the way in which the Government felt was best practice in teaching mathematics - which itself was heavily influenced by pedagogical approaches in high performing Asian countries.

This research is specifically framed around the use of the textbook series titled ‘Maths - No Problem!’ In primary schools in England, concerning both the intended and enacted MNP curriculum which will be discussed in more depth shortly. The background and context to the introduction of the MNP textbooks is complicated and wide-ranging in its reach, hence this study will focus herein predominantly on MNP itself, rather than the influences and origins of it becoming a DfE approved text for primary schools in England. However, MNP is considered in this research to be a model of what the DfE perceives as the best way to teach mathematics, hence whilst the focus is on the curriculum package, the package itself is viewed as a manifestation of the DfE’s actions to influence the way in which mathematics is taught in schools in England. Hence the intended ways of teaching as demonstrated by MNP are in essence considered to be the intended ways of teaching as desired by the DfE.

Within the context of the MNP curriculum package, this study considers the impact of teacher agency on the prescriptive nature of the program, and whether this affects the intentions of the program either ‘positively’ (is considered to enhance or uphold the intent) or ‘negatively’ (is considered to detract from or undermine the intent). As such it is important to consider not only the context of the introduction of the program from a political perspective, but also the way in which the mastery agenda is perceived by teachers themselves. Hence it is important to appreciate the national contextual influences at play which in turn are likely to influence more localised contexts in schools, providing useful insights into explaining the situation schools find themselves in. For many schools, mastery approaches are relatively new as they seek to adopt the strategies and techniques promoted by the DfE whilst simultaneously redesigning curriculum content to align with the curriculum reforms that came in around the same time.

Both the desire of the DfE for teachers to adopt a mastery approach, and the prescribed nature of textbooks which traditionally set out curriculum content, sequencing, exercises for practice and assessment, and even lesson plans and teacher notes could be seen to be
overly prescriptive and undermining the agency of teachers. Ruth Merttens (2015a) professor of primary education, wrote in an article in the Guardian that the heavy promotion of Singapore and Shanghai style mathematics textbooks and teaching styles was “profoundly undemocratic”, “unlikely to improve children’s learning” and undermined creativity, originality and difference for the sake of uniformity. Merttens goes further in her 2015 plenary lecture for the Mathematical Association, asserting that the imposition of prescribed textbooks and pedagogical styles imported from abroad is primarily an unviable tool of governmental teacher control with no understanding of the broader cultural and moral roles of high-quality teaching (Merttens, 2015b). The teacher response then to the substantial changes to maths education, particularly the introduction of ‘borrowed’ pedagogy, appears at the very least to be mixed. As will be discussed in the literature review for this study, the social and political context surrounding teachers adopting a curriculum package appear to affect the way in which they deliver it.

Finally, whilst it is not a focus of this study, it is worth briefly noting that mastery approaches to teaching have been found to have a positive effect on learning outcomes for students. Recent studies such as Jerrim and Vignoles (2015), performed two clustered randomly controlled trials over two years where mastery approaches were introduced using the Ark Curriculum Maths Mastery package to a selection of 90 schools in England. The study found that East Asian teaching methods did have a modest positive effect on student outcomes compared to control groups, which comes at a relatively low per-pupil cost. However, there are critics of the study, for example Didau (2016) highlights that determining pupils have ‘mastered’ a topic can risk not revisiting topics despite them being just as likely to be forgotten as anything else we learn over time. It is also interesting that the Ark program is not dissimilar to the MNP program in so much as it contains a range of comprehensive lesson plans, slides and resources, as well as teacher guidance and associated training materials, hence the term ‘textbooks’ can be misleading when referring to MNP, as it is actually a full suite of resources and support materials, of which a traditional ‘workbook’ for students is merely one component. The full MNP package will be discussed in more detail later.

In support of the findings of Jerrim and Vignoles, the Education Endowment Foundation (EEF) concluded after looking at 80 different studies on mastery learning, that the approach
was both cost effective and impactful on student outcomes in a positive way (ie it accelerated student progress in learning), particularly in primary school settings, and particularly in mathematics (EEF, 2021). However, the homogenisation of the term ‘mastery learning’ is perhaps problematic, with definitions of mastery often differing across sources such as Bloom (1968), NCETM (2016a) and McCourt (2016). Whilst peripheral to this study, the efficacy of mastery approaches will inevitably contribute towards ‘buy-in’ from teachers. In other words, if teachers believe in what they are delivering and it meets their expectations in raising attainment in classrooms, that may be an important consideration regarding how teachers respond to and adapt the use of MNP in the classroom. Whilst this study is focused on the intent of the MNP curriculum package, and the ways in which it is interpreted, gaining a sense of ‘buy in’ from teachers may help with my own interpretation of teacher actions and intent.
Chapter 2: Research Aims and Questions

Research Aims:

1. Identify specific pedagogical approaches in curriculum packages endorsed by the Department for Education for primary mathematics in England.

2. Determine how the teacher agency of enthusiastic adopters affects the enactment of the intended curriculum provided by DfE endorsed curriculum packages for primary mathematics in England.


Aims Rationale:

1. Identify specific pedagogical approaches in curriculum packages endorsed by the Department for Education for primary mathematics in England

Why this aim?
Whilst we have seen from the previous section that it is explicitly stated that a ‘mastery approach’ is endorsed by the DfE, the definition of what a mastery approach actually is, or entails, is less clear. ‘Mastery teaching’ is a somewhat fuzzy term, but this research aims to develop a clearer set of tangible strategies alongside an underpinning philosophical stance regarding how mathematics should be taught and learnt through the lens of the ‘Maths - No Problem!’ program. This in turn, it is argued, can be viewed as the DfE’s perception of best practice in mathematics teaching, due to the fact that at the time of this study, MNP was the only government endorsed textbook for teaching mathematics in primary schools in England.

Why is it important?
This research aim will contribute towards a greater understanding of what the DfE consider
to be high quality teaching in mathematics at primary school in England. It will provide insight into the ways in which teachers are expected to transmit knowledge to students, and add clarity to what is meant by the DfE when referring to Mastery Teaching, or Teaching for Mastery. It will also contribute towards wider research into governmental interventions in the teaching profession both generally and specifically regarding preferred pedagogical approaches in the classroom as well as wider research into the impact of international league tables in education.


**Why this aim?**

An in-depth analysis of the ways in which students practise maths and of how maths is presented to them will enable me to better comprehend what MNP presents as ‘mastery’ in mathematics. For example, there may be a particular emphasis on the use of pictorial models, worded scenarios as a basis for problem solving, or long periods of ‘drill’ practice carrying out the same procedure repeatedly as part of lesson structures. This in turn helps formulate a more accurate picture of the intent of the MNP curriculum to analyse as a comparative measure to the enacted curriculum, ie, what the teacher actually does in their teaching. For example, does a teacher emphasise the same models as MNP intends, do they replicate the same proportion of question types etc, or more generally, does a teacher ask questions (and do students practise questions) in the same way as they are presented and intended by MNP? Hence this research aim will gather suitable data to inform analysis of both the intended and enacted curriculum.

**Why is it important?**

This research aim contributes new knowledge towards understanding what the DfE means explicitly when promoting mathematics mastery. It is one of the first research projects into the DfE’s endorsed textbook series and will shed light on both how the MNP program is constructed, and what such a construction infers about what maths teaching should look like, not only through the lens of MNP, but through the lens of the government. It contributes towards wider research and debate regarding what maths mastery means
(definitions and interpretations differ eg Bloom, 1968; NCETM 2016b; McCourt, 2016 all
differ slightly in their definitions of the term) - an issue highlighted by NAMA (The National
Association of Mathematics Advisers) in their 2015 paper ‘5 Myths of Mastery in
Mathematics’ (NAMA, 2015) who argue that Mastery is an applied set of principles that are
interpreted differently by different bodies.

3. Determine how the teacher agency of enthusiastic adopters affects the enactment of the
intended curriculum provided by DfE endorsed curriculum packages for primary
mathematics in England.

Why this aim?
At this time there are substantial changes taking place in mathematics education in England
initiated by the DfE, from multiple angles such as curriculum content, assessment changes,
and promoted pedagogical approaches. The MNP textbook program endorsed by the DfE
provides a heavily prescribed curriculum package that can be seen as a way to (or an
attempt to) control how teachers teach mathematics in the classroom. As such, it is of
considerable interest to the researcher to observe and document how the role of teacher
agency interacts with the prescription of content and delivery from MNP. For example, how
are activities and ideas interpreted by the teacher in the classroom (enacted) and how well
does this align with how they were intended to be used? How closely do teachers follow the
materials, and are there any consistencies across different teachers regarding how much
curriculum coverage is delivered, adapted or omitted from MNP?

Why is it important?
This research aim will contribute new knowledge as one of the first studies to investigate the
ways in which the first state sponsored maths textbooks impacts upon teacher pedagogy in
England, and how the teacher agency of enthusiastic adopters works alongside these
prescribed resources and planning. It holds currency in English mathematics education in
that, as of 2022, there are now only 2 endorsed mathematics textbooks, and maths mastery
is still a very prominent element of mathematics teaching. This objective also has local
currency in that the government has, in 2022, announced a new National Academy that will
provide a fully resourced curriculum freely accessible to schools which appears to be an
extension of the idea of prescribed resources already endorsed for mathematics. This aim will contribute towards wider research considering the agency of teachers and impact of curriculum packages on teachers in the classroom. Specifically, there is a range of research into whether singular textbooks or indeed fully resourced curriculum packages can change teacher pedagogy to align with the intent of the resource, to what extent, and what factors encourage or dissuade that pedagogical change from occurring. There is also a lack of research in English schools on the influence of textbooks on teaching and teacher agency. This can also be seen more broadly as a contributor towards research into organisational change. This research also contributes to mathematical textbook research in general and could be used as a specific resource for comparative studies of the content of maths textbooks either in England or across nations.

**Research Questions:**

The research questions outlined below are intended to support the research aims and help determine and steer the kind of data that will be collected such that the aims can be met effectively. A more in-depth analysis of how data will be gathered to inform the research questions so that they can be answered effectively will be presented in the methodology section.

1. What strategies are emphasised and prioritised in curriculum packages endorsed by the DfE for primary mathematics in England?

2. How are textbooks used as teaching aids in DfE approved curriculum packages for primary mathematics in England?

3. How do teachers facilitate knowledge acquisition using DfE approved curriculum packages for primary mathematics in England?

4. How is teacher agency impacting upon the interpretation and delivery of the intended curriculum in DfE approved curriculum packages for primary mathematics in England?
Research question 1 informs research aim 1, research questions 2 and 3 inform research aim 3, and research question 4 informs research aim 2.
Chapter 3: Literature Review

Introduction

The literature review explores the relationship between the intended and enacted curriculum in mathematics. It analyses previous case-study research that investigates similar phenomena and circumstances to the situation in England in which MNP was introduced. The purpose of researching similar cases was in part to uncover the different ways in which agency has affected how the intended curriculum, via textbooks and textbook associated prescribed resources such as lesson plans, workbooks for students, presentations and teacher scripts, have been enacted. Whilst the theme of this research could arguably be broadly defined as *enacting organisational change* - upon which there is an overwhelming wealth of research already, this literature review aims to contain and limit the scope of research reviewed specifically to mathematics education in order to keep it specific to the area of study. The overriding purpose of the literature review is to better understand what research already exists in this area, how it was conducted (to gain insights into suitable potential methodologies for this research), help sharpen the focus of enquiry (ie, what specifically to look for, observe, analyse or ask when conducting the research) and help position this research in the wider context of how agency affects an enacted curriculum. The literature helped me understand key influential factors and circumstances that shape the ‘success’ of enacting the intended curriculum - in other words, it helped determine the extent to which the enacted and intended curriculum align with different teachers and identified themes that appear across a variety of research cases.

This chapter will also identify the most suitable method(s) to analyse the content of the MNP textbooks to confidently and accurately address research aim 3 (*Determine the structure and formats of practice underpinning curriculum programs endorsed by the DfE for primary mathematics in England*) and contribute towards research aim 1 (*Identify specific pedagogical approaches endorsed by the DfE*). This was achieved by comparing and contrasting existing research that focuses on analysing the content of school mathematical textbooks. Literature in this area tended to fall into two categories:
1. In depth analysis of a single textbook

2. Comparative studies of textbooks from two different sources (usually from two
different countries).

An intended outcome of this chapter was to determine a suitable toolset with which to
analyse the content of the MNP texts in a way that is both robust and pragmatic, but also
tailored to answer the research questions efficiently. In simple terms, this section sharpened
what to look for in MNP textbooks and how to find it. The terms intended, and enacted
curriculum package will be discussed and defined here. Firstly, it is important to understand
that ‘curriculum package’ will be used to help differentiate the National Curriculum (what is
stipulated by the DfE regarding what mathematical content must be taught to each key stage
in schools in England) from the Maths, No Problem! set of resources that were designed to
implement the National Curriculum through their lesson plans, workbooks, textbooks and
slides. As such, the National Curriculum will be referred to as ‘The National Curriculum’, and
‘curriculum package’ refers to the MNP suite of resources: the sequenced organisation of
subject content to be delivered, including planned exercises and experiences, for students –
which is the focus of this study. Unfortunately, the concept of an intended curriculum is
fuzzy within the literature studied in this section. For example, for different literature, the
intended curriculum may be a simple list of content (e.g. early models of the UK National
Curriculum), or a single textbook of sequenced exercises aimed at students - however in
other studies, ‘curriculum’ may also incorporate much more substantial guidance for
teachers such as a suite of resources including detailed lesson plans, scripted teaching and
questions with anticipated responses, homework activities, teacher videos, slides, online
support and training opportunities. The latter example more closely relates to the resources
offered by MNP, referred to as the MNP curriculum package. The range of possibilities
regarding what is meant by ‘curriculum’ in the literature makes comparisons and conclusions
across sources for this review challenging, along with the rise in digital online support in
more modern curriculum packages rendering older sources less relevant, or at least making
it important to carefully distinguish more recent research. Hence within the literature
review, attention will be drawn to what form the intended curriculum takes in specific
research where appropriate. Furthermore, a detailed analysis of what is provided and what
constitutes the intended curriculum for MNP will be incorporated into this study.
For the purpose of this research, the interaction between the intended and enacted curriculum is of particular interest (research aim 2), specifically how much influence teacher agency has on the enactment of the intended curriculum. The intended curriculum refers to what is intended by the creator/author (Harwood, 2017), whereas the enacted curriculum refers to the teacher’s interpretation of that curriculum, and how it is enacted live in a classroom. Similarly, the intended curriculum *package* is seen here as what is provided by Maths, No Problem!, whereas the enacted curriculum *package* is what teachers actually deliver to their students (how they interpreted MNP). Ball and Cohen (1996) refer to the enacted curriculum as being “jointly constructed by teachers, students and materials in particular contexts” influenced by a number of factors such as what teachers think about how their students will react to the materials and how teachers comprehend the materials themselves. For example, how a teacher takes a resource, interprets its intent, and uses it (with or without adaptations) in the context of their classroom is distinct from how the curriculum package authors developed it – and the way in which the teacher uses it is likely to be shaped in part by how their students react to it. The two instances (intended and enacted) are likely to differ, perhaps significantly as discussed shortly. Similarly, Eisenmann and Even (2011) refer to the enacted curriculum as the *implemented curriculum* but further simplify this to ‘classroom instruction’, noting that the implemented curriculum is often significantly different to the written (intended) curriculum. In simple terms, the intended curriculum is the provided instructions or guidance for delivery, and the enacted curriculum is what is actually delivered, or what Harwood (2017) helpfully defines as ‘what happens in the classroom’.

**Defining Teacher Agency**

Janine Remillard (2005) emphasises the importance of the agency of teachers in the context of curriculum use, framing them as *active designers* of the enacted curriculum, rather than passive implementers of predetermined plans. She positions teachers as having their own agency over any prescribed curriculum, meaning they actively engage in shaping and modifying it based on their professional judgement, beliefs, and experiences (Remillard, 2005, 2012). She explores the factors that influence the choices teachers make in the
classroom and how these choices play out in actual teaching practices, including the examination of representation of tasks and concepts in the curriculum and how teachers interpret and adapt these for their students, acknowledging that high fidelity to the written curriculum is not always possible or desirable. Instead, the level of fidelity towards curriculum intent leans towards an interpretivist approach, where teachers are seen as meaning-makers who draw on their beliefs and experiences to create meaning from curriculum materials (Remillard & Bryans, 2004). This approach sees curriculum use as a participatory relationship wherein teachers and curriculum materials are two parts of a dynamic interrelationship - with the teacher possessing the power to shape human activities (Remillard, 2005) and therefore construct the enacted curriculum. In this research, this decision making is considered to be the definition of the teacher’s agency in the context of curriculum use: highlighting the active, interpretive, and participatory role of teachers in shaping the curriculum based on their application of professional knowledge and judgement considering the needs of their students.

The Intended Curriculum: Textbooks as drivers of pedagogical change

Indications of the positive influence of strong textbooks in the classroom are evident around the world (Fan & Kaeley, 2000). Textbooks are used as an integral and institutionalised part of most courses and syllabi globally (Harwood, 2017; Yerushalmy, 2014) and they are considered to influence how a teacher teaches (Fan et al., 2013) as well as dictating the content to be taught and what order to teach it in. As such, textbooks can be perceived as authoritarian from both the perspective of the authors or national authority imposing expectations around delivering content and pedagogy, but also around the teacher’s authority onto the text and themselves (Remillard, 2005). The influence of textbooks, and the power they can yield on both what is taught and how it is taught is taken very seriously in a number of countries such as Japan and China where they are tightly controlled (Watanabe, 2001). For example, in Japan, textbooks are used in a significant majority of maths lessons, only a small selection of government approved textbooks can be used, and entire districts use the same nominated textbook series (Watanabe, 2001). This level of influence can also be seen elsewhere around the world. In Brazil for example, textbooks are the main resource for planning classroom activities and are cited by the
ministry of education as directly helping conceptual development of teachers by helping them build their knowledge and encouraging autonomous learning and self-assessment (Borba & Selva, 2013). Given the influence of Asian approaches to textbook use and pedagogy on the English educational push for maths mastery (Gibb, 2016), it is perhaps unsurprising that the Department for Education further emulated Asian counterparts with what appears to be a partial textbook governance plan by beginning its own procurement process to endorse and partially fund the adoption of any winning textbook series that passed their quality threshold measuring how well each series reflected a ‘teaching-for-mastery’ approach (Schools Week, 2017; NCETM, 2017). Given this, it is conceivable that teachers newly adopting MNP in the classroom may perceive its use differently to early adopters prior to the government endorsing it. Specifically, a school adopting MNP because the DfE want them to, might affect teacher agency and adoption of the curriculum package differently if it is seen as a way of coercive control or imposition. This in turn led to the decision to observe enthusiastic adopters of MNP prior to their announcement to partially fund it which is discussed in the next chapter.

Whilst textbooks form a direct link between author, authority and teacher (Yerushalmy, 2014), Freeman and Porter (1989) argue that this assumption is disrupted by teachers rarely following the prescription of textbooks - a finding more recently repeated by Pepin & Haggarty (2001) and again by Remillard & Heck (2014). It appears that alignment of both the intended and enacted curriculum is often loose rather than tight - and that views of the teacher as a conduit for the curriculum (Walker, 1976) have evolved more into teachers as shapers of the curriculum (Remillard & Heck, 2014). Indeed, how teachers shape the MNP curriculum is a key focus of this study captured by research question 4. Brown (2009) presents a model of teacher appropriation of affordances in textbook use as a continuum, with teachers at one end of the scale simply offloading agency of lessons to the textbook - in other words, literally teaching ‘by the book’ and tightly aligning the enacted curriculum to the intended curriculum, and at the other end, teachers improvise all of the lesson from resources away from the book. In the middle, teachers adapt textbook resources and share responsibility for the delivery of the lesson. By paying for and adopting the MNP program, it seems highly unlikely that MNP teachers will not use the resources at all, but where they sit on such a scale is pertinent and can help define how much a teacher aligns with the intended curriculum. Furthermore, how MNP frames their curriculum package is also
important. Is their intent for teachers to sit at one extreme end of the scale and offload lessons as they are presented, or do they expect or anticipate a more shared responsibility and present their resources expecting teachers to be the ‘shapers’ that Remillard & Heck (2014) refer to?

How textbooks are viewed by teachers themselves is a key factor in their uptake. Fan argues that the influence of textbooks (and associated materials) can be both positively and negatively influential and can be perceived by teachers as either a facilitator or barrier to pedagogical change. For example, Fan et al. (2013) found that textbooks were highly regarded and used by Shanghai mathematics teachers and were believed (by the teachers) to be instrumental in facilitation of instructional change, whereas only around 10% of students were reported to use textbooks regularly in 2014 (Oates, 2014), and an anticipated drop to 8% by 2020 was reported at the International Textbook Summit (International textbook summit, 2018 June 14). The near wholesale rejection of the regular use of textbooks in England suggests they are not nearly as highly regarded as in other high performing countries. Historically, this is sometimes suggested as being a result of ‘educational establishments’ (ie, the DfE or Ofsted) not favouring their use (Askew et al, 2010) which Bokhove and Jones (2014) speculate came from an Ofsted publication in 1993 that referred to over reliance on textbooks in their analysis of nearly 10,000 Ofsted reports - however, beyond this single reference, they found little evidence of any deterring of textbook use using their text analysis tool. Echoing this, Ball and Cohen cite educators ‘disparaging textbooks’ and suggest reform-oriented teachers often idealise professional autonomy which in turn leads to a cultural view that good teachers do not follow textbooks (Ball and Cohen, 1996). Bokhove also speculates that textbook uptake for maths in England has perhaps been disrupted by the introduction of multiple exam boards diluting the textbook market, high costs to schools and a dip in quality of more modern texts (Bokhove, 2021). Fan’s claims of textbooks being facilitators or barriers, and Bokhove’s speculation about low quality texts are further supported by a number of other research findings. Ding (2016) found Chinese textbooks supported and developed teacher knowledge about the number zero for example, whilst Davis (2009) found US textbooks had almost no effect on teacher understanding of exponents, and a Spanish textbook was found to be a potential obstacle to learning topics of measure due to their ‘skewed’ approach to the subject
(Mengual et al, 2015) - a finding similar to Vincent and Stacey (2008) that popular Australian textbooks were in part to blame for simplistic teaching of some topics due to their poor structure and presentation of topics. For this research, how textbooks are perceived by the teachers or by influential stakeholders in their community (for example, school leadership, or parents) may well affect the ways in which the curriculum package is enacted.

In fact, this general concern around textbook quality falls under one of five key themes that became apparent whilst conducting the literature review. Each theme can be seen as an influence that contributes to how closely teachers align the enacted and intended curriculum:

1. Trust in textbook integrity
2. Teacher understanding and interpretation
3. Perception of students
4. Political landscape
5. Social landscape

Trust in textbook integrity

As previously discussed, Vincent and Stacey (2008) and Mengual et al (2015) both point to issues with the quality of textbooks in Australia and Spain respectively. In Australia, Vincent and Stacey found that some of the most popular textbooks in mathematics contained a significantly disproportionate amount of repetitive question types of low complexity, and they observed that teachers closely delivered this content to their classes, resulting in almost no reasoning or verification of mathematical results in lessons, which Stacey dubbed as ‘shallow teaching syndrome’. This offers two key insights, firstly, the perceived poor quality of the content of the textbook may as Ball and Cohen (1996) suggest, contribute towards apathy towards using them over time, but also, the poor quality in the book manifested into poor teaching in the classroom. The influence of the textbook appears clear albeit detrimental. In similar, yet more positive findings, Mayer et al. (1995) found that the multiple representations used to explain addition in three different Japanese textbooks were effectively emulated in TIMSS video study data where teachers spent 54% of problem solving on making connections across representations.
There appear to be two factors at play here. Firstly, and perhaps most importantly for this study, the textbooks in these studies influence changes in teacher pedagogy either positively or negatively. Secondly, the perceived quality of the textbooks appears to reduce ‘buy in’ and could manifest what Brown (2009) referred to as improvisation of tasks that do not align with, or even undermine, textbook intent.

The cultural perception of textbooks in England is important to this study, as it may influence the way in which teachers adopt (and adapt) MNP if, for example, there is a cultural resistance to textbook use generally. Teacher beliefs in the integrity of the MNP curriculum package may also help determine whether changes made to the program are intended to be faithful to its principles, or to undermine them as they may not be valued.

**Teacher understanding and interpretation**

Whilst there are proportionally fewer studies on textbook use (Fan et al., 2013), there are interesting findings that appear to confirm the influential nature of regular textbook use on pedagogical approaches in the classroom, however the picture does not appear to be incontestably clear. The following papers are taken from a sample discussed by Harwood (2017), which highlight significant insights around the complexities affecting textbook adoption and adaptation. Furthermore, two of the papers he reflects upon are particularly relevant to this study in that they prompt important questions around teacher agency and the impact of teacher support materials. The two most pertinent papers to this study are discussed below, and highlight factors that can determine how agency affects the delivery of a prescribed curriculum package (research question 4).

**Collopy (2003)**

Rachel Collopy studied the impact of teacher materials on the pedagogical development of teachers. ‘Teaching materials’ here refers to a textbook that not only provides exercises for students, but also pedagogical development for the teachers themselves. She studied two elementary teachers using these materials for a year, but who received no additional training beyond the materials provided. The materials provided to the teachers were similar in nature to more typical textbooks used in countries like Japan and
China, where significant content for teachers is integrated into resources (Ma, 1999). Collopy highlights how complicated the factors are that influence the impact of teacher resources on affecting change in teacher pedagogy. She points to teacher goals, interests, values and expectations of curriculum materials, as well as subject-matter efficacy and teaching self-efficacy as key factors that inevitably sway the impact of teaching materials on a teacher-by-teacher basis. Each factor holds some degree of influence, and a varying degree between different teachers. This is, arguably, an acknowledgement of the uneasily quantifiable influence of teacher identity on teacher agency. A particularly interesting part of this study was the decision to allow for the participants to enact the curriculum on their own, with no guidance or support beyond the supply of the materials to reflect the reality of how new curriculum materials are often introduced in schools. This is important, as the MNP program has substantial training materials and opportunities available to teachers, however it seems unlikely that a primary school would send its entire teaching staff to training sessions that cost money per teacher to attend (Maths No Problem, 2019d), hence it may well be the case that a significant proportion of teachers delivering the programme have not attended training from MNP directly. This is a factor that will be investigated in this research and will be discussed in the conclusion chapter.

Another interesting component of this research is the choice of participants. Both are described as ‘veteran’ elementary teachers. It would be tempting to conclude with a novice teacher that any change in teaching style from the use of a particular set of learning materials would be influenced by their lack of experience in teaching, however such an argument cannot be used so readily here.

The textbooks themselves included a number of investigations (Collopy, 2003) and a notable shift away from procedural teaching of mathematics towards a more inductive approach not unlike the MNP approach referred to earlier. Each section included two pages of teacher support, including occasional scripting (word-for-word suggestions of what to say in the lesson), and teacher notes in response to questions from teachers in previous iterations of the guidance. Similarly, MNP’s curriculum package includes lesson plans that are several pages long and also include occasional scripting, making this study of particular interest.

Collopy’s results are interesting - one of her teacher participants repeatedly misinterpreted tasks, or altered them to the extent that the purpose of the task was negated
(for example, a task emphasising that speed of calculation was not important, was adjusted with speed emphasised). The teacher grew increasingly frustrated with the materials and eventually abandoned them completely. The teacher used her own experience and beliefs about mathematics as a tool to redesign tasks offered in the resources to suit her own convictions. Here, teacher experience and identity appeared to inform agency resulting in acting in opposition to the purpose of the guidance materials, essentially taking the intended curriculum and turning it into something entirely different. This brings several questions to light. Firstly, would the teacher’s enacted curriculum have been closer aligned to the intended curriculum without any guidance materials? Were instructions too explicit or narrow for this particular teacher? The answer is likely to be a range of factors working together, but for the purpose of this study, it is important to note that teacher agency directly affected the efficacy of the provided resources and severely misaligned the intended and enacted curriculum. A point of interest that directly relates to this study, is whether the guidance for MNP lessons is framed more as instructional, or whether more general philosophies are communicated to help teachers understand (and presumably align with) the purpose and intent of the programme.

In stark contrast to this, the second teacher in the study became entirely immersed in the teaching materials and enacted them as the resources intended, despite it being a ‘dramatic’ change in her approach. Her justifications for her change in practice aligned with those outlined in the intended curriculum. A key difference between the two participants was their own self belief in their mathematical ability. The participant who abandoned the resources felt very confident in her abilities, whereas the participant who embraced the resources and adapted her teaching style accordingly was much less confident, and wanted her students to feel enjoyment where she had not. She saw the resources as an improvement to the norm, whereas the other participant did not. Again, the interplay between teacher identity and agency appear to have shaped how closely the teacher’s enacted curriculum aligns with the intended curriculum. Here it appears that the integrity of the textbook was seen as high, rather than low, despite it being exactly the same book. Hence what is important here is the perception of integrity by the teacher themselves, rather than any objective or external value of quality. Whilst it appears only one teacher demonstrated a willingness to change, it is important to consider that both teachers were
willing participants in the study, suggesting that despite one teacher later abandoning the resources provided, they were not entirely closed to the idea of adopting new resources and strategies at the beginning of the study.

Remillard and Bryans (2014) also conducted a longitudinal two year study of textbook use in an elementary school, finding that three teachers (Zoe Kitcher, Peter Jackson, and Kim Reston) provided contrasting examples of how orientation toward a curriculum influenced its use. Kitcher, with views compatible with the curriculum, used it as her primary resource and followed the guide closely. Jackson, who held more traditional views on mathematics teaching, used the curriculum intermittently and structured activities differently than suggested. Reston, sceptical of packaged curricula, used investigations alongside other resources and adapted it to fit her established teaching practices.

In further school-based case studies, Remillard (1999, 2000) reported similar findings while exploring how a reform-oriented textbook can contribute to teacher learning. She examined the learning of two fourth-grade teachers during their first year of using a new mathematics textbook, focusing on activities in teaching and using texts that created learning opportunities. These activities included analysing students, mathematical tasks, and making decisions on how to proceed, suggesting materials most likely to foster teacher learning are those that engage teachers in these processes.

The study is set against the backdrop of mathematics education reform in the United States, following the National Council of Teachers of Mathematics' (NCTM) 1989 publication of Curriculum and Evaluation Standards. These standards advocated for increased emphasis on mathematical reasoning, understanding, and problem-solving, and decreased emphasis on memorising rules and computational procedures. The reform led many school districts to adopt new textbooks, aiming to revamp the content and pedagogy of mathematics instruction.

Remillard (2012) and Remillard and Bryans (2014) challenged the notion that textbooks alone can lead to substantial curricular change, and pointed towards teachers' beliefs and knowledge about teaching, learning, and subject matter influencing their decisions more
than the content presented in texts. This recognition of the need for substantial teacher learning in the process of pedagogical change aligns with the focus towards teacher development and well-designed curricular guidance that are embedded as part of the curriculum package by MNP.

Akin to Collopy’s study, Remillard (2000) found that teachers reacted differently to textbook use depending on their experiences and setting. One teacher, Catherine McKeen, taught at a school with limited focus on professional development and had conventional ideas about mathematics learning, centered on computational mastery. The other teacher, Jackie Yarnell, taught at a school that had become a professional development school and was actively involved in several related activities, leading her to reexamine her beliefs about teaching, learning, and mathematics. Remillard found that teachers’ reading of the text differed significantly, and their interpretations were influenced by their own perspectives, leading to different opportunities for learning. Catherine viewed the reform agenda as adding additional topics such as problem solving to the existing mathematics curriculum and saw the text as a source of new mathematical tasks. Jackie, on the other hand, used the text as a source of mathematical and representational ideas, from which she adapted and invented her own tasks. This difference in approach significantly influenced their learning experiences.

Remillard concluded that tasks in the textbook initiated classroom activities, but it was the teachers who actively engaged in reading and acting on these events. The growth experienced by both teachers occurred when they engaged in pedagogical reading and decision-making. This suggests that textbooks might contribute more effectively to the enacted curriculum by fostering teachers’ reading and subsequent decision-making, showing that while textbooks can provide a framework and resources for mathematics instruction, the actual learning and adaptation of the curriculum depend heavily on the teachers’ engagement, interpretation, and decision-making processes. Teachers' active involvement in reading, interpreting, and applying the curriculum material in the context of their classroom and students appears critical for effective teaching, learning and development.
Whilst these findings are interesting, it is important to consider how this may or may not have changed since more integrated digital resources have been introduced to the textbook market. As mentioned previously, ‘textbooks’ as a term can mean different things in different studies. In Collopy’s study, the associated resources were detailed written teacher guides that encouraged note taking by the teacher. However, MNP’s program is more substantive, including lesson guides, slides, digital video support, online access to detailed scripted components of lessons, detailed training programs for subject leaders and a well-resourced website that includes blogs around the general philosophy and implementation of MNP.

In the paper, “Keeping an Eye on the Teacher in the Digital Curriculum Race”, Remillard (2016), focuses on understanding the impact of digital curriculum resources on teachers and their teaching practices. The paper examines how these resources can organise content and engage learners in exploring mathematics in new ways, while also presenting new challenges to teachers. Remillard notes for example, that teachers must navigate two types of digital environments: those shaped by digitally-designed curriculum resources and those formed by the vast collection of potential curriculum resources available online. Digitisation and the web have expanded the availability and access of new resources, creating new challenges for teachers - particularly around nonlinear structures that develop from hand picking resources from different authors - whilst they allow for personalisation and multiple modes of engagement by the user, making them more interactive than print materials, the vast quantity of digital resources available, particularly in mathematics, presents challenges in determining the quality and appropriateness of what is used, as they vary in their curricular aim, nature of prior learner experience, and are often not designed to be adaptable or personalised.

Lindorff, Hall and Sammons (2019) studied the impact of a Singapore-maths style textbook called Inspire Maths (and associated resources that were similar to those provided by MNP) in England in a study closely aligned both in its locality, demographic (primary school) and subject matter to this study. Furthermore, the textbooks, like MNP, are based heavily upon existing Singapore textbook series. The study took place across 12 schools that were randomly assigned to either adopting the textbook and associated resources
immediately at the start of the school year, or a delayed adoption at the beginning of the second term. The study focused on a number of areas including pupils’ knowledge and skills, their attitude toward mathematics, classroom practice (via observations), teacher perspectives (from interviews) and intervention specific professional development (to support the use of the introduced textbook) with a view to observing the changes that occurred after the introduction of the textbooks and associated resources and training. Interestingly, in almost all of the schools, the changes that occurred after the introduction of the textbooks aligned, and were sustained into term 3 of the study. In other words, in the schools where the textbook was immediately introduced, the styles of pedagogy and attitudes of teaching aligned to the books, but different significantly to those schools where the textbooks had not been introduced, yet after the introduction to the second group in term 2, teaching across all the schools began to align and conform to the intended curriculum. Notable changes included a change in teacher language and vocabulary during instruction towards more frequent use of technical vocabulary and a more dominant use of follow-up questions involving asking how or why something was true in maths (moving beyond accepting answers alone), and a general improvement in praise, warmth and enthusiasm towards students - although it is not entirely clear how these fuzzy concepts were measured in the study beyond anecdotal commentary and example quotes. The researchers noted that whilst there were minor variations between teachers and schools, there were no ‘pervasive patterns’ of differing teaching approaches. Using the texts also appeared to improve teacher organisation, with many teachers in the study beginning to use ‘caddies’ of manipulatives (physical tools to represent concepts) on desks and worksheets more readily available at the start of class. Perhaps most notable was the change in teacher perceptions towards student ability, which shifted from a more global pupil focused attitude (ie, a student is unable to grasp mathematics) towards a more localised concept specific perception (eg multiplication is an area of development for this student) which the researchers placed partial responsibility for upon the textbook series’ philosophy of mixed ability grouping on tables - something none of the schools previously did before introducing the books. Again, these changes appeared to be widespread across the study group (12 different schools) and came about after the introduction of the textbooks. Whilst it should be noted that the research for this particular paper was commissioned by Oxford University Press (a subsidiary of which published Inspire Maths), the authors are University researchers
and whilst the results seem incredibly positive in terms of showcasing the impact textbooks can have on teachers and pedagogy, there were some interesting elements of the study that point to the influence of the political landscape in shaping the enacted curriculum.

In a further longitudinal study, McNaught at al.(2010) undertook a 3 year study into the implementation of a mathematics textbook in the US, looking at two specific measures: content implementation and presentation implementation. Content implementation was concerned with how much of the textbook content was actually delivered, and by proxy how much was left out during each school year. High content fidelity would therefore be attributed to teachers who taught textbook content in full, but the order of content was also taken into consideration (specifically, whether a teacher delivered their content in the same order as the book recommended). Conversely, presentation implementation was concerned with how the content was presented to students, and the way in which students were expected to engage with that content during lessons. Teachers were asked to complete a ‘textbook use diary’ and researchers observed three lessons per teacher across each academic year. Most interestingly in this study, the presentation implementation was measured in part by interviewing the authors of the textbooks themselves to gain a clearer insight into the implementation intent. In other words, textbook authors were given a voice in the study to explicitly state how they intended the content in their books to be taught. This added layer of intended curriculum is intriguing as, for the researcher at least, they are able to confirm or dismiss any interpretation of anything considered ambiguous in the books themselves and add a further layer of communication between author and researcher, offering a kind of triangulation of intent between author, textbook and teacher. The ‘textbook-use diaries’ captured whether teachers taught each lesson (listed in the book) primarily from the textbook itself, with supplementation, from alternative resources, or did not teach the lesson at all. Thus the study managed to capture an array of information around textbook use in the classroom from multiple perspectives, and the results are equally interesting. In general, teachers tended to spend fewer lessons on each topic than was recommended in the textbook, and assigned far fewer problems to students than the number recommended by the authors. In fact this became even more pronounced during the second year of the study, and the trend continued into the third and final year. Problems were categorised into four areas: Modelling, Organising, Reflecting and Extending.
The latter two categories were consistently the most underused by teachers, and by the third year of the study, no teachers were assigning problems in the ‘extending’ category at all. Regarding the extent of textbook implementation, the study found that roughly a third of textbook lessons were not taught, but of those that were, 51% used the textbook as the primary source, 31% used some supplementation, and 18% used alternative resources. Across the 109 teachers participating in the study, the manner in which lessons were taught (presentation fidelity) was significantly less consistent with the author’s expectations than was the content of lessons taught, with a moderate relationship between higher content coverage and higher presentation fidelity. In other words, teachers that used the book more taught lessons more aligned with author expectations. The authors speculated that much of the variance in implementation of the textbook was related to the influence of testing in each district - topics more likely to be tested were focused on more heavily at the detriment of other topics.

McNaught’s 2010 paper is insightful and a helpful contribution towards the research aims of this study. Firstly, it highlights a successful strategy towards measuring the effectiveness of a textbook in the classroom, focusing on both content coverage and content delivery, and offering ways to quantify the latter through categorisation of different problem types (and assessing how much each was used), and speaking directly to authors. Whilst this study will not take place over years, and a longitudinal view of the textbooks will therefore not be possible, it will be helpful to consider how teachers might deviate from the Maths, No Problem! Textbooks, and what decisions they make around each exercise and chapter in the books. For example, a teacher might purposefully skip particular exercise types, or focus more heavily on others. They could miss out parts of the book that do not closely align with their own curriculum plans - or might adjust their curriculum plans to align more closely with Maths, No Problem!. There may be patterns of behaviour regarding what is and is not opted into across MNP’s resources that could be captured through observing content coverage and textbook reliance. Beyond McNaught’s methodology, the findings of the study have equally interesting implications. Whilst teachers tended to use the textbooks either as a dominant or supplementary resource in class for most of their lessons, the way in which they used them was often significantly different to the author’s intent. This feels hard to capture, as a fuzzy
concept such as ‘intent’ is difficult to quantify, however what is clear is that teachers almost unanimously shied away from certain types of activity promoted in the book and relied more consistently on others. ‘Modelling’ and ‘organising’ problems are arguably more traditional or common mathematical tasks and align with common teaching observed in Ofsted’s 2012 Made To Measure report on how mathematics was taught in England at the time (Ofsted, 2012), whereas ‘reflecting’ and ‘extending’ appear to be more open ended or less formal. For example, a reflective task might not have one fixed answer, and extension of a task or concept might take students in different directions.

It is interesting that the common lack of interest in specific tasks, appear to align with the findings of Eisenmann and Even (2011) who also found teachers regularly opted out of task types rather than subject matter. It seems that the teachers in McNaught’s study were not selecting task types based on their perception of what their students would enjoy most (Smarogrinsky’s finding) simply because so many teachers made the same decisions about what to opt out of. It could perhaps therefore be related to teacher preference or training needs - it is not clear what support ran alongside the textbook in order to help teachers implement certain tasks or explain the pedagogical reasoning behind doing them. Perhaps then, the training program and support provided in the MNP package will improve what McNaught calls presentation fidelity, and in turn ensure a closer alignment of curriculum intent and curriculum enactment. This also prompts questions around how tightly the MNP program is tied to the DfE’s Programme of Study for maths - which itself dictates not only what should be taught in each key stage, but also what should be taught in each year group for key stages taught at primary school (DFE, 2014). For example, if there are sections of the MNP content that do not align closely to what is stipulated in the DfE Programme of Study, it seems probable that teachers may opt out of some content or activities. Furthermore, McNaught’s speculations about the effect of regional testing structures in the US also resonate with this study, with Year 2 and Year 6 students in schools in England having similar national tests in mathematics that may well affect the way in which teachers utilise the MNP program and resources or deprioritise traditional teaching models and structures (Bradbury et al. 2021; Ward & Quennerstedt 2019, Bolden & Newton, 2008).
Whilst much of the previous research offers a lot of insight into curriculum reform and textbook use in mathematics, it is important to also review and compare more recent findings in a digital era where internet resources and communities are culturally embedded in modern living. Research by Van Steenbrugge and Ryve (2018) analysed the impact of curriculum resources (textbooks and teacher guides, referred to in their entirety as *curriculum programs*) over a number of years in Sweden on planning and enacting the curriculum by monitoring 11 teachers who taught the same classes over sequential school years. Contextually, teachers in Sweden rely heavily on textbook use (Boesen et al, 2014) yet authorities in Sweden do not formally quality control or encourage the use of either textbooks or broader curriculum programs. This in turn has resulted in curriculum programs being largely authored by teachers themselves alongside commercial publishers. This state of play is similar to that of the English education system prior to the mastery agenda introduction in 2014.

Teachers involved in the program were observed and interviewed by the researcher and regularly completed diaries that captured how they modified the content of the textbooks during their teaching, and how they might alter lessons in the future after reflecting upon each lesson taught. Van Steenbrugge found that teachers predominantly began by using the curriculum programs almost exactly as intended by the authors, which included some fairly radical shifts away from traditional ‘textbook exercise’ dominant teaching to more explicit didactic methods as well as more interactive whole class discussions. However, the students in several classes struggled to cope with the new changes in approach, particularly the more frequent teacher interactions and steering. The teachers themselves found that managing so many more transitional moments in class between independent activities and whole class teaching was particularly challenging. Within the teacher guidance provided, teachers particularly valued the instructions on how and when to move from each segment of teaching to the next, as they struggled with timings for what some considered to be a larger number of goals per lesson than they were used to. Similarly, teachers and students struggled with the new lesson structures, where lessons were now multi-phased and involved teacher facilitation of discussions about and explanations of student work - although they mentioned that prompts for class discussions were useful to help guide discussions more successfully. Finally, examples of student work were also highly valued as
they helped teachers discuss student methods when their own classes had approached some problems in a more uniform manner. Teachers generally found particular tasks difficult to manage or implement. Those task types tended to be more open and discourse based, which aligns closely to the findings of Eisenmann and Even (2008, 2011) as well as other similar studies (Wood et al. 2001; Leinhardt & Steele 2005, Sherin, 2002).

As the study reached its final year, teachers had become much less dependent on the teacher guidance and relied more heavily on the provided lesson resources (slides) to indicate how to teach. Slides were developed for the program as support materials but in fact took on a central role as 80% of participants claimed they used the slides as their main source of planning in year 3 of the study. Teachers in the third year of the study often mentioned in interviews that they did not focus on any of the additional support for the lessons, despite the fact that the slides were not intended as explicit guidance for the lesson, and were therefore circumventing much of the advice and support offered by the full curriculum package. Interestingly one teacher confessed to printing out the slides and writing notes on them as their planning technique, but did not read any of the notes that were already provided in the lesson guidance suggesting a way of feeling more ownership over the lesson in what is arguably a more arduous task than reading the notes provided by themselves.

In interviews, teachers cited that the dominance of slides over other more explicit guidance materials was largely attributed to time saving. For example, in the first year of the study, many teachers highlighted how long it took to prepare lessons in this new style and diligently read all the suggested reading for the approach and for specific lessons, however it appears that practicality and the realities of day-to-day teaching (teachers specifically mentioned how other subjects that they taught were impacting the amount of time they could dedicate to lesson preparation) meant that teachers found their own way towards making the resources work as best they could in their own circumstances. This specifically draws interesting attention to the impact and effect of workload implications and agency over time. It appears that the teachers in this study began to shed what they considered extraneous elements of the lesson guidance to both try and preserve what they perceived to be the core elements of the program, and to make the program work within the framing of
their own work-life balance. A combination of factors appear to be at play in this study, but each can still be divided into the aforementioned categories. First, it seems that in struggling to enact some of the activities promoted in the textbook, teacher interpretation and understanding of how to deliver the activities was potentially misaligned with the intended curriculum, which in turn caused difficulties in class, leading to a perception that the activities wouldn’t work with ‘my students’, falling into the ‘perception of students’ category. Finally, this seems to result in a shift in perception of the integrity of the textbook - certain activities appear not to work and become side-lined by the teacher.

Relating directly to this study, capturing teachers using the MNP program a few years into its use is potentially more useful to determine how agency has affected the implementation of the intended curriculum over time. This would possibly come at the cost of losing insight into how teachers initially adapted to the program, but would instead capture how they had settled into it and developed their own ways of working with or adapting it - or alternatively, are teachers still using the program entirely as prescribed after a few years of use?

It’s clear that across several of the aforementioned sources, tasks that involve more interactive discussion with students to explore mathematical concepts are often dropped over time, left out altogether, or attempted and highlighted as difficult to implement. Despite this, the value in these types of tasks both from the resource authors and wider academic community is clearly high. For example, in England, the Association of Teachers of Mathematics states “The power to learn rests with the learner. Teaching has a subordinate role. The teacher has a duty to seek out ways to engage the power of the learner.” and “Encouraging a questioning approach and giving due attention to the ideas of others are attitudes to be encouraged.” as two of their guiding principles (Association of Teachers of Mathematics, 2018). It appears the rejection or abandonment of this kind of teaching stems at least in part from the difficulty in facilitating it successfully. Arguably, a teacher changing the tasks, content or sequencing in their teaching is easier than changing the way in which that content is delivered, especially when it requires facilitating productive conversations and enquiries with the students themselves. Indeed, Leindhart and Steele (2005) studied what they referred to as ‘instructional dialogues’ in depth and observed a number of subtle
skills and micro decisions involved both in planning and delivery of successful practice. Leindhart also conceded that the ‘intellectual climate’, which might be more commonly referred to as the classroom culture, is a ‘critical issue’, and that creating a positive environment that enables ‘instructional dialogues’ to thrive and be successful, takes time and skill - it cannot be developed quickly, and so any teacher adopting strategies that require it as a prerequisite, will likely struggle. This all leads to another useful consideration for this research. Are ‘instructional dialogues’ being adopted and implemented successfully, or are they being deprioritised or removed by teachers similar to other recent studies that sit in the same research area? Furthermore does the experience of the staff delivering the program have any correlation with this?

Perception of Students

A further interesting study by Eisenmann and Even (2011) looked at the way in which a single teacher taught the same lessons, using the same resources, to two different classes. What Eisenmann and Even found was that the teacher varied their teaching across the two classes substantially, and that entire units were skipped - more for one group than the other. Furthermore, the teacher opted out of several activity types for each class, claiming that the students would not be able to work well with particular tasks, or that their knowledge was not at a level where some tasks would be beneficial to them. Whilst it is difficult to verify these reasons, it is clear that the teacher was confident enough to assess what was, and was not suitable for her classes based on her knowledge of them. Whether her decisions were appropriate or not is somewhat irrelevant to this study, but what is pertinent, is that the intended curriculum was substantially adapted by the teacher in different ways for each of her classes - suggesting that the changes made were less about personal pedagogical preferences, as Collopy (2003) and Smarigrinsky et al. (2002) suggested, but more about how the teacher determined the suitability of the resources for the students themselves - or at least a combination of both. Specifically, the teacher omitted a particular type of task for both classes, as well as adapting or removing some elements for only one class or the other. In other words, the teacher in Eisenmann and Even’s study adapted different parts of her provided resources for different students rather than consistently removing or changing particular components.
Eisenmann identifies the primary reason for these adaptations as the teacher’s attempts to be attentive to students and their mathematical behaviours and performance. Put simply, the teacher was reacting to how the students themselves reacted to the tasks she gave them. If students were less responsive or enthused by a particular activity type, that type of activity fell out of favour with the teacher and was emphasised less or skipped entirely from the teaching resources. This is distinct from Collopy’s study, in that the motivation to adapt comes from how students receive the task rather than the teacher’s perception of its worth.

The units not taught to either class included an investigative unit involving computers and an enrichment unit. Whilst the paper does not explain why those were not used, it perhaps could have been related to accessibility to IT and time constraints. Similarly, the investigative elements of the teaching resources were used more for one class than the other. This element of teacher agency is particularly interesting - could it be then, for teachers implementing ‘Maths, No Problem!’, teachers in the same school each adapt the provided resources in different ways depending on their classes? Or is it more likely that teachers adapt a particular component of the resources consistently across a school. For example, a particular teaching element that is suggested for all lessons in MNP might be replaced or removed by all teachers in a department as policy, rather than individuals tailoring individual lessons or components possibly without consultation of other deliverers of the program. Indeed, Eisenmann cites a number of motives for teachers to adapt the intended curriculum,

Two further studies by Even and Kvatinsky (2009) and Eisenmann and Even (2008) used the same method of observing a single teacher delivering identical materials to two different classes. They found in both cases that the teaching experiences for each class were roughly the same, albeit with some minor adaptations to the source materials. In all three of Eisenmann’s studies, teacher agency has proactively adapted the intended curriculum either to align with pedagogical preferences, or to tailor to the perceived needs and suitability for the students in front of them. This is also a cautionary consideration in that for my own research, how a teacher teaches an observed group may well differ from how they teach other classes if they teach different groups maths in their school.
Political Landscape

Elements of local (for example, within schools or multi-academy trusts) and national (Governmental level) politics inevitably influence the enacted curriculum in schools (Yerushalmy, 2014). In the case of Lindorff, Hall and Sammons, there was a small cluster of teachers in their study in England who made notable substantial deviations from the intended curriculum by, for example, rearranging the sequencing of content to match upcoming school assessment dates, making significant substitutions of content with their own ideas and resources, or, at perhaps its most extreme, reducing the amount of use of the textbooks in lessons in keeping with a perceived wider school ethos around ‘child-led learning’. When interviewed, these teachers hinted at external pressures around covering content at a particular pace, ensuring that enough of the curriculum was covered by certain calendar dates, and target related pressures around assessment in general. At face value, these can all be attributed to the local political landscape - external pressures to the classroom, but internal to the school. Reys et al. (2006) found similar results in that half of the teachers they surveyed were influenced heavily by state-determined materials and assessment dates which undermined their engagement with textbooks.

In fact, in England often these assessments are related to externally enforced assessments at a governmental level such as SATs or GCSEs (Bradbury et al. 2021; Ward & Quennerstedt 2019, Bolden & Newton, 2008).

Furthermore during one on Lindorff’s interviews, a teacher stated that textbooks made it easy for teachers to ‘not think’ and just do ‘what the textbook says’, but objected to the idea saying ‘I don’t want to do that’. This is an example of a teacher perceiving prescribed resources as deskill the teacher, or removing their autonomy or sense of teacher identity. It appears there is a tension between adoption of textbooks and a perception of ‘forced’ adoption from a higher level (School or Government).

The teacher has interpreted the intent of the resources as to remove their agency, and rejected it. This perceived level of control is also emulated elsewhere around the world. In the US, Taylor (2013) refers to the ‘teacher-proof curriculum’ - a phrase she says is commonly used and that, by its very nature, suggests an intent to remove agency and one
that feels harmful and derogatory towards teachers. This framing raises questions around the textbook authors’ perception of the role of the teacher in the classroom, and by proxy, how the textbook is intended to be used in the classroom - and how that affects design. Specifically, is MNP designed to be teacher-proof or framed from a position of authority, or is it designed to be perceived as teacher-enabling (does it actively encourage agency and adaptation?). In his paper ‘Challenging the authoritarian role of textbooks’, Michal Yerushalmy (2014) points to newer digital textbooks with online support offer more liberal perspectives of participation, flexibility and personalisation by promoting a less linear and more interactive experience for teachers but highlights that more ‘traditional’ models of textbook are more rigid and authoritarian in both their design and perceived inflexibility.

More widely, textbooks more generally are described by Lerch et al. (2017) as the lens through which models of society, institutionalised understanding and conformity are formed and communicated by the authority of national states and other elites. In Brazil for example, textbooks are the main resource for planning classroom activities and are cited by the ministry of education as directly helping conceptual development of teachers by helping them build their knowledge and encouraging autonomous learning and self-assessment. (Borba and Selva, 2013). Politically the textbook is held in high regard and is considered influential in directly shaping pedagogy. The direct intervention of government in England to introduce an approved ‘Singapore style’ textbook series alongside a wider mastery agenda and curriculum reform all at once has, perhaps unsurprisingly, caused similar concerns around control and deskillling teachers and ultimately a rejection in principle to the adoption of such ‘approved’ methods and their inferred superiority. This concern and scepticism is perhaps best captured in England in Professor Ruth Merttens’ (2015a; 2015b) writings for the Guardian and the Mathematical Association where she declared the reforms as an unviable tool of Governmental teacher control.

There are inevitably further complications too. Taylor (2013) highlights that even though we may be able to categorise some of the reasons why teachers adopt and adapt textbook resources, these actions are often time dependent - in other words, a teacher is likely to adjust how much they adhere to a textbook based not only on factors such as textbook integrity and social landscapes, but that those motivations will differ at different moments in
time - and teacher adaptations may not even be deliberate (Drake 2006; Drake & Sherin 2009). In fact, Taylor’s research found correlation between a teacher’s time spent with a textbook and how much they adapt the tasks and instruction guidance within it - teachers stayed closely aligned with textbook exercises initially, but over time grew further towards adaptation of tasks as they gained more confidence and experience with them.

Interestingly Taylor found that task adaptation and supplementary tasks were more likely than textbook task replacement - although her sample included only a small number of teachers. Taylor suggests that we shift away from the derogatory narrative of a ‘teacher-proof curriculum’ and towards a more collaborative model of a ‘curriculum-proof teacher’, whereby teachers are seen as inevitable adapters of curriculum who will maximise the effectiveness of any curriculum they adopt. As part of this argument, Taylor highlights that some curriculum models do take an approach that is intended to be flexible and purposefully adapted (the ‘mutual adaptation approach’, rather than a stricter fixed inflexible ‘remote control’ approaches. Where MNP sits on this spectrum will be interesting, both in terms of how it is intended to be used (is it intended to be flexible or inflexible?), and how it is enacted (does its intent match with how it is interpreted?).

Social Landscape

A further paper considered particularly relevant to this study is Smagorinsky, Lakly and Johnson (2002) who investigated the tension between University training for teachers and real life practice in schools, which they cited respectively as being Liberal, and Conservative. Universities often took liberal views to pedagogy (stressing the need for diversity of practice and direction), whereas schools tended towards conservative approaches (protecting what has traditionally been done). Smagorinsky et al. followed the pedagogical journey of six new teachers over both their training and induction years as they navigated this tension and observed whether personal teacher identity changed or became a source of conflict between the two approaches of teaching as they attempted to enact a student-centred approach in a district that supplied a heavily scripted curriculum that closely aligned to school assessments. One of the teachers, Andrea, was closely monitored every two weeks for two years with data collected via observations of teaching and post-lesson interviews.
Andrea found the prescribed materials dull and uninspiring but teaching from them was mandated. She initially developed a hybrid approach of teaching the materials to preserve some of her own pedagogical values (which differed from the prescribed texts) whilst ensuring that the material covered met the demands of the standardised tests and prescribed resources. However, this hybrid approach began to lean more heavily on her own preferred style of teaching as she learnt that colleagues were also resisting using the textbook as it was intended. She was soon redesigning tasks or creating new ones entirely, whilst always ensuring that the content matched the prescribed curriculum. Despite this, she still felt unhappy with her teaching, as she found much of the curriculum dull (as did her students) despite her efforts to engage them with more personalised tasks and teaching approaches.

What this study highlights is that not only do a teacher’s personal beliefs appear to strongly influence the ways in which they adopt prescribed teaching resources (in alignment with Collopy, 2003), but also the interaction with peers and their respective beliefs and practices. As Andrea found like-minded peers who also interpreted the prescribed guidance in more liberal ways, she found new confidence to further depart from the intended ways in which the prescribed resources suggested she teach. And so despite the fact that both the content and ways in which it was meant to be taught were both mandated to her, she found ways to introduce her own teaching identity and pedagogical beliefs without abandoning altogether the provided content. Between this case and the study by Collopy, it’s clear that teacher agency and in fact the agency (and influence) of colleagues appear to directly affect the enacted curriculum whether the teacher’s beliefs are generally aligned to the intended curriculum or not. More generally, the context in which the intended curriculum is enacted is important and affects how closely the enacted and intended curricula resemble one another. This aligns with conclusions made by Kilpatrick (2012, p.569), who determined that “at the crux of any curriculum change is the teacher. The teacher needs to understand the proposed change, agree with it, and be able to enact it … situated in a specific educational and cultural context”. Hence for the purpose of this study, it will be important to consider the context in which MNP is being enacted both nationally (which will be identical for each
school participating in the research, however the perception of it within schools may differ) and locally (the school-based context).

In England, Bolden and Newton (2008) investigated the epistemological beliefs of primary teachers teaching mathematics through observations and semi-structured interviews of teachers over a six month period, and found that whilst the teachers had a desire to adopt a more investigative, enquiry based approach to mathematics teaching, they felt external pressures from their school leaders and school culture were incompatible with these approaches, focusing instead on assessments (SATs) and curriculum coverage, fearing they would be held accountable for any change in approach to the norm, or shift in focus away from teaching to the test. Whilst the study was small (only three teachers took part in the study), the results are echoed more substantially by Bradbury, Braun and Quick (2021) who’s sample in England included interviews of 20 primary headteachers, and a further online survey of 288 primary headteachers who, in the researchers’ words, ‘careers depend on their SATs results’ primarily because a schools’ SATs results are a key part of the Ofsted inspection process and provide information for national school league tables. Comparatively poor results can trigger Ofsted inspections which can in turn result in schools being converted to or adopted into academies or academy trusts, with a new leadership team in place. The study looked at school organisational behaviours in response to the pressure of high stakes tests (SATs).

The paper highlights the significant impact of SATs assessment schedules on schooling, specifically including widespread adoption of grouping by ability sets - particularly for maths, an intervention culture of removing students from classes to focus on maths and English (the subjects assessed in SATs), an accountability regime (directly echoing Bolden and Newton) and educational prioritisation of ‘borderline’ students, who often receive additional work, interventions and general attention from teachers. Conversely, this means that those students below the borderline are not given more support in what is arguably a perpetual issue related to ‘closing the gap’ regarding underachievement in education (Bradbury et al. 2021; Ward & Quennerstedt 2019, Bolden & Newton, 2008).
Closing comments

The literature review has highlighted key studies around the intended and enacted curriculum of textbook design focus on case studies of a small number of teachers, often contained within one or two schools. This in turn has allowed for more detailed qualitative data collection that gives insights into some recurring themes that affect the way in which teachers deliver the curriculum which will inform the methodology section with regards to what and how data will be captured in order to answer the research questions. Furthermore, a number of tools have been identified that each offer ways in which to capture task intent in textbook design which again will be used to inform the methodology for this study.
Chapter 4: Methodology

This chapter will begin by discussing the theoretical framework in which this research is positioned in order to demonstrate how the research questions and subsequent methodology were developed. The chapter will outline how decisions were made to select appropriate data collection tools and the methods used to gather and interpret convincing, valid and robust data to answer the research questions effectively. Data samples and sizes will be discussed and defined, as well as any ethical considerations and implications for this research project. I will conclude with a discussion around how the data collected will be analysed.

Epistemological Framework: The Interpretative Paradigm

In order to gain suitable information to answer the research questions for this study, the data collected will be used to inform a comparative study of the intended and enacted curriculum of the mathematics curriculum package provided by ‘Maths, No Problem!’. This curriculum package is a set of resources for teachers to use in the classroom to deliver lessons in a sequence written by ‘Maths, No Problem!’ which aligns itself with the National Curriculum requirements for maths in England. It includes lesson plans, textbooks, student workbooks, slides and online materials for teacher development. As such, there will be layers of interpretation by different stakeholders throughout this study such as how I interpret various sources of data to develop an understanding of the MNP intended curriculum, how a teacher interprets materials in the classroom, how I interpret their interpretation through observation notes, and potentially how they themselves interpret their own teaching in reflective interviews. Further to all of this, I will be making comparative observations of the extent to which teachers adapted or deviated from the intent of the MNP curriculum.

Epistemologically, this in essence ensures that any perceived knowledge I have around the intended and enacted curriculum will be a social construct, created through language, interaction, and behavioural observations - open to analysis and deconstruction, but inevitably also open to interpretation. Specifically, the enacted curriculum will be live
teaching recorded by hand with only myself as the external observer - and so my written
records of those lessons will be the only physical evidence of what happened regarding
teacher behaviours, actions and responses. The motivation and strategic intent of
behaviours are just as important as the actions and behaviours themselves. In other words,
why things are said, asked, ignored or exaggerated, is arguably more important than what
things occur. ‘Truth’ in this study is constructed differently across the two areas of research.
When studying the intended curriculum, much of the evidence to support what MNP want
teachers to do, or how they think about mathematics pedagogy, is explicitly documented in
their own materials, training and website - through necessity - it is an integral part of the
success of their product to be able to ensure teachers deliver their program as accurately or
closely as possible to their intent to preserve its integrity. Conversely, evidence to support
interpretation of the enacted curriculum will be developed through conversation, language,
actions and behaviour, but also through an underpinning social structure alongside an
evolving sub-structure within individual lessons themselves. These elements can be
categorised holistically as discourse which herein refers to the conversations, language,
actions and behaviours observed in the classroom and in teacher interviews.

**Researcher Bias and Positioning within the Study**

Under an interpretive framework, there is a limit to the feasibility of true separation
between researcher and research. Indeed, the interpretive approach fully acknowledges
subjectivity, and views the interpretation of what is ‘true’ as something that depends upon
the relationship between study participants and research, and reaction to characteristics
and positioning of each party (Finlay & Gough, 2008). As such, to ensure such limitations are
in no way invisible, it is important to enable transparency and acknowledge potential bias
and underlying assumptions prior to the research being undertaken. This exposition of self,
or reflexivity is indeed part of the process of data production. Specifically, I, as researcher,
have a substantial amount of experience as a mathematics educator in secondary schools,
and as a teacher trainer at that level. I am positioned advantageously within this arena -
being a mathematics educator ensures familiarity in pedagogical structures, strategies and
purposes, as well as pedagogical language and behaviours, and I have a strong
understanding of the conceptual structures of the mathematics and mathematical
vocabulary being taught - this allows me to more accurately interpret what is happening in
the classroom, and identify behavioural intent in both teacher questioning, and response to
students, as well as being able to identify how well students are understanding or not
understanding what is being taught. Similarly, I carry personal historical knowledge,
experience and understanding of the introduction of mastery and Singapore style
mathematics pedagogy through my own experiences, and my observation of (and
interpretation of) how mastery approaches have been received and understood by the
secondary mathematics profession. Whilst these experiences and this knowledge may seem
advantageous, it is also a lens through which confirmation bias and false assumption can
creep in, even subconsciously - but acknowledging I am an integrated element of the
mathematics education ‘world’, even when studying others, is a step towards rigour and a
more robust awareness of self with regards to this study.

This awareness of self and my proximity to maths education does not end at the point of
research design, but continues throughout the data collection process, subjecting myself to
the same level of scrutiny as my research subjects as a method of validation and
legitimisation of the research design process (Finlay & Gouch, 2003: 5). Specifically, I will
seek to address or highlight where I am interpreting actions or intent based upon my own
experiences of teaching rather than any specific commentary or cue made by teachers
themselves when observing, and I will make space in this research to outline in detail how I
interpreted elements of task design or question design when analysing both the MNP
textbooks and workbooks, and any questions or tasks aimed at students in observations.
Hence, I will value my experiences and the advantages they give me in this research, but I
will ensure that I am transparent in where I have used those advantages in an attempt to
better understand the why alongside the what.

This awareness of self involves a further dynamic to consider. Not only should I be aware
of my own beliefs and experiences, and how they interact with my interpretation of data, I
also need to be acutely aware of my interpersonal influence on others (Hesse-Biber & Leavy,
2006). That is, I must acknowledge the influence and interpersonal dynamics between
myself and research participants (observed teachers) - and how this shapes the creation of
knowledge in a study. For example, should an observed teacher have a fractious relationship
with me, or feel as though they were intimidated, or under scrutiny or judgement, then outcomes regarding how a teacher behaves will likely differ considerably to an environment where teachers are at ease in my presence.

Green and Thorogood (2018) add a further dynamic to the influence of ‘self’ in research, which is the consideration of social setting, and the potential impact of where the research is conducted on knowledge creation. Contextually, researching teacher pedagogy outside of the ‘natural environment’ of the classroom would likely impact the way in which teacher and student responded to one another, but similarly, should the research be conducted in situ, then it would need to be acknowledged that both teacher and students are subject to the norms and limitations of the school system, which affects behaviours and strategies. For example, students may get restless towards the end of a lesson, because they are aware of the timing of a break or lunch, or teachers may begin lessons hastily should they be expected to be performing a break duty leading right up to the beginning of the lesson itself.

Bigger contextual influences may also be at play, such as how teachers feel about the context in which they have become teachers of mastery (has the MNP program been imposed on them from a senior figure in the school for example?) which again may impact on how MNP is delivered - however this may be difficult to accurately or convincingly capture. It is not inconceivable that a school leader may have directed the change to a prescribed curriculum without any real investment from teachers themselves and, should that be the case, it may well impact not only upon the way in which teachers perform in the classroom, but also how they respond to the researcher. Elements of social setting such as these (lesson timings, school or social events impacting on the lesson structure or culture) should be considerations when interpreting the behaviours and actions of pupils and teachers in lessons.

Conversely, there is a risk of over-indulgence in reflective thinking and worrying about the influence of self on research (Hammersley & Atkinson, 2007). Analysis could feasibly become too loose or inconclusive due to an unwillingness to commit to any themes or diversions from the MNP program simply because I am too concerned that my interpretation is overloaded with preconceptions and uncertainty around the motivation or intent of any
actions observed in the classroom, or that note taking becomes sterile to the point of giving no insights into purposeful actions and behaviours.

In light of this conscious awareness of self, bias and potential preconceived notions of practice, it was decided early on that the research undertaken would be focused not upon secondary mathematics education where the vast majority of my own experience lies, but rather primary mathematics where I am less likely to unconsciously make assumptions around pedagogical decisions without evidence or without, for example, seeking confirmation from teachers in interview regarding motivational or response choices noticed in teaching observations. This would therefore limit, to some extent, personal concerns regarding the legitimacy, objectiveness and subsequent robustness of this practitioner research (Merton, 1973).

**The intended and enacted curriculum**

The research will be divided into two distinct sections: the intended curriculum of ‘Maths, No Problem!’ and the enacted curriculum – how teachers *actually deliver* the program. The reasoning for this split approach is to work separately on distinct elements of MNP in relation to the research questions. Specifically, research question 1 asks: *What strategies are emphasised and prioritised in curriculum packages endorsed by the DfE for primary mathematics in England?* Firstly, there was only one DfE endorsed curriculum package for primary mathematics at the time that this research was conducted: ‘Maths, No Problem!’ Secondly, the question asks what the program wants to be delivered, not what is actually delivered. As such, observing teachers is not a suitable data collection method to answer this question, as it adds a layer of interpretation (the teacher) that potentially dilutes the intent. Hence, to remove that layer, only the source materials for the program should be investigated to respond to this research question: the textbooks, workbooks, website and lesson plans.

In the *intended curriculum*, analysis will be focused on the intent of ‘Maths, No Problem!’ in terms of both the structure and philosophy of the resources and lesson design. I will determine what Maths, No Problem! say they want to achieve and deliver in the classroom,
and triangulate analysis of the resources provided in their textbooks and workbooks through that lens to assess whether they align with what is said. For example, if ‘Maths, No Problem!’ advocate the Concrete, Pictorial, Abstract model of teaching mathematics, then how, and to what extent, is that reflected in the resources they provide? The approach to this data collection and analysis is outlined later in this chapter.

Conversely, in section two I will explore the enacted curriculum of MNP. Here I will explore the enactment of MNP resources in lessons by the teachers who are using the program. This approach is to address the remaining research questions:

2. How are textbooks used as teaching aids in DfE approved curriculum packages for primary mathematics in England?

3. How do teachers facilitate knowledge acquisition using DfE approved curriculum packages for primary mathematics in England?

4. How is teacher agency impacting upon the interpretation and delivery of the intended curriculum in DfE approved curriculum packages for primary mathematics in England?

For research question 2, there is a crossover of useful evidence regarding how the textbooks are designed to be used, and how teachers are actually using them – both factors are of interest in order to answer question 2, but also inform question 4. Similarly, question 3 requires analysis of the philosophy and design principles of MNP to look for explicit and implicit evidence regarding what pedagogical approaches they promote to facilitate knowledge acquisition – for example, by direct instruction, rote learning or constructing knowledge through investigation. But it also requires observation of teachers to determine whether MNP approaches are being omitted or adapted in any way. Again, evidencing question 3 will also produce relevant and useful evidence to answer question 4.

As such I will explore how closely the intended and enacted curriculum align, and what role teacher agency has in adapting lesson design and resources in an environment where MNP has become embedded practice.
The Intended Curriculum

An analysis of the intended curriculum will aim to determine what processes, theories and structures inform the MNP program, followed by an in-depth analysis of what assumptions are made by MNP with regards to the type of teacher they are intending to work with and shape. This teacher type can be thought of as MNP’s presupposed alignment of affordances with teacher beliefs, knowledge, orientation and goals (Peppin et al., 2017; Remillard, 2005).

A key resource in the extrapolation of the ‘Maths No Problem!’ intent and principles of design is the series of textbooks and workbooks produced and written by them. The textbooks rely on the lesson phases and question prompts for the class, and the workbooks highlight the independent practice for pupils and subtly point to task design principles (identifiable to me as a research practitioner) that may be useful evidence to support research questions 1 and 2.

Valverde et al. (2002) describe textbooks as a translation of policies as intended by their authors, which is an important distinction to note as they are not actually the curriculum, but rather an interpretation of it. This additional layer of interpretation however is of no interest for this study. This is for two key reasons: firstly, the Maths, No Problem! curriculum is a fully resourced package that includes not only what content to teach, but also how to teach it, and what to teach it with. This is a much more explicit and structured model than the DfE’s programme of study which serves only a handful of core principles alongside a list of content by key stage. The endorsement of the Maths, No Problem! programme, and the rejection of all other applicants for approval to the DfE, infers that MNP’s programme is an accurate interpretation of the DfE’s curriculum as well as being the singularly approved ‘mastery teaching approach’ (NCETM, 2017) as a curriculum package. As such this is an incredibly valuable resource and area of study in the field of mathematics education.

Secondly, I consider the breadth of this study to be large for this thesis, and there is a tension between elements that are included and considered important, and elements that are interesting but which risk it becoming bloated, unfocused or unmanageable. Therefore, the study is purposefully curtailed to only focus on MNP’s curriculum package design and how it is taught in the classroom.
Website analysis

To capture the principles and philosophy of MNP, two key sources of data were utilised for analysis. The first was the MNP website, https://mathsnoproblem.com which hosts both marketing materials outlining the program and its links to maths mastery, and over 200 blog posts written by MNP staff and teachers working in MNP accredited schools. The blog posts in particular gave strong insights into the structure and design of individual components of the MNP design, influences on the program (including individual posts on academic influences, and how they relate to MNP's design), and how several of the philosophies and principles play out in teaching scenarios or specific examples of how they underpin resource design. As such, exploring the MNP principles through the website was the starting point of the intended curriculum analysis, followed by analysis of blog posts related to those findings to further gain explicit examples of how they manifested in resources and pedagogy.

Following this, tools (explored later in this chapter) to analyse the textbook and workbooks could be customised to focus in on any determined features or principles as, in theory, the books should echo the philosophies outlined in the website. For example, should the website and blog posts declare that narrative question design was a key component of their program, then the subsequent analysis of the textbooks and workbooks would specifically seek to find evidence of narrative questions design being used. In keeping with the interpretative paradigm, how I interpreted and subsequently categorised types of tasks in my textbook and workbook analysis is outlined later in this chapter.

Textbook analysis

In order to ascertain what is, and is not prioritised by the MNP program, it was essential to identify preferred ways of assessing understanding of mathematics. For example, questions may be assisted with visual models quite frequently, or chapters may emphasise real-world applications of mathematics to communicate mathematical ideas. These decisions are unlikely to be random if they are a consistent theme, and they subtly communicate how MNP perceive ‘good teaching’ whilst also demonstrating what is considered less of a priority (ie, what is not included in the texts, or what types of questions are used less often). The categorisation of task types can of course be done in a number of
different ways, which is why it is important to review literature on this area to help
determine what is both feasible and most useful for analysis. Furthermore, any modelling of
worked examples in the textbooks - ie, fully developed questions and solutions used as a
teaching tool in the classroom will also give insight into preferred pedagogy. It may be for
example, that worked examples rely heavily on real world scenarios rather than abstract
mathematical models, or that pictorial representations of maths are frequently presented
alongside questions, or questions themselves are framed to prompt thinking about methods
rather than to pinpoint a specific answer.

Fan and Kaeley (2000) point to some specific uses and insights that textbooks can
provide regarding curriculum intent, for example, the way in which lessons are structured
and sequenced, alongside the strategies around question positioning and general
pedagogical approaches should provide important insights not only into how the curriculum
is intended to be delivered by a teacher, but also into what MNP presents as model teaching
and learning strategies. The textbooks and workbooks provided in the program offer a rich
source of information both explicitly (how lessons are presented) and implicitly (why lessons
are presented in a specific way) regarding curriculum intent. The way in which those
chapters and questions are written can give indications as to the principles of the ways in
which mathematics is intended to be taught by teachers, and intended to be learnt by
students. Author decisions determine how the mathematics is presented, and how it is
considered best for students to practice. Furthermore, the ways in which the textbooks and
workbooks are structured should echo the principles and philosophies outlined by MNP in
their documentation, website, and blog posts – this is particularly important to consider, as
the website and blog posts may be explicit in their aims, however it is the textbooks and
workbooks that teachers will be using in the classroom.

Similarly, the way in which information is presented in the MNP books will infer
assumptions about not only the role of the teacher, but also what kind of teacher the book
wants them to be. This intended teacher will be discussed during the analysis stage of this
study by considering a range of evidence across the MNP books and lesson guidance.
Specifically, how the blog posts and website position the role of the teacher, their agency,
their subject knowledge and principles they adopt around pedagogy will be discussed. For
example, do blog posts encourage agency, or infer a more rigidly structured approach to following the MNP program. Does evidence point towards a ‘teacher-proof curriculum’ approach to delivering the program, or a ‘curriculum-proof teacher’ where a teacher is anticipated to be enabled to adapt a provided curriculum package, maintaining its principles whilst maximising its effectiveness for the students in front of them (Taylor, 2013).

The types of tasks that students work on to learn mathematics have been a point of interest for a number of years. Sullivan et al. (2012, p.14) for example, reason that ‘mathematical thinking comes from students working on a succession of problem-like tasks, rather than following the teacher’s instructions step-by-step’. For example, an emphasis on more open-ended mathematics activities, or a reliance on particular visual models would be deliberate decisions made to support an overarching theme of what is considered good mathematics teaching and good mathematics learning.

As Fan et al. (2021) highlight, the majority of research conducted around school textbooks concern content analysis rather than impact analysis. Often this forms a comparative analysis across two different books from different regions or countries, concerning, for example, content sequencing (Wang & Lu, 2018; Kim, 2012), problem types (Zhu & Fan, 2006; Mayer et al. 1995) or an analysis of differences and similarities within a particular topic (Hong & Choi, 2013; Lee & Smith, 2011; Wang et al. 2017; Alshwaikh, 2016; Cady, Collins & Hodges, 2015). However, the aim for this part of my research is not to compare between texts, but rather to determine the structure and formats of practice provided by MNP. Within this, I will look at how that structure aligns with the aims and principles that underpin MNP design, specifically stated or inferred on their website and through their blog posts. Despite this, the techniques and analysis tools used for comparative studies, such as the Mathematical Task Analysis Guide (Smith & Stein, 2000) are still useful and worth exploring in order to ascertain how to analyse MNP texts in a meaningful and specific way so as to address the research questions competently.

Task categorisation has to balance a number of factors including practicality, usefulness, and ease of identification - for example, if categorisation is overly specific, inevitable difficulties will arise with question codification where questions themselves cross over a
number of categories. It is not unreasonable for example, to incorporate pictorial elements into a worded narrative question. Furthermore, codification requires interpretation, which adds an element of unreliability or potential bias. In order to add a layer of rigour to this process, it will be important to incorporate a ‘coding agreement’ (Campbell et al., 2013) whereby an external maths specialist also uses the same codification system as the researcher and applies it to a sample of the texts in order to compare and calculate an intraclass correlation coefficient (ICC) to measure how strongly both researcher and external codifier are aligned.

Each of these scenarios point towards different preferences and attribute value to certain styles (and conversely, devalue others). Analysis of these themes over a number of books would therefore develop a sense of MNP’s idealised pedagogy in the teaching of mathematics albeit specifically regarding the choice and presentation of examples and practice. This in turn would contribute significantly towards research aim 1 (‘Identify specific pedagogical approaches in curriculum programs endorsed by the Department for Education’) and would form the evidence base for research aim 3 (‘Determine the structure and formats of practice underpinning curriculum programs endorsed by the DfE for primary mathematics in England’).

**Berisha and Bytyqi’s (2020) Textbook Analysis Model**

In a paper titled ‘Types of mathematical tasks used in secondary classroom instruction’ (2020), Berisha and Bytyqi developed an amalgamated textbook analysis tool that borrows from Stein’s Mathematical Task Analysis Guide (TAG) (Stein et al., 1996), Zhu and Fan’s conceptual framework for task analysis (Zhu & Fan, 2006) and Gracin’s five-dimensional analysis of textbook exercises (Gracin, 2018). The model presents five dimensions of focus that present a strong overview of the composition of tasks within textbooks, enabling insights into specific points of interest for this study such as the frequency of narrative-based tasks (listed as ‘fictitious-application tasks’, and tasks that utilise pictorial representations (visual presentations) therefore aligning potentially with the pictorial element of the Concrete, Pictorial, Abstract model promoted in mastery teaching (NCETM, 2018). It should be noted that while Gracin’s model is also five-dimensional, they are not identical to the five
dimensions in Berisha and Bytyqi’s variant, in part due to the fact that Gracin’s model utilised some dimensions to specifically reflect Austrian School Standards criteria.

However, the detailed nature of the framework may require considerable time and expertise to apply effectively and consistently, alongside inevitable elements of subjectivity in determining categories for data, and the scale of the task to analyse several complete texts may offer too many data points so as to lose focus. Thus there is potential risk of diminishing the accessibility and applicability of the tool.

Schoenfeld (2018) offers a contrasting yet complementary methodology for evaluating the effectiveness of mathematics textbooks and classroom practices, introducing a Teaching for Robust Understanding (TRU) Framework. This framework assesses classroom environments across five dimensions to ensure students become knowledgeable, resourceful thinkers, and problem solvers. Unlike traditional teacher-focused evaluations, TRU emphasises the quality of student engagement with mathematical content, aiming to improve teaching practices through professional development tools tailored to enhance these dimensions. Schoenfeld’s TRU framework focuses on the dynamics of classroom interactions and the opportunities provided to students for engaging deeply with mathematical concepts, whereas Gracin’s (2018) analysis offers a structured method to evaluate the potential of textbooks to support such engagement. Schoenfeld’s approach underlines the importance of reflective and adaptive teaching practices that respond to student needs which would be more useful in this study for analysing elements of the enacted curriculum rather than the intended curriculum, while Gracin’s (2018) work provides critical insights into how textbooks, as key resources, can either facilitate or hinder the development of robust mathematical understandings. Together, Gracin and Schoenfeld suggest that enhancing teacher agency in mathematics education involves both the critical evaluation of classroom practices through frameworks like TRU and the analytical assessment of textbooks to ensure they offer diverse and challenging mathematical tasks.

An outline of the model in full, focusing on five different dimensions of design (Contextual features, forms of presentation, answer forms required, mathematical activity
involved, and level of cognitive demand) taken from Berisha and Bytyqi (2020) is discussed below.

The tool itself was decided upon and customised following the analysis of MNP’s website and blog posts, so that fields were tailored to better suit what was being investigated.

**Dimension 1: Contextual Features**

This dimension will measure the context in which a task is presented. For example, whether a task has fictitious or authentic application by specifically checking whether a task is:

- **a) Non-Applicative** - no connection with reality-related contexts (entirely abstract)
- **b) Fictitious application** - connected to reality-related contexts created by the author (eg narrative)
- **c) Authentic application** - utilising real-life data collected by students

This dimension is borrowed from Zhu and Fan (2006) who aimed to distinguish between real world style problems and more abstract ‘non-application’ problems that were considered to be unrelated to the real world. Fictitious problems were considered by Zhu and Fan to communicate why the maths required was useful in real world situations and require students to extract the mathematics from a more narrative driven presentation, whereas authentic style tasks required students to gather information about themselves or their experiences around them in their daily lives, with an objective to help students make sense of the world and contextualise the maths they are learning.

This dimension was considered important due to findings during the website and blog analysis discussed in depth in chapter 5, however for clarity in this chapter, the dimension helps explore how many questions and tasks in MNPs textbooks included a narrative, which is a specific design element cited as important in MNPs principles of design. It also captures whether a task involves pupils capturing their own data (‘authentic application’) which again aligns with a further principle of design cited by MNP regarding the importance of play and
inquiry in knowledge development (discussed in depth in chapter 5). An example of what may be considered a narrative driven task is outlined below:

“Jolene goes to bed three hours after dinner. Yesterday, after dinner, she spent 1 1/2 hours on her homework and 2/3 of an hour on the telephone. How much time did she have left before bedtime?”

This is an example of a narrative driven mathematical question (often referred to as a ‘worded problem’). It contextualises the mathematics required and gives the maths a less abstract purpose beyond a calculation. Furthermore, the mathematics for this question is not explicit in that the calculation required is not directly provided to the student.

It should be noted that this is a usefully identifiable component because it has been outlined as a key feature of MNP’s design, and that the efficacy or suitability of such elements of curriculum design, including their positioning within a sequence of learning, are not a focus of this research. For example, whilst motivational theories may point to a sense of meaning and applicability aiding the motivation to learn something (McCrea, 2020), one might argue that cognitive load theory points to students learning something more effectively if they only have to focus on very few things at once - and interpreting maths through narratives adds an unnecessary burden at early stages of learning a new idea (van Lieshout & Xenidou-Dervou, 2020).

**Dimension 2: Forms of Presentation**

This dimension will measure whether a task is presented in either:

a) **Symbolic** form

b) **Textual** form

c) **Visual** form

d) **Combined** choice of two or more of the above.

In this section, evidence supporting the Concrete, Pictorial, Abstract (CPA) model can be captured. Again, this is a key design principle cited in MNP’s website and blog posts which
will be discussed in more detail in chapter 5, however, an outline in how these different types of questions were identified and interpreted is shown below. For transparency, the questions within the textbook and workbook were interpreted and categorised by me, and validated only by cross referencing the accuracy my interpretations with a single chapter analysis performed by a second maths expert, specifically addressing the need for a ‘coding agreement’ principle outlined earlier (Campbell et al., 2013). This process is discussed in more detail in chapter 5.

**Symbolic Example:**

“Simplify $1 - 4^2$”

This is an example of a symbolic, entirely abstract mathematical question. It has no supporting visuals, or narrative context and the calculation required is explicitly provided.

A further example cited in the MNP blog posts of an abstract maths problem is provided in Figure 4.1:

![Figure 4.1 An example of a symbolic mathematics question](image)

**Pictorial Example:**
Figure 4.2 shows an example of a *pictorial* mathematical question. Whilst the purpose of the question is without context or narrative, the abstract symbolism of $6/8$ is supported by the pictorial representation of the fraction in a rectangular array, with further scaffold provided using the subtle difference in line width to partition pairs of squares. As part of my duty to add clarity regarding my own interpretational influence on this study, it is important to note that I interpreted a pictorial mathematical question as including a *mathematical* picture such as a model (shown in the example above), countable objects, or mathematical tools such as a part-part-whole diagram. Pictures that added no insight into the mathematics were not included (for example, a picture of a character’s face simply to illustrate them asking an abstract question).

Furthermore, in an adaptation to this model, ‘combined’ was subdivided to better understand what combinations of symbolic, textual and pictorial elements of presentation were preserved so as to be able to identify potential patterns of where, for example, pictorial models were supporting textual problems. These combinations were considered to offer a more accurate insight into how scaffold in tasks was added or removed in that a task that involves, say, five abstract questions, might begin with two questions that include pictorial support to symbolic representations, but that pictorial support may be removed as confidence is assumed to grow as the lesson progresses. Such patterns would potentially be missed with a collective ‘combined’ category.

-an example cited by MNP is provided in Figure 4.3:
Dimension 3: Answer forms required

This dimension simply captures the way in which pupils are expected to provide an answer to a question or task.

a) **Close-ended** tasks - have only one answer (for example a calculation like ‘3 x 4’ which has only one correct answer).

b) **Open-ended** tasks - Have several correct answers. (for example, ‘which method do you think is the most efficient to solve this problem?’ or ‘draw a model to represent your answer’).

c) **Multiple response** tasks - Respondents select only correct answers from the choices offered – in other words, the question is presented as a multiple-choice question.

Zhu and Fan (2006) utilised this category inspired by the PISA (2003) Framework distinguishing closed constructed response, open constructed response and multiple-choice. The closed constructed response can be easily judged as either correct or incorrect. Multiple-choice problems offer limited defined response-options with suitable distractors. This section helps capture themes around MNP’s interpretation of *instrumental and relational understanding* and *metacognition*. These themes are discussed in depth in chapter 5, however for clarity here, an element of MNP design focuses on allowing for play, investigation, and allowing for time to develop ideas or investigate multiple methods to
solve problems as a strategy to develop a deeper understanding of ideas, referred to in chapter 5 as *relational understanding*. As such, open-ended tasks felt particularly relevant as something to capture within the textbooks and workbooks. Similarly, close-ended tasks contribute in part to evidencing more procedural knowledge development (although it cannot be inferred that a close-ended task immediately must be a procedural task), referred to in chapter 5 as instrumental understanding.

**Dimension 4: Mathematical activity involved**

a) **Representation and modelling** task - Requires the presentation of mathematical data in different forms or the translation of mathematical data from one representation to another

b) **Calculation and operations** task - Requires performance of mathematical operations, calculations, transformations, geometric constructions etc

c) **Interpretation** task - Requires recognition, reading and contextual interpretation of mathematical relations or data presented in different forms

d) **Argumentation and reasoning** task - Requires elaboration, description and stringing of the right arguments that lead to a conclusion

In this section, further evidence supporting metacognition, relational understanding and the CPA model can be captured particularly through the categories of ‘representation and modelling’ and ‘argumentation and reasoning’ as well as the promotion of *communication* as outlined by MNP’s principle designer, Dr Yeep Ban Har, as a key element of a strong mathematician (which is discussed in chapter 5).

**Dimension 5: Level of cognitive demand**

a) **Memorisation** task (Low Demand) - Involves the reproduction of previously learned rules, facts, formulae or definitions

b) **Procedure with no connections** (Low Demand) - Involves performance of general procedures and algorithms without making connection to underlying concepts or meaning

c) **Procedure with connections** (High Demand) - Involves performance of procedures and algorithms making connections to underlying concepts or meaning.
d) **Exploratory task** (renamed) (High Demand) - Involves complex and non-algorithmic thinking, exploration and understanding of mathematics concepts, processes or relationships.

**Issues around Instrumental and Relational Knowledge as a Dichotomy and Level of Cognitive Demand**

In this research, a) and b) will be considered to require and promote **instrumental knowledge**, whereas c) and d) are considered to require and promote **relational knowledge**. However, it is important to acknowledge the subjectivity and challenge associated with both categorising questions as a procedure with / without connections, and, related to this, associating tasks to promoting either instrumental or relational knowledge.

Whilst instrumental knowledge distinguishes an ability to follow a memorised rule or procedure *disconnected* from reasoning why it is suitable or how it relates to other areas of maths, being able to identify where a question or task falls into this category of knowledge development is challenging. For example, Braithwaite and Sprague (2021) highlight the interconnectivity between instrumental and relational knowledge in solving *any* maths problem, suggesting in fact that even classically abstract questions that appear to require only instrumental knowledge do in fact often cause students to rely instead on relational knowledge where the problem is unfamiliar or if they cannot solve it. Similarly, Herheim (2023) argues against a strict dichotomy between conceptual and instrumental understanding, proposing instead a continuum where these approaches overlap and interact which in turn would help teachers navigate the complexities of the interplay of each type of knowledge and inform more effective teaching practices - avoiding observed difficulties in teachers trying to diagnose and subsequently design suitable interventions when handling students getting stuck on different types of maths problem.

However, the distinctions in task type are useful in this research as tasks that are identified as ‘high cognitive demand’ specifically require knowledge of, or use of interconnected elements of mathematics, which in turn aligns directly to what MNP distinguish as relational knowledge, stating that relational knowledge “focuses on establishing connections, …[and] applying concepts to other problems” (Maths No Problem, 2019e). As developing
instrumental knowledge is a key strategy in the design of the MNP program, it makes sense to identify tasks that, by their definition, match that objective.

Furthermore, it is important to acknowledge that whilst this tool will attempt to capture the intent of individual tasks and questions throughout an MNP textbook or workbook, it will not capture more longitudinal elements of mathematical knowledge development across units of work in the way that the curriculum is sequenced over time. For example, the strategic decisions to build up, say, deep understanding of the properties of multiplication cannot easily be recognised through individual question analysis, but instead would be more suited to a study of the choice of curriculum sequencing and specific analysis of what a series of questions or tasks over a unit or units is attempting to achieve. Whilst this is further important and interesting data that could be gathered to contribute towards or complement this research, it was considered to be beyond the scope of what was feasible to achieve for this thesis. However it is acknowledged that further research into longitudinal intent within the textbooks could impact upon how some individual tasks are viewed and categorised with such additional context.

**Reflecting against the research questions and aims**

Gathering data on the types of question and task, and how they are presented to both the teacher and pupils provides supporting evidence to help answer the following research questions and aims:

Research Aim 1: Identify specific pedagogical approaches in curriculum packages endorsed by the Department for Education for primary mathematics in England.

Pedagogical approaches promoted by MNP (the only curriculum package endorsed by the DfE) combine philosophies and ideas with physical resources that should, in theory, align with those principles. As such, the tasks and structures of an MNP lesson or sequence of lessons need to be analysed against the explicit principles MNP outline in their website and blog posts to determine whether the resources teachers take into the classroom mirror what MNP prioritise as their pedagogical approach. How the resources are presented is likely to
affect how they are interpreted by teachers, and therefore is likely to affect how aligned the enacted curriculum is with the intended curriculum.

Analysis of both the workbooks and textbooks can, for example help distinguish task types, frequency of task types and distribution of task types. Elements of interest can be quantifiably categorised and analysed to help determine differences, or support alignment with both the pedagogical intent of the program, and the ways in which knowledge is intended to be developed or acquired by the learners. For example, Leshota (2020) discusses quasi-deductive and quasi-inductive task categories as useful distinctions. Quasi-deductive tasks commence with a definition of a concept, which is ‘exemplified through one or more worked examples and followed with practice exercises for the learner’, whereas quasi-inductive approaches commence with ‘an investigative activity that provides learners with an opportunity to make conjectures before the textbook elaborates on the concept with notes and illustrations’ followed by worked examples and practice exercises. Leshota uses this framework to categorise entire units of work as either quasi-deductive, or quasi-inductive rather than individual lessons, and then analyses how teachers ‘mobilise textbook affordances’ - in essence, how they enact the curriculum.

The Enacted Curriculum

Case Studies

In response to research questions 2, 3 and 4, data would need to be collected regarding the ways in which teachers taught using MNP. For referral, the research questions are:

2. How are textbooks used as teaching aids in DfE approved curriculum packages for primary mathematics in England?

3. How do teachers facilitate knowledge acquisition using DfE approved curriculum packages for primary mathematics in England?

4. How is teacher agency impacting upon the interpretation and delivery of the intended curriculum in DfE approved curriculum packages for primary mathematics in England?
Whilst the intended curriculum section of this chapter covers some elements of research questions 2 and 3, how textbooks are actually used by teachers may well differ from MNP’s intent. Similarly, the intent regarding the style of teaching to facilitate learning in the classroom may well differ from how teachers interpret or adapt the program with reference to research question 3. Finally, research question 4 requires data for a comparative analysis of what MNP intend to be delivered (and how) in the classroom, and what teachers actually do. Hence each of these questions demands insight into both the intended and enacted curriculum and as such, it was considered critical to be as close to the act of teaching as possible, in order to limit the layers of interpretation that would come without close proximity to the act of teaching itself. For example, should the data collected be taken from teacher interviews as the primary source, then arguably several layers of interpretation would be present, allowing for potential compromises regarding validity of data and a blur between what I, the researcher, determine as agency and adaptation, or faithful reproduction of MNP’s program, and instead capturing how the teacher perceives their own agency or indeed understanding of the MNP program itself.

Whilst a teacher’s own perception of their agency and the intended curriculum is interesting, it is not the focus of this study. Risks associated with additional layers of interpretation include teachers interpreting their own actions through their subjective memory of them, rather than in the moment (and secondly, I would then interpret meaning from those interpretations). These additional layers of subjectivity risk a reduction in the accuracy or usefulness of data collected (Miller, 1987). Furthermore, without the researcher being present for the delivery of MNP to pupils, the noticeability of points of interest, possible themes, and intent vs action would all potentially require teachers themselves to be trained extensively on the purpose of the research (which in turn could bias and affect their performances in the classroom), as well as require them to be able to accurately communicate ideas through memory. All of these factors risked data integrity and reliability.

Therefore, for researching the enacted curriculum, a case study approach was undertaken as the primary research method to develop understanding of how MNP is interpreted in schools. Case studies are widely used in education studies, including those looking specifically at textbook use and mathematics education such as Boaler and Staples
(2008), and, as previously discussed in the literature review, Eisenmann and Even (2009), Collopy (2003), Lindorff Hall and Sammons (2019), Norberg (2019) and Ward and Quennerstedt (2018). Each of these studies uncovered acute and specific behaviours and dynamics amongst individual teachers, and were able to discuss them in fine detail due to their intense focus on specific case studies rather than holistic larger scale data collection.

A small sample of ‘critical cases’ were analysed and discussed with a dominant focus on each individual case as a unique entity, however some comparative assessment and commentary supported any notable points of interest or emerging themes.

A strong justification for case study research as a tool of investigation is highlighted by Flyvbjerg (2006) in his paper ‘Five misunderstandings about case study research’, which addresses both the common criticisms and misunderstandings around case study research, as well as unpicking the problematic assumptions that underpin them. Flyvbjerg highlights how incredibly important intimate knowledge of individual cases is in the development of expert activity and understanding, particularly when studying human behaviours. For example, whilst most of us may demonstrate expertise in everyday skills like riding a bike or communicating to each other in an acceptable manner, only a few reach expertise in more specialised skills such as playing an instrument or playing chess - but what is common to such experts is their intimate knowledge of thousands of concrete cases in their area of expertise. Hence their ability to draw upon context-dependent scenarios as their experience base enables them to skillfully develop a much deeper understanding of their craft and a more proficient navigation of new circumstances due in part to features with close familiarity to previous experiences.

The knowledge and understanding developed through case studies is therefore, Flyvbjerg argues, at the very heart of learning. For this research, a case study approach enables examination of different teachers under different circumstances connected by the commonality of utilising the same intended curriculum, and therefore serves as a useful tool to examine whether any emerging themes of adaptation or divergence from the MNP program emerge independently of, or potentially because of circumstances surrounding an individual institution or multi-academy trust. These external, circumstantial or situational...
features of each case study help position a school within the research. Flyvbjerg’s emphasis on the need for intimate knowledge of individual cases has influenced this study and persuaded me to spend time with each research participant, and thus study their craft over a number of days rather than for a single lesson or instance of teaching, and supports the decision to interview teachers during visits, so as to explore in finer detail the motivations and decisions made within lessons that are sometimes implicit rather than explicit during delivery.

Reflecting upon the literature research conducted for this study, many of the papers were deeply rooted in case study knowledge rather than theoretical perspectives or large, wide scale research projects across districts or principalities. Instead, they are more contained, often to single schools or small groups of localised teachers such as Eisenmann and Even (2009), Collopy (2003), Lindorff Hall and Sammons (2019). This containment offers more specific examples of behaviours and individual changes made to curricula, and give insights into what circumstances or relationships within an individual setting may have instigated or accelerated any desire to change, maintain or dismiss altogether the resources provided to teachers. For this research, it is exactly this kind of precision that will enable me to accurately comment on how teachers adapt, engage or disengage with MNP content. This in turn provides a strong evidence base to be able to answer research questions 2, 3 and 4, each of which relate directly to teacher practice and behaviours in the classroom. It should be noted for balance, that it is not always clearly stated why case studies are preferred in these studies. For example, there may simply be factors around resource constraints at play, however the value of each case study is clear and has indeed contributed towards understanding manifestations of the intended curriculum and in turn aiding the development of this research.

Case studies should be treated as a route towards understanding and depth of knowledge - giving insights into what Flyvbjerg calls a “more nuanced view of reality” of human behaviour which cannot be meaningfully understood by rules. Campbell (1975) fully pivots towards endorsing case study research after years of being a critic of the approach, declaring naturalistic observation is the only route to knowledge despite how difficult it may be to interpret and how resistant it is to generalities (Campbell, 1975; Morrow & Brown,
1994). Case study research is not without criticism, for example, Geertz (1995) describes it as having an inherent flaw in not adhering to scientific methods and declares case study research as being more susceptible to subjectivism due to the coercive nature of being close to the field - a nature that is impossible to circumvent. However, Flyvbjerg asserts that this perceived propensity to confirm bias is in fact a problem in reverse - that case study research is more likely to falsify than confirm initial assumptions or hypotheses - and can be viewed therefore as a more rigorous approach to test initial predictions for researchers due to the nature of case study research involving close proximity to those under study – a factor that was specifically appealing for this research, allowing them to correct their subjectivism more quickly and readily than when researchers are not close to those being studied (Miller, 1987).

It is worth noting, that this argument appears to simply flip the narrative on features both sides of the argument agree are unique to case study research - where one academic may feel being close to those being studied as advantageous, another finds it as problematic. Despite these disagreements, it feels entirely inevitable that by choosing a method of teacher observation, and of a sequence of lessons over three sequential days, that the researcher will feel close to the research, both literally and figuratively. Justifiably, a warm and approachable researcher with empathy and understanding of the role of the teacher (based on their own experience as a practitioner) is perhaps better positioned to capture a teacher at ease and therefore less performative for the researcher and more likely to demonstrate their more typical teaching strategies, attitudes and approaches (Dadds, 2008). Similarly, using the interpretative paradigm as a positional foundation for this research incorporates a more explicit acknowledgement of the position of self, and interpretational influences on the study so that all assumptions and interpretations are more transparent. Of similar note is that perceived problems arising from interpretation, researcher bias, confirmation bias and data validity are not unique to case study research and are dealt with and reduced in much the same way as other research methods.

Finally, case studies are often considered to be a subset of qualitative research methods (Yin, 2018), however, this study will gather data from a range of sources and will in fact integrate mixed methods (as noted from the more quantitative analysis discussed previously in the intended curriculum section). In fact, even at the level of each case study, the
observation toolset used a more quantitative *Innovation Configuration Map* (Huntley, 2012) as part of the analysis (discussed later in this chapter).

**Defining the boundaries of each case study**

For this research, it is important to both define and bound each case study (Yin, 2018) and ensure that both validity and reliability are planned for and integral elements of design. Perhaps most pertinent to this study, is what Creswell and Creswell (2018) refer to as the selection of ‘instrumental cases’ to ensure that the most accurate and reflective data (in light of the research questions) is presented and accessible to the researcher. Instrumental cases are difficult to both determine and access, but they are considered ideal for capturing data that is representative of the field being studied (Creswell & Creswell, 2018; Flyvbjerg 2006) and efforts should be taken to determine what characteristics they might have in advance of any field study. In this research, an instrumental (often referred to as *critical*) case is specifically determined to be one that focuses attention on the *established* use of the MNP program with minimal distraction. ‘Distraction’ is defined as to mean attention taken away from comfort in use, predominantly through inexperience and unfamiliarity.

There is an anticipated learning process that has taken place *prior* to this research in which the teachers have familiarised themselves with the program and potentially adapted the materials and approaches promoted, or their own teaching behaviours (or both) to suit their students and context in which they are teaching. In other words, a criterion for identifying a critical case is that teachers are *experienced* in delivering the program, to alleviate the risk of teachers developing their own understanding and interpretation of the intent of MNP. Hence a condition or *bound* for each case is that MNP has been used *before* which explicitly means that schools and teachers have used the program for a minimum of two school years prior to the study. This is a prerequisite intended to minimise teacher unfamiliarity with MNP materials and create optimal conditions for teachers to determine *how* and *why* they want to use MNP in the context of their classes.

A case study is an in-depth exploration of a bounded system (Creswell & Creswell, 2018; Yin, 2018), and what Creswell calls a multiple instrumental case study approach takes several
cases that share properties indicating that they fit under a bounded system providing insight into a singular issue. For this research, each case study will include a maximum of two teachers utilising the MNP program in a single school with a number of predetermined bounds that qualify the participants to be a best-fit usage case of MNP based predominantly upon the established use of MNP (ie, MNP has been introduced in the school at least two years prior to this research), and the teachers themselves have used the program in previous years. Therefore, the teachers in each case study should be considered enthusiastic adopters due to their prior commitment to the program without the influence of Governmental persuasion (due to adoption taking place prior to the announcement of MNP as a part funded recommended textbook by the DfE). For this research, meeting each of these bounds qualifies a case to be considered critical or instrumental. We have visited one criterion previously (teacher experience), however, sampling is detailed fully in the next section.

**Sampling**

**Schools in England:**

As the research focus is on English primary schools interpreting curriculum packages endorsed by the DfE, the decision was made to select primary schools in England that are considered well positioned to teach in the style and philosophy intended by the textbook series ‘Maths, No Problem!’ created by Andy Psarianos and Dr. Anne Hermanson - which strongly promotes what is commonly referred to as the ‘Singapore Method’ of teaching mathematics (Maths No Problem, 2019c). The textbook series was, at the start of the research period, the only resource endorsed by an expert panel selected by the DfE to teach ‘mathematics mastery’ (Maths Hubs, 2017a). For context, an open tender was launched in 2017 to textbook companies to be granted formal approval and endorsement from the DfE, which included funding for schools to adopt any of the winning textbook(s). Winning bids were selected by an expert panel who looked at contributions by numerous established textbook writers and education companies including Oxford University Press, Pearson, Scholastic, and Collins Learning - the latter of which submitted direct translations of the Singapore Maths books used in Singapore itself (Lewis, 2017) However, the tender resulted in only a single company winning endorsement: ‘Maths, No Problem!’ The monetary incentives provided by the DfE for schools to use these books could arguably be conceived
as the closest the DfE have come to providing a state sponsored text – a commonality across
the top performing countries in PISA mathematics tests (Watanabe, 2001). And so, whilst the
research questions refer specifically to interpretation of DfE approved curriculum packages,
this research arguably captures a strong approximation of what the DfE considers to be
effective ‘mastery teaching’.

Indeed, by looking at schools that used ‘Maths, No Problem!’, this research is consciously
positioning itself to look at what is considered the best possible interpretation of what ‘DfE
approved’ teaching of mathematics in primary schools should look like. As such, it is
important to capture teaching under optimal conditions to do it in the way it was intended
by ‘Maths, No Problem!’ (and therefore, the DfE) using the criteria outlined herein to
establish critical cases. Furthermore, whilst the DfE has consistently promoted the notion of
teaching mathematics mastery since 2014, there have arguably been no clearer signals as to
what a fuzzy term such as ‘mastery’ actually looks like than the explicit teaching materials
and instructions provided by the ‘Maths, No Problem!’ resources. Indeed, the managing
director of ‘Maths, No Problem!’ , Martin Casimir, went on record to air his opinion on the
outcome:

The DfE’s announcement today should go some way to weeding out the
plethora of providers out there in the market-place jumping on the mastery
bandwagon but whose methods are not founded in research. They simply
serve to confuse teachers and hamper the roll-out of the mastery method.

(TES, 2017)

Selecting schools that did not use these resources would create an additional variable to
consider – namely, how schools have chosen to interpret mastery approaches, which would
almost certainly vary from school to school and remove an element of inconsistency
(Mccourt, 2016). This additional complexity would arguably blur the intentions of the
research questions, as it potentially widens the gap further between what is being said and
promoted by the DfE, and how that is being interpreted by schools. Observing teaching in
those schools would potentially refocus the research to, as Casimir put it, look at the ‘confusion’ of approaches in those schools less enabled. Whilst that element of the DfE’s encouragement of specific teaching styles for mathematics is indeed independently interesting, it is not the focus of this research. In summary, this research intends to explore what ‘DfE endorsed’ teaching looks like in primary schools considered most enabled in England through the lens of ‘Maths, No Problem!’.

Enthusiastic Adopters

As the participants will have already had experience with MNP, and presumably successful adoption and integration of the program into their teaching (due to continuing to use and finance the program), the ‘optimal conditions’ described in the section above are considered to be a lens of enthusiastic adoption of the MNP curriculum package. That is, teachers are likely to be positioned as keen users of the program which in turn may increase the likelihood of deeper engagement with the materials and related professional development delivered by the authors of MNP in order to align teachers closely to the intent of MNP in terms of both content and style of delivery. As teachers who have been using the MNP package for years prior to this study, and, crucially, prior to the Government’s incentives to adopt MNP (arguably bypassing potential scepticism or suspicion of wider government intent), teachers are arguably more likely to exhibit a strong willingness to engage in professional development, embrace the program and demonstrate belief in the potential of the materials to enhance student learning.

Furthermore, as the critical cases each integrated MNP into their schools prior to any Government initiative to promote and persuade the adoption of MNP, this research does not concern itself with how curriculum materials such as MNP might be interpreted in cases where adopters were less likely to be enthused or influenced by the program which in turn could affect the ways in which adoption differed or aligned with MNP’s program intent. Hence it is considered likely that in this study each teacher will be in a position of wilful adoption rather than coerced adoption, and therefore any insights into how teacher agency affects the enactment of the intended curriculum of MNP should be contextualised accordingly.

Advantages of Focusing on Enthusiastic Adopters
The study of 'enthusiastic adopters' provides a nuanced lens through which to explore the intersection of teacher agency and the intended objectives of a prescribed curriculum package. By focusing on teachers who have voluntarily engaged with MNP for an extended period, the research sheds light on how individual choices, motivations, and interpretations can shape the implementation and outcomes of MNP in order to enhance it, or at least intend to enhance it, when factors surrounding the imposition of a package upon a less willing teacher are significantly reduced or removed. This approach also navigates the complexities of attributing changes in educational practices to the agency of teachers versus the inherent characteristics of the program itself.

Enthusiastic adopters exemplify how teacher agency can positively influence the adoption and adaptation of curriculum reforms. In this study, commitment to MNP for two or more years suggests a proactive engagement with the program, highlighting how teachers' personal and professional values, beliefs, and pedagogical choices play a critical role in the successful integration of new teaching methods. The case studies will offer insights into how teachers can creatively interpret and integrate curriculum reforms into their existing practices, and their experiences may reveal the ways in which teachers negotiate between the program's objectives and their pedagogical beliefs, adjusting and tailoring the curriculum to fit their students' needs and their educational contexts, investigating the factors that drive teachers to go beyond superficial compliance, exploring how their sense of professional identity and commitment to student learning influences their willingness to embrace and sustain changes in their teaching. Furthermore, observing teachers who are already familiar with MNP through years of experience, may offer insights into how reflective practice and participation in learning communities and training affect agency with the program, presenting a unique case in the broader discourse about the adoption and enactment of educational curricula.

Limitations of Focusing on Enthusiastic Adopters

Overlooking the experiences of teachers who may struggle with or resist the curriculum package in this study could give the perception of the specific MNP program being communicated as an overly effective or successful program for others to adopt. Despite the
program’s impact on students not being a focus of this study, it should be considered that the implied efficacy of MNP to effectively indoctrinate teachers to its pedagogical beliefs and style of teaching as a result of this study could be interpreted as endorsement or critical review instead of focusing on a broader discussion around the adoption of prescribed curriculum packages in general.

Enthusiastic adopters may share similar characteristics, such as a predisposition towards innovation or positive attitudes towards mathematics, which might not represent the broader teaching population. Insights into resistance can highlight potential flaws in the program or its implementation strategy, offering opportunities for improvement and a more balanced view of the specific program in this study. This homogeneity can limit the generalisability of findings to other contexts or demographic groups, however this narrower lens instead sheds light on a specific circumstance that contributes new knowledge to a growing area of interest in mathematics education.

As such, this study should not be considered to be an endorsement or critique of the ability of MNP’s specific program to either positively or negatively affect pedagogical approaches to teaching mathematics, but rather the study is concerned with the fidelity with which teachers enact it. Remillard (1999, 2000, 2012), emphasises the dynamic nature of curriculum enactment, where instructional materials are not merely implemented but are interpreted, adapted, and sometimes transformed by teachers. Enthusiastic adopters, with their positive stance towards new curricula, are likely to engage deeply with the materials, exploring and understanding them in ways that go beyond superficial compliance, and thus bypass, or be more likely to bypass, some of the common pitfalls of curriculum adoption.

Furthermore, as the schools being selected for this study will have utilised the MNP program for several years, they effectively bypass what MNP describe on their website as problems within the primary mathematics teaching profession. Specifically, MNP state that many teachers:

- Are ill-prepared to teach maths effectively
- Don’t have the necessary skills or resources
- Are generalists
● Lack confidence in their ability to teach maths
● Have a fixed mindset rather than a growth mindset
● Don’t accept that problem solving is at the heart of mathematics.

(Maths No Problem, 2019d).

Whilst there appears to be no clear data sources utilised to support these claims, it seems that these perceived barriers to good mathematics teaching, which could feasibly also be barriers to successful adoption and implementation of the MNP materials, are absent in the case studies being investigated.

Type of School

Beyond the requirements of a school being both a primary school and one that uses the DfE approved textbook series ‘Maths, No Problem!’, further decisions needed to be made regarding the type of school, its demographics and geographical location in order to narrow down the potential sample space to select from, whilst ensuring that additional criteria did not risk reducing the sample space so much as make access or school identification become an issue, or that too few schools met any defined criteria so as to make a meaningful representation of what ‘enabled’ means. In other words, if so few schools are considered ‘enabled’ within England, then the criteria used to define the term would be too strict and not representative of enough schools to be valid. With these cautions in mind, it would still be preferable to visit schools where the use of ‘Maths, No Problem!’ had become established practice. In this research, the term established practice is taken to mean that schools have used the texts for at least two full academic years, and as such teachers are very familiar and confident with the methods and strategies encouraged within the texts, rather than, say, a school that was using the texts for the first time.
There will of course be some disadvantages to this approach. For example, it could be that by actively excluding schools that are only beginning their journey into using the books, the research may be less likely to uncover any teething issues, teacher frustrations with new changes, compromises made, or a teacher’s awareness of the distinct changes between their original style of teaching and what is being introduced with MNP. However, again, the focus here is on schools that are implementing MNP in what are considered optimal conditions in order to fulfil being a critical case. A school that is learning how to use the program is, in this context at least, decidedly less enabled to implement MNP as it is intended than one that has established the teaching strategies and philosophies across the school (or at least several year groups). A risk associated with this approach was that the announcement by the DfE to endorse MNP only came within a few months of the start of this research, and so intuitively some schools using it were perhaps more likely to have only used it for a short time. However, what makes this research particularly useful, especially for those schools that have just begun their journey into using the texts, is the study of what teaching looks like over an extended period – and potentially how schools have reinterpreted the resources to suit their staff and students over time. In other words, a by-product of this research may be that it sheds light on the likely evolution of agency in schools that are just beginning to adopt the MNP program as a result of it being endorsed by the DfE.

In summary, the criteria for school selection to be considered a critical case are:

1. The school is a primary school in England
2. The school is using the Maths, No Problem! texts to teach mathematics
3. The school has been using the Maths, No Problem! texts for two full sequential years or more.

Geographical Locations

As the focus of this research question is on how primary schools deliver the MNP curriculum package in England, it made sense not to constrict the study into a smaller region such as a county like West Yorkshire or a region such as South-West England. Widening the geographical locations of the schools reduced the risk of making assumptions or false
conclusions about observations that may in fact be localised, and indeed influenced by locality itself (Atkins & Wallace, 2012). For example, the validity of an assessment made about the ways in which mathematics education is being delivered by schools ‘in England’ where the entire sample was based around, say, Norwich, would be questionable. Furthermore, with the critical case restrictions regarding the use of the texts two years or more, the likelihood of finding enough schools in a small geographical location would arguably be diminished.

However, this approach also created risks for the research. Approaching schools from further afield could have proven difficult or more unsuccessful due to a lack of formal or informal relationships with them, and the logistics of visiting schools far away from home, for several days (see ‘Number of Observations’ below) would have implications with regards to managing time, and monetary costs to being away from both work and family commitments during term time.

In summary, the criteria for geographical location to be considered a critical case are:

1. A majority of schools in which teachers selected as case studies should not collectively be from the same geographical area

Demographics

As the demographic makeup of a school was not a key focus within the research, no particular demographic was sought to use specifically. However, in order to reduce the possibility of an unvarying demographic undermining the validation of the study, schools were selected with differing demographics where possible. Specifically, should the demographics of the students of each case study be very similar, this could be considered to influence the way in which the teachers teach (Bobbett, 2001), and therefore distract from the specific influence of MNP on teachers. Allowing for varied demographics should allow for a more balanced insight into how schools in any area within England utilise or adapt their text resources regardless of factors such as affluence/social deprivation, ethnic makeup, or school/class size (Creswell & Clark, 2007). Again, this had the potential to cause some geographical complications with regards to access and distance from home. Insights into the
demographics for each case study were collected from the latest Ofsted reports for each participating school.

**Number of Observations**

When balancing the potential difficulties with regards to travel and location, visiting a small number of schools would be manageable and realistic. Considering also that the available pool of schools that met the critical case criteria was likely to be relatively small, it was decided that between three and six schools and six teachers in total was a realistic number to be able to visit within a time frame of around eighteen months. Furthermore, a large number of case studies risked reaching sample saturation (Hennink et al., 2019) whereby new schools added little insight into, or contribution to, already stabilising patterns of findings.

In fact, 8 teachers were observed including one for a single observation as part of a pilot study (discussed later in this chapter). Further restrictions would also need to be considered with regards to the timings of the visits, as schools have intensive exam preparation periods, end of term activities off timetable, and term breaks throughout the academic year. Furthermore, the commitments of the researcher needed to be taken into account, which fell mostly within the same term dates as schools. This meant that the times most suitable to the researcher were in fact times when schools were not open.

As the purpose of the study of the enacted curriculum was to gain insight into the methods and strategies used to deliver the MNP program, it was considered fundamental to research design that enough time was given to fully understand the decisions made within lessons (Denzin & Lincoln, 2008a), and how they impacted subsequent teaching either during a single lesson or across sequential lessons. Furthermore, an extended observation of 3 sequential lessons for each teacher addresses criticisms of case study research which often cite an ‘inability’ to convincingly identify meaningful data in single cases (Campbell & Standley, 2015; Dogan & Pelassy, 1990).
It was therefore considered unsuitable to see single lesson delivery, but rather, a small sequence of lessons by the same teacher with the same class. This would allow for deeper insight into how each lesson informed the next, and if or how teachers adapted to what happened within a single lesson. If only a single lesson was captured, potential deviations or adaptations to the MNP program would be at risk of being hidden from the observer, giving a less accurate picture of how teaching is taking place. Multiple observations would also eliminate the risk of documenting only unique circumstances such as staff absence or unforeseen situations within school that might adversely affect the way in which a lesson was delivered (Denzin & Lincoln, 2008b). As such it was decided that a sequence of three lessons would be considered optimal. Observing up to two teachers in a single school was considered an acceptable margin and useful to potentially draw comparisons between two teachers under the same situational setting.

A majority of primary schools in England teach mathematics once daily (OECD, 2016), and so the researcher would need to visit a school for three consecutive school days in order to carry out the desired observations. Again, this would have further implications regarding available time and travel. A further complication was envisaged in that mathematics is often taught at the same time across all year groups. This would mean that in order to see more than one teacher in a school, the timetable might need an inconvenient restructuring for the visit, or the observer would need to visit for six days rather than three. As such, seeing two teachers in one school was determined not to be a priority, but a useful luxury if it were possible.

Observation Structure

A minimally intrusive observational structure was considered the most suitable approach for the study. Minimally intrusive observations are characteristic of interpretative research design (Mujis et al., 2018). Since the study intended to capture a natural lesson, or sequence of lessons, any intervention from the researcher, such as a change in location, time or topic
would be inappropriate. Hence observations were intended to capture a moment, and the experiences of teacher and students within that moment, without the moment itself being contrived or altered for the benefit of the researcher. Despite this, it should be noted that the physical presence of the researcher in the room may in fact have affected both students and teacher to some extent (Mujis et al., 2018).

Lessons would be documented using pen and paper and the primary foci would be on the resources used, how they were used and what actions the teacher took within the lesson (Hammersley & Atkinson, 2007). Particular focus would be around the questions asked, how they were asked and how the teacher responded to the students.

Other observational methods were considered, but more extensively detailed observation methods such as video recording would make access arrangements more complicated with regards to ethical approval, data protection and parental permissions prior to school visits. As the study was focused primarily on the style of teaching, and not its perceived effectiveness, the decision was made not to evaluate the specific progress of the students – either within the allocated observations, or over longer periods by, for example, examining assessment data. Instead, the observations would concentrate predominantly on teacher resources and actions. This constraint would also alleviate any ethical or data protection related issues regarding the sharing of potentially sensitive student progress data.

**Interviews**

Whilst observations could capture actions, questions and responses within each lesson, there is an associated risk that the intent associated with those actions and responses could be incorrectly judged by the observer, particularly when the observer is a practitioner in mathematics education. Hence to better understand the nuance of the intent of those actions and responses by the teacher, interviews were incorporated into the data capture design so that I could more accurately view how teachers facilitate knowledge acquisition (research question 3), and how teacher agency impacts the interpretation and delivery of MNP (research question 4). As an example, if a teacher delivered a lesson that included
phases or tasks not included in the MNP program, there may be a multitude of reasons for this, each with differing consequences for how I, the researcher, perceive their intent. It may be that a specific task is removed from the MNP design because the teacher doesn’t understand it, or that they see no value in it, or it may be that they included that task in a previous lesson, or that they used their own agency to determine that the task was superfluous considering how much their students had evidenced the depth of their understanding of the topic taught up to that point. Not understanding the purpose of the task could be linked to subject knowledge, or poor communication of task intent on behalf of MNP, however, considering it superfluous when examining the evidence of pupil progress could feasibly demonstrate strong subject knowledge and align closer with MNP principles of design. The reasoning behind the decision would not be clear to the observer without explicitly addressing it with the teacher and asking them to explain their motivation. Hence while the primary data collection method in schools was decided to be observations, informal interviews would also be used to help clarify elements of the implementation of MNP lessons by teachers.

Thus the main purpose of staff interviews was to better understand decisions and strategies made within the observations, and to comprehend how teachers are able to articulate their understanding of the underlying principles of Maths, No Problem! As such, interviews would require a relatively loose structure based around themes and ideas, rather than strict pre-planned questions (Hennink, 2013). Specifically, should a decision be made in a lesson that was unclear or needed elaboration for the purpose of data capture (eg, a teacher deviates from an activity, but it is unclear why they did), then the interview would focus in on this specific element of lesson delivery. Similarly, if a teacher responded to a specific student query or misconception in a way that meant they deviated substantially from the lesson plan (for example, a majority of the class unveil a misconception through questioning, and the teacher decided to effectively stop the planned lesson and pivot onto tackling the misconception in depth), then the interview would focus, at least in part, on that development – and it could not be feasibly foreseen in advance of the lesson.
This would allow for more flexibility and responsiveness to pursue interesting lines of enquiry as they appeared in conversation and enable specific questioning of details that arose within the lessons themselves. Conceptually, actions would be recorded in lesson observations, and reasoning would be captured through interviews. Should common themes arise across different schools or teachers, then a loose structure would also allow for adaptability to follow new lines of enquiry. Analysis of interviews and observations would largely be qualitative, however should themes become apparent across participants then there may be suitable cause to codify and quantitatively analyse responses in some areas of questioning without significant loss of context or finer details. Potential risks associated with delivering teacher interviews existed around access, considering the busy schedule of teachers, and the risk of making school visits too formal or demanding for teachers to accommodate. As such, interviews were seen as a secondary data capture that would be flexibly worked around teacher availability, and should teachers be unavailable, or should interviews be short based upon time constraints, then only a brief follow-up via email would be pursued so as not to burden the research participants. The structure therefore of interviews would focus on two key elements: their understanding of the MNP program’s intent, and a looser structure improvised around the happenings within the lessons observed. Standard questions asked to all participants are listed below:

1. Can you confirm which lessons in the MNP program you taught?
2. Did the lesson(s) go as you intended?
3. What are the key features underpinning MNP lessons?
4. What do you consider to be the most important aspects of the MNP approach?
5. Is there anything you particularly find difficult to deliver using MNP lessons?

Question 4 was designed to elicit any insight into the teacher’s beliefs or alignment with the MNP principles, whereas question 5 was designed to prompt discussion about any elements of MNP lessons that were either desirable but difficult, or undesirable. In turn the responses to those question could offer insight into how agency impacted upon the interpretation and delivery of the intended curriculum of MNP (research question 4).

Due to time constraints for the teachers involved in the study, interviews lasted roughly thirty to forty minutes and took place informally over lunch times. This was in part to ensure
access to teachers and reduce risk of cancellation, but also to try and ease the burden on participants so that positive relationships were maintained. The research was not intended to become burdensome to the participants. Should it become so, it was felt that the research may risk less immersive responses from participants, or an unwillingness to divulge useful insights into strategy and perceptions regarding the lessons. Furthermore, overly long interviews risk becoming repetitive or deviating from the purpose of the study (Creswell & Clark, 2007).

In addition, interviews were primarily to avoid any misinterpretation of purpose during the observation or follow up on any queries arising from observed behaviours and strategies. As such, the interviews would be deemed more likely to yield accurate analysis of the lessons if they followed on shortly after the lesson had finished. In several cases in this study, this was conveniently able to take place almost immediately after the lesson, albeit not by design. Similarly, reflections between case studies were undertaken to determine whether saturation had been reached regarding the usefulness of any specific data captures such as interviews, which proved difficult to facilitate and extract useful data from, which is discussed in more depth in the final chapter.

Year Groups

No preference was given to specific year groups for the study, as long as the classes were primary school age, ie Years 1 to 6. As such, year groups were determined by the primary schools themselves with regards to which would be the most suitable to be observed. It was predicted that this would be determined in part by the staff member(s) who had received training directly by MNP and therefore might be considered the ‘lead’ on the program, which turned out to be true in all case studies.

Gaining Access

Teaching schools and lead schools in Multi-Academy Trusts were considered most likely to be willing to participate as they may be invested in research and wider participation with other schools in furthering professional development – hence the idea of being observed by
visitors would potentially be seen as less intrusive due to the commonality of such experiences in the day to day running of the school. As a practitioner in education, I understood the difficulties in contacting the appropriate member of staff in order to gain access to schools. As such, it was felt that a more personal approach through social media would be more immediately successful (Henderson & Bowley, 2010) than any formal approach without a personalised point of contact. Schools were identified by following the Maths, No Problem! Twitter feed as well as approaching their accredited schools listed on their website. Schools would often be retweeted by the MNP twitter feed when they publicly celebrated the use of the texts, or participation in training events run by the company. Those schools were then contacted, via twitter, to informally enquire as to how long they had been using the texts and whether they would be willing to participate in the study. If responses to those initial messages were encouraging, schools were then formally approached via email directly, outlining the purpose of the study and the desired observations or criteria required.

By using this approach, three schools (Pilot School, School A and School B) were initially approached via social media, and all three granted access for the study. A further two schools were accessed through networks developed within the initial three schools. Specifically, Maths No Problem! encourages participant schools to attend network meetings, and as such, original participants in the study offered suggestions of further schools to approach, and the specific details of headteachers or classroom teachers as points of contact. Neither of these school contact methods involved social media. This resulted in a further two schools (School C and School D) being successfully accessed from recommendations independently suggested from School A and School B.

In total, five schools and seven teachers participated in the study, and a total of nineteen lessons were observed. The pilot study involved a single observation, whereas the main study involved observing a sequence of three lessons per teacher over a period of three consecutive days each. Watching 19 hours of teaching using the MNP program, including six sets of 3 consecutive hours of teaching, suitably delivered enough information and data to convincingly answer the research questions, evidenced in part by the saturation levels reached by the final visit, at which point patterns of adaptation and behaviours had become
minimally distinctive from previous school visits. Furthermore, visiting teachers for 3 consecutive lessons proved invaluable as discussed in chapter 7, due in part to the fact that some lessons were designed as responses to previous lessons, and should the lesson observed have been one of those response lessons in isolation, very little data would have been useful from that case study. In relation to the prescribed curriculum from MNP, which for those lessons was not used at all.

The total time taken to arrange and subsequently visit all eight teachers in five schools took a period of fifteen months. School visits took place between February 2018 and March 2019, and Maths No Problem training was attended in June 2019.

**Ethics**

The ethical considerations outlined by BERA (2011) were adhered to and suitable consent was sought from adults participating in the study. It should be noted that all school visits were conducted prior to the updated BERA guidance published in June 2018, as demonstrated in the completed sample University permissions form in Appendix f and documented in Table 4.3 below.

There was no direct communication with school pupils during the study, nor was any assessment or personal data about pupils sought, seen or stored. Schools had no direct prior connection with the researcher to avoid any conflicts of interest or confidentiality. Any significant details regarding the names of schools and staff, as well as school addresses, have been omitted, and school teachers pseudonymised so that each may be referenced in discussion, and observations have been framed descriptively and without judgment. At no point during school visits did the researcher require access to children beyond lesson observations where members of staff at the school were present. Secure informed voluntary consent was a compulsory requirement before research commenced in schools from participants, and they retained the right to withdraw from the study throughout the process. As students were not the focus of the study, no data was gathered or seen about them. As such it was deemed unnecessary to seek parental consent in addition to the school headteacher’s approval.
While it is acknowledged that interviewing teachers took time away from them and thus potentially added to their weekly workload, all efforts were made to ensure time commitments from participants were kept to a minimum. Interviews, for example, took place at times convenient to the interviewee. Interviews conducted in schools were not shared with other members of staff, and any subsequent publishing of materials from the interviews has been pseudonymised. Teachers reserved the right not to be interviewed or to withdraw from interviews at any time.

No participants were coerced into the research, nor were they persuaded to contribute via any form of manipulation or bribery. The researcher’s positioning within mathematics education was not formally announced or mentioned beyond introduction as a PhD student and formal job title (Secondary PGCE Course Leader), however it should be noted that being an experienced teacher and lecturer in education in itself could potentially influence findings, due to ‘insider knowledge’ of the teaching profession, with specific acuity in mathematics pedagogy. Participants in the study were not made aware of which other schools were taking part, however the two schools that were recommended were aware of those making the recommendation. Specifically, schools A & D were aware each was participating in the study, as did schools B & C but none knew which teachers were specifically observed. Where two teachers were observed in the same school, each knew of the other.

Following the updated BERA ethical guidance incorporating General Data Protection Regulation (GDPR) in 2018 (BERA, 2018), the storage of data collected was reviewed to ensure new guidelines were adhered to.

**Participant Schools**

Geographical locations purposefully differ, with the exception of two schools based in Birmingham. The second Birmingham visit (School D) was based upon a recommendation from the first, School A. The demographics for each school was suitably varied. An outline of demographic makeup can be seen in Table 4.1, taken from the most recent Ofsted reports for each school:
Table 4.1 Demographic information for each school where teacher case studies were located

The teachers were nominated by the lead maths coordinator in each school, and were selected in consultation with the researcher regarding specific criteria preferences. As such, each teacher had a minimum of two years’ experience teaching using the Maths, No Problem! texts, resources and lesson guides – in other words they were familiar with the maths no problem scheme and had experience teaching with it. Teachers C2 and D2 were also the maths leads in their schools. Teacher participants were of varying ages and at different stages in their teaching careers. A summary of the teachers and lessons observed for each case study is displayed in Table 4.2 below.

All participants signed ethical research participation forms in compliance with University and BERA guidelines.

<table>
<thead>
<tr>
<th>School</th>
<th>Region</th>
<th>Class</th>
<th>Teacher Code</th>
<th>Lesson Codes</th>
<th>Visit Dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>Cambridge</td>
<td>Year 1</td>
<td>α1</td>
<td>1B_20_3</td>
<td>07.02.2018</td>
</tr>
<tr>
<td>A</td>
<td>Birmingham</td>
<td>Year 5</td>
<td>A5</td>
<td>5A_6_14b 5A_6_15 5A_6_15b</td>
<td>27.02.2018 - 01.03.2018</td>
</tr>
<tr>
<td>B</td>
<td>Selby</td>
<td>Year 1</td>
<td>B1</td>
<td>1A_9_2 1A_9_3 1A_9_4</td>
<td>13.03.2018 - 15.03.2018</td>
</tr>
<tr>
<td>B</td>
<td>Selby</td>
<td>Year 5</td>
<td>B5</td>
<td>5A_6_7</td>
<td>13.03.2018 - 15.03.2018</td>
</tr>
</tbody>
</table>
Table 4.2 Lesson and teacher information for each case study

Documentation

There were several types of documentation utilised and analysed in this research study – the documentation used was a comprehensive list of what was available to me, but all documents were sampled only rather than analysed across the entire available set, which would include, for example, 12 textbooks and 12 workbooks across years 1 to 6. The Maths, No Problem! resources include a number of texts for utilisation in the classroom, as well as an explicit teacher guide for those delivering the program.

For any given year group, a typical lesson would include the use of a year-specific textbook (for example, in Year 2, there is Textbook 2A, and Textbook 2b) which outlines and duplicates resources displayed on the teachers’ board. A second book is also utilised each lesson, referred to as a ‘Student Workbook’. Again, these are divided into Workbook 2A and Workbook 2B, as they include exercises for students to complete in the book in tandem with the materials set out in the textbook. Hence for any given lesson observation, there are two key texts of relevance – the textbook that includes the lesson, and the matching workbook that students use to carry out exercises from. Both texts were considered critical to the research in order to gain a deeper understanding of lesson design and structure, and to help contextualise this design against other potential methods of delivery. Research question 2, ‘How are textbooks used as teaching aids in DfE approved curriculum packages for primary mathematics in England?’ required deep familiarisation with not only the use of the texts,
but also their content and design features. Unlike in some other educational settings in other countries, these texts are in fact freely available for purchase online, and were subsequently acquired to aid the study.

The Maths, No Problem resources also included a relatively in-depth teacher guide to each lesson, outlining several priorities, assessment strategies and lesson objectives for teachers to use. These documents were more difficult to locate, as they are not available online and are not obtainable without first purchasing a full class set of resources and site license using a school account. Several requests to purchase the guide, or receive a trial version made directly to Maths, No Problem were declined. However, upon visiting the schools, participants kindly printed the relevant lesson resources from their own accounts to aid the research. Despite the difficulty in obtaining these documents, they were incredibly useful and offered a number of interesting insights into lesson design and preferred pedagogical approaches from the perspective of Maths, No Problem! They also enabled a more accurate assessment of teacher agency by allowing for a clearer contrast between the prescribed strategies and resources, and the way in which a lesson was conducted live with students.

Further documentation included Ofsted reports for each of the schools in order to gain insight into the general demographic of the school, and the potential climate within the school itself. Specifically, Ofsted ratings were noted, not in judgment, but to ascertain whether all schools sampled had the same Ofsted rating or not (they did not). Furthermore, Ofsted reports provided information regarding the ethnic makeup of the student population, as well as the proportion of EAL (English as an Additional Language) and FSM (Free School Meal) students (an indication of the relative wealth of the school intake). This information was obtained purely to help distinguish commonalities and differences between the sample schools in order to better understand how similar each school's makeup was prior to visits.

Finally, during several of the school visits, it was recommended by participants that the researcher should attend training hosted by Maths, No Problem! to gain further insight into understanding the purpose of the recommended teaching strategies utilised in the program – and the research that has informed them. As such, a Maths No Problem training program
was attended in Manchester run by the company itself, and subsequent slide handouts and slide notes were incorporated into the research.

The entire data collection involved in this study to construct the multiple case studies was therefore:

1. Five Ofsted reports for participant schools
2. Nineteen Maths No Problem lesson plans
3. Nineteen sets of field notes of observations
4. Six Maths No Problem textbooks (which mirror teacher-led examples)
5. Six Maths No Problem workbooks (which students write in to answer independent practice questions)
6. Eight informal interviews and notes
7. One set of Maths No Problem training slides and notes
8. The Maths, No Problem website and associated hosted blog posts by staff members and teachers from accredited schools

Each case study therefore comprised of three observations of each teacher within a school, one informal interview with each teacher where possible, a set (one workbook and textbook, and three lesson plans) of Maths No Problem resources, and an accompanying Ofsted report for the school.

**Pilot Study**

In order to test the methods developed and assess what data would be gathered, a pilot study was conducted in a primary school in Cambridge (School α). For the pilot study, a Year 1 class was observed where the teacher used the Maths No Problem! texts, lesson plans and resources. The class teacher also had several years of experience teaching this particular program, and was interviewed following the observation.

Following the data collection, a reflexive assessment was conducted to determine whether the data collected during the observation was useful. In order for the data to be
useful, it needed to convincingly communicate the way in which the lesson was conducted, and inform the researcher regarding what both the teacher and students did and said sequentially. Furthermore, it needed to give some signalling into the more subtle elements of the lesson such as positional design, response design, task design, specific vocabulary used, and any indications of teacher emphasis, student struggle and teacher adaptation.

To test and refine the proposed methods of data capture outlined in Chapter 4, a pilot study was conducted at school $\alpha$. This study consisted of a single day visit and single observation of a teacher ($\alpha 1$) teaching a Year 1 lesson (1B_20_3), followed by a semi-structured interview to attempt to capture some of the thinking behind decisions made during the lesson. Following the visit, a number of key findings helped inform and improve the data capture approaches for the finalised case studies:

1: Familiarise with lesson content prior to the observation.

Whilst it was known that the lesson would be with Year 1, the content of the lesson was not known before the observation. A consequence of this was that observation notes occasionally focused more on what was being presented rather than how it was being presented, or the dialogue happening between teacher and pupils. Following this observation, the lesson content would be identified and studied prior to the lesson observation to help focus data collection on the actions of the teacher, and in turn help draw attention live in observations towards any noticeable deviations from MNP content. However, access to lesson plans for MNP was behind a paywall for members only, and membership to MNP at the time of writing could only be granted via subscription, which only a verifiable school could obtain. This meant that access to lesson plans was not possible for several of the observations until later in the study, and so only the textbooks and workbooks were used to identify content and intent at the time of observation.

2: Accessibility for teacher interviews, even in the pilot study, was difficult due to the intensity of the school day. As such, interviews were short and time pressured, and whilst follow-up was offered by the teacher, the time-dependency of accurate and meaningful responses to questions regarding decisions made during the lesson (ie, remembering why decisions were made) meant that interview questions became a secondary opportunistic
data capture during case studies rather than a primary concern. As a result, despite interviews being conducted for all teachers for some of their lessons, the observation data became the sole consistent data capture across each case study.

3: Time stamps indicating the start of each lesson phase (In Focus, Let’s Learn, etc) were added to observation notes after the pilot study due to a difficulty ascertaining how much time-emphasis a teacher had placed on particular lesson components during analysis of observation notes post-lesson.

4: Although it was not pre-planned, lesson observation notes tended to develop into a predominant focus on the words and interactions used by the teacher, and the responses of students. Hence it became particularly important to be able to identify who was saying what to whom, and so clear identifiers were utilised as a result of the pilot study, labelling each quote that was recorded to better record the flow of conversation between teacher and pupils, and pupils amongst each other during group discussions.

Observation Structure

Observations loosely followed a framework introduced by Brown (2009) intended to accommodate the changes in interpretation of the intended and enacted curriculum in the classroom. Brown’s framework considers the relationship between curriculum materials and teacher practice and emphasises being attentive towards how teachers perceive and interpret representations that curriculum texts use to develop and communicate ideas. This aligns closely with the purpose of this study and will produce data that can be analysed to indicate how agency is affecting curriculum intent. Brown (2009) visualised teaching as a design activity and that a significant role of teachers was to navigate what he saw as tension between agent and tool, and he notes that teachers notice and use curriculum materials differently based on experience, intent and ability. Brown categorises three different types of curriculum interpretation: offloading, adapting and improvising. Each category is seen as an observable interpretation of curriculum intent inside the classroom by the teacher, on a
task-by-task basis. Each of these categories can be more explicitly understood with the Brown’s explicit example:

Consider a teacher deciding to set up a classroom experiment based on guidance from curriculum materials. Rather than setting up in the way indicated in the materials, she allows the students to design their own set-ups, but coaches them in the ways indicated in the guidance to ensure their results match the structure and format of the lesson. To explain the ways in which students must analyse their data, she relies on the scripted instructions in the lesson plan. Finally, she initiates discussion with her class using questions in the materials, but diverges into a debate with two students about something they said around interpreting the results of the experiment which departs entirely from the curriculum guidance (Brown, 2009).

In this example, we can see all three categories of curriculum interpretation (offloading, adapting and improvising) at work. When allowing students to design their set-ups, the teacher adapted the existing guidance. She then offloaded agency when relying on the scripted instructions, and then improvised away from the materials as she decided to engage with an interesting debate in her classroom related to the learning. Attempting to identify these three categories of agency and interpretation is likely to require post-observation analysis rather than in-class codifying due to the fast pace of dialogue and interactions in lessons. Hence live note taking in lessons will focus on literal descriptions of what is seen and heard, enabling effective post-lesson analysis utilising Brown’s (2009) analytical framework effectively, and cross referencing with the curriculum guidance and lesson plans provided.

Capturing the actions, questions and techniques used in the classroom, combined with cross referencing against the provided lesson plans and subsequent results of Brown’s framework, will give a strong set of data to help analyse and discuss not only where teachers implemented their own agency, but also the framework helps capture the degree to which they ‘took over’ from the materials (ie, whether they adapted, or improvised). This in turn provides a useful comparative measure between teachers and may prompt some interesting
questions around any significant discrepancies around which teachers do, or do not, significantly adapt lessons provided by MNP in their practice.

However, Brown’s (2009) model highlights only three categories of interpretation regarding a prescribed curriculum package, and each appears, to some extent, to preserve the intent of the curriculum package provider. In addition to this model, a further category needed to be utilised to capture where teachers might deviate from the MNP program in a way interpreted (by me) as against the task intent or principle. For example, if the intent of a task was for pupils to discuss an open problem to derive ideas for discussion with the teacher afterwards, but is delivered in a format that does not facilitate the time required or discussion intended (for example, the teacher allows pupils to think on their own for a few seconds then offers an answer), then the task would feasibly be classed as ‘adapted’ in Brown’s (2009) framework, however that does not accurately convey the impact upon the task intent – which is an important element to consider when answering research question 4: how is teacher agency impacting upon the interpretation and delivery of the intended curriculum in DfE approved curriculum packages for primary mathematics in England? In the hypothetical case described, the impact of teacher agency was to disrupt the intent of the task, not enhance or maintain it. This would not necessarily be captured by focusing solely on a level of detail that highlighted simply that the task was adapted.

With this in mind, the principles of Brown’s model were considered useful but not complete, and a further model, the Innovation Configuration Map outlined later in this chapter was also integrated into lesson analysis.

**Innovation Configuration Maps**

Mary Ann Huntley (2012) developed a process to analyse and evaluate the enacted curriculum called Innovation Configuration Maps. She describes how studying curriculum implementation can be categorised into two areas: quantity and quality of coverage of the intended curriculum. In other words, how much of the content is actually taught or missed out, and how closely what is covered aligns with the author’s intent. How much content is covered is less open to interpretation, in so much as it can generally be measured by whether a teacher does or does not work through a particular chapter, worked example,
practice activity or ask questions provided by, say, a teacher guide. This can be measured both cross-sectionally (within a single or small sequence of lessons) or longitudinally (for example over a term or school year) and therefore feels specifically useful to this study, which as previously mentioned, will be cross-sectional in nature (school visits will be relatively short and contained).

The more difficult element of curriculum implementation to measure or capture in any meaningful way is the quality of coverage of the intended curriculum. Interestingly, Huntley (2012) underscores the risk of viewing the quality of coverage as a dichotomy, which she argues is a crude approach and a weakness of some studies. Specifically, she believes that determining whether a teacher ‘does’ or ‘does not’ adhere to the intended curriculum fails to acknowledge the inevitable nuance of minor adaptations that still remain faithful to the author’s original intent in a dynamic classroom. This spectrum of acceptable changes is referred to by Huntley (2012) as ‘fidelity of implementation’, ranging from practice that modifies major characteristics of the curriculum intent, to keeping major features intact within a given ‘curriculum envelope’ envisaged by the curriculum designers themselves (the intended curriculum). This in turn leads to an inevitable problem referred to by Fullan (2007) as the ‘fidelity-variation dilemma’. Fullan’s work informed Huntley’s (2012) model, and his fidelity-variation dilemma is expanded upon by Ben-Peretz (1990) into a problem of determining what lies within, and outside of a ‘curriculum envelope’ (perceived acceptable changes that remain faithful to the curriculum intent). The boundaries of such an envelope refer to specific content or instructional strategies that are considered integral and fundamental to the intended curriculum. This aligns with what Remillard (2000) calls ‘meaning making’ as a way of enacting teacher agency to become an ‘active designer’ of the curriculum materials and move beyond simple regurgitation of the MNP curriculum towards a more personalised implementation that remains fundamentally faithful to the MNP principles of design. This state of teaching - where the principles of design are effectively indoctrinated in the teacher and utilised in adaptations that work to enhance and personalise the MNP program is considered in this research to be one extreme of what Huntley refers to as the fidelity of implementation spectrum.
Hall and Hord (2006) developed an innovative diagnostic tool they called an Innovation Configuration (IC) Map as a kind of rubric to assess implementation of an innovation on a spectrum. The maps present descriptors of both the ideal way of implementing an innovation, and variations away from that ideal. Specifically, this entails creating a table of important innovation components (per row), and increasing deviations away from each component along the row, emulating a kind of spectrum of acceptable change, with the leftmost value being the ‘ideal’, and rightmost descriptor indicating that deviation is far away from the creator’s intent. Huntley (2012) took this model and adapted it for use in measuring the fidelity of mathematics curriculum implementation. She created IC maps for several areas of curriculum design: “Textbook use” (determining how closely student practice and modelling followed the textbook), “standards-based instructional practice” (related to the national curriculum, not the textbook series she was investigating), “launch”, “explore”, and “summarise” (which each corresponded to identically worded specific phases of the curriculum package lesson design. Each area was subdivided into smaller foci in what equates to several rows of data in the corresponding IC map. For illustration, an example is shown below in table 4.3:

<table>
<thead>
<tr>
<th>1. Textbook use during instruction</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problems from the textbook serve as the basis for classroom instruction. There is no or very minimal supplementation or omission of material from the textbook. Some problem contexts may be altered by the teacher to suit local circumstances but the mathematical content remains the same. The textbook is followed</td>
<td>Problems from the textbook serve as the basis for classroom instruction. There is <strong>some</strong> minor supplementation or omission of material from the textbook. Some problem contexts may be altered by the teacher to suit local circumstances, but the mathematical content of altered problems is the same. The textbook is followed</td>
<td>Some problems from the textbook are used for classroom instruction. However there is considerable supplementation from other sources and/or omission of material from the textbook. Up to 50% of instructional time is used for mathematical learning goals different than the ones outlined for the lesson</td>
<td>The textbook is used minimally, accounting for less than 50% of the classroom instructional time.</td>
<td></td>
</tr>
</tbody>
</table>
from one page to the next in order.

<table>
<thead>
<tr>
<th>textbooks is followed from one page to the next in order.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Textbook use for homework</td>
</tr>
</tbody>
</table>

… … … …

Table 4.3 Example categorisation for an IC Map

As shown in table 4.3, the general pattern across a row is that the descriptors demonstrate gradual deviation away from curriculum implementation fidelity. Whilst there are four statements (A-D), this is not a compulsory component of IC maps, nor does the number of statements need to be consistent across each section (row). In the example above, the authors and researcher determined that anything categorised as A or B fell inside the ‘curriculum envelope’, whereas C and D did not. Hence while B is considered more of a deviation from the intended curriculum than A, it is within an acceptable enacted spectrum that adheres to the principles of intent and is deemed to still have high fidelity.

IC Map Development

The IC maps themselves are carefully constructed by firstly identifying the ‘innovation components’ of the curriculum by reading the descriptive materials provided in the resources and, where possible interviewing the developers of the materials. For this research, the MNP materials will be analysed, including lesson plans and blog posts on the MNP website that both outline curriculum intent and provide insight and explicit overviews of the core principles of the MNP program. Furthermore, a training program delivered by MNP will be attended to further develop understanding of the core principles of the program. In turn, this insight will inform the development of the IC maps that will be completed from lesson observations of teachers using MNP in school settings.

Following this step, the second phase of design is to pilot the IC map and revise and identify any further additional components and variations before determining that the IC maps are complete and ready for a pilot study. Whilst this is feasible for this specific study, Huntley’s guidance suggests that the pilot study in the second phase should encompass a
‘wide range’ of implementers – inferring that the pilot study itself should utilise a large number of participants. Whilst this would be an ideal scenario for MNP research, it would widen the size of the research participant pool and create additional issues around access and identification of schools in what is already perceived to be a relatively small sample space that may be difficult to access. As such, for this research, a pilot study will include only one school rather than several.

Huntley suggests that IC maps are ready when the researcher can confidently answer the question ‘what does implementation of the curriculum look like and how do you know when you see it?’ which she borrows from Hord et al. (2006).

**IC Map validity**

Utilising IC Maps for this research is a manageable and clear way in which to highlight some clear and relatively straightforward comparative measures between the intended and enacted MNP curriculum in schools. It also enables comparative measures across different schools in different settings by using a consistent tool developed against the framework of the MNP program. However it is important to acknowledge the limitations and risks associated with such a tool. Firstly, there are a number of points of interpretation to consider. In the initial design phase of IC Maps, the researcher interprets the intent of the curriculum by analysing suitable materials from the curriculum provider (in the case of MNP, textbooks, workbooks, lesson plans and web support materials such as blog posts and website information). Huntley’s model suggests that this interpretation is then validated by the authors of the materials themselves before being field tested in a pilot study – this is an unlikely scenario in this research, as the authors are numerous and span across different countries. Furthermore accessing them would prove difficult, and getting them to commit and contribute to this research even more so. As such this element of Huntley’s model, and subsequent layer of validation will not be utilised in this research. However the extensive materials available for the MNP program, coupled with access to specific face to face training, should provide a much richer insight into curriculum intent than in Huntley’s examples that provide only a textbook and teacher notes.
In summary, Huntley’s Innovation Configuration Map is a useful tool that will be used in this research as a way to measure the alignment of the enacted curriculum with the intended curriculum that complements Brown’s measure of how the curriculum was adapted, by measuring how that change impacts the intent of the curriculum package. The model will be adjusted to cater for the small scale and scope of this study by reducing the size of the pilot study used to refine the model prior to finalising it, and the model will not be directly informed by the authors of the curriculum themselves. The IC Map will not be the only tool for this study, but will instead form part of a wider discussion and analysis that also utilises in-depth lesson observations and teacher interviews. Categories for the content of the IC Maps were determined through analysis of the MNP curriculum guidance and associated resources and discussed in chapter 5.

Analyzing Classroom Discourse

Traditionally, discourse analysis concerns the use of language and how it is used as a means to achieve a goal – and that the way in which language is used, in a setting with shared, mutually agreed meanings and symbolism constructs our understanding of reality (Starks and Brown, 2007). It defines social roles and serves as an enabler of specific identities. (Chandler, 2018; Lyons, 1995).

Careful analysis of language helps realise social norms, themes of identity and power, and subconscious rules of cultural engagement (Gee, 2014).

This is particularly relevant to this research, in that the discourse between teacher and student will inevitably form a basis for interpreting the way in which mathematics lessons are being taught. The role of teacher and student is often interchangeable between adult and child, as well as between children themselves. The rigidity of the ‘teacher role’ and ‘student role’ can be restrictive and oppressive to deconstruct effectively in order to gain real insight into the nature of strategy, teaching, learning and the evolution of a lesson or series of lessons. Furthermore, discourse as a solely linguistic focus feels too simplistic through the lens of a lesson observation, where there are a number of discrete and explicit communications, prompts and actions both purposeful and subconscious – referred to as ‘micro transactions’ by educationalist Bill Rogers (2004). To wilfully disregard things as
fundamentally important as say, gesture, kinesic and proxemic behaviour, task design or resource presentation to focus entirely on linguistics would potentially lose valuable data and insight into pedagogical strategy. Castañer et al. (2013) found that kinesic and proxemic behaviour evolve as a teacher gains experience, and that novice teachers demonstrate specific attributes with both their gestures and proximity to students that changes over time, to the benefit of their teaching. As such it is important for this research to maintain diligence regarding the subtleties of discourse in order to be able to interpret meaning and intent, supported by interviews with teachers to help align thinking and preserve validity.

**Critical Discourse Analysis**

There is much to consider when gathering data from informal observation settings in schools. It may seem straightforward in that one can take notes regarding what is happening in each lesson and walk away once the lesson has ended, but a trained eye will know the importance of the nuances of information being displayed during a sequence of teaching lessons. Herein we refer to the term *discourse* and *semiosis*. For the purposes of this research, discourse is taken to be the process of determining meaning through a particular social process (here, specifically teaching). Not least speech and vocabulary, but also the importance of roles within the room and how they inform decisions, words and status during encounters and general teaching. Gee (2014) describes this process as seeing interactive communication through the lens of socially meaningful identities. In other words, each action and reaction from both teacher and students is in part informed by their identity and purpose within a larger structure. We are therefore not only concerned with the actions of the teacher, but the societal identity of the teacher, their role as the holder of knowledge to be imparted – or facilitator of learning, and how their identity helps form their decisions and style of pedagogy. Of interest therefore may be clothing, gestures, body positioning and posturing, learning environments, pedagogical tools and technologies, and any other potential social display of beliefs and values. The intertwining of these different aspects of discourse adds a layer of complexity that, whilst certainly important, risks being lost under the relatively language-dominant label of ‘discourse’. As such Fairclough encourages the use of the term *semiosis* to emphasis those more external elements alongside more traditional views of ‘language’ – in all its forms, associated with discourse.
Furthermore, the role of the student in a teacher-student relationship is likely to alter within single lessons as well. Gee (2014) refers to ‘socially significant kinds of people’ as important factors to consider in discourse analysis. For this study, the socially significant roles are those of both teachers and students, and as such time needs to be spent understanding their relative positions within a mathematics lesson, as well as in a wider societal context. Below are three important elements of discourse covered in more depth that are particularly important in the context of this study with respect specifically to how teachers facilitate knowledge acquisition (research question 3).

**Teacher Language – Position Design**

The term *position design* is used hereafter to refer to the design and implementation of language with the goal of shaping how a teacher wants students to think, feel and behave (how they want to *position* the student). Specifically, for this study, the way in which a teacher poses initial questions, presents problems to be solved, delivers instructions and allocates time to enable students to carry out tasks both performance related (ie “*do something*”) and tacitly (ie “*think about something*”) (see also task design below). By understanding the positional design of questioning and tasks presented by the teacher, insights into how teachers facilitate knowledge acquisition (research question 3) can be developed. Again, this focus helps interpret how teachers facilitate knowledge acquisition, but also helps support how aligned teachers are with some of MNPs less ‘tacit’ principles such as how relational understanding is developed, and how pupils are given time to develop ideas through play and knowledgeable others (the principles of MNP design are explored in depth in the next chapter).

**Teacher Language – Response Design**

The term *response design* is used hereafter to refer to the ways in which teachers respond to the data in front of them live (and preempted) in the classroom. Specifically for this study, we are referring here to features such as planning for misconceptions (pre-response design) and any live data from the lesson itself (eg student work, student questions, student errors or gaps in understanding, and behaviour management. Whilst
behaviour management alone is not seen as intrinsic to mathematics pedagogy, it could potentially inform task and position design, as well as tools and resources. For example, a teacher guide may suggest using materials deemed unsuitable for a particular class that involve say, a large amount of ‘mess’, tidying up, or potentially risky equipment such as a pair of compasses. In such a scenario, task design, resources and position design may have been informed directly or indirectly by response design.

**Task Design**

Task design refers to specific tasks asked of students throughout a lesson. Some may be explicitly planned beforehand, such as exercises from the textbook, starting activities, or pre-planned questions to go through as part of teacher exposition. The notion of task design is extended beyond this to include ‘improvised’ tasks during a lesson, for example, a child’s error may prompt an additional short task, or a teacher may feel that students are not ready to move on and create additional activities or extension activities during a lesson. It may also occur that teachers decide to plan an additional lesson or part lesson based upon the way in which the previous lesson was reviewed (ie whether it was determined to be successful or requiring more time). Task design will also incorporate the strategies used to implement the task. For example, a prompt question would be part of a task design, but so also would be the way in which students were instructed to complete it – it could be a timed task, an open task, an independent task, a group task, a written task, a purely thinking based activity and so forth. In some ways, task implementation has the potential to give more strategic insights into position design and the idealised purpose of the role of the student / teacher.
Chapter 5: The Intended Curriculum

In this section I will determine the key features of the intended curriculum package of ‘Maths, No Problem!’ This will be informed by analysing the following four sources of information:

1: The MNP website and blog

The promotional website for Maths, No Problem! includes key information on both the academic influences that helped shape the program, and the key principles that underpin their philosophy. The website also outlines the structure of MNP lessons and explains the reasoning for each phase. Furthermore, many of the principles, structures and influences are elaborated on in more detail through the MNP blog (an extension of the website), which includes more in-depth posts on several elements of MNP design and intent, as well as practical examples of the application of ideas and philosophies in the classroom (Maths No Problem, 2019d). As such, the resource is an incredibly useful insight into MNP design and intent.

2: Analysis of 2 sample textbooks and 2 accompanying workbooks

Identifying key features of design from both the website and training materials in turn determined the features to be examined and identified in the textbooks and workbooks themselves. This was seen as a critical stage of determining the intended curriculum as it is one of the most direct, and most frequent sources of interaction between teachers, pupils, and lesson delivery. The books present the questions, tasks, sequencing and infer the pace of lessons, and so it is important that there is a stage of analysis that attempts to determine the alignment of what is said and promoted in the website and training materials, and what is actually written in the books used in the classroom.

3: Analysis of a sample of 18 MNP lesson plans and guidance notes

At the point at which the lessons to be observed were determined, the accompanying lesson plans provided by MNP were identified and analysed prior to the observation of teachers delivering the lessons. This served as a key data source for the intended curriculum of MNP which, combined with the previously stated sources, acted as part of a wider picture
of how MNP intend their curriculum to be used in class, and what the underlying pedagogical principles are that underpin it. In turn, this was used to inform research aims 1. and 2., namely:

1. Identify specific pedagogical approaches in curriculum packages endorsed by the Department for Education for primary mathematics in England.
2. Determine how the teacher agency of enthusiastic adopters affects the enactment of the intended curriculum provided by DfE endorsed curriculum packages for primary mathematics in England.

Lesson plans typically provided partial scripts, assessment opportunities, differentiation, objectives, and misconceptions alongside a general walkthrough of the lesson materials (see Appendix 1). These will be discussed in depth later in this chapter.

4: Sample MNP training materials

During the research period, an opportunity arose to attend an MNP training course titled *Putting Theory into Practice* which promised to explore:

- The importance of the 5 core competencies
- The development of Mastery Learning
- The application of key learning theories in the classroom
- Teaching through the CPA sequence
- Developing Relational Understanding

Cross referencing information and ideas from the website with, for example, training materials exploring the application of key learning theories would further strengthen understanding of the ideology and underpinning principles of MNP and thus help shape understanding of the curriculum package intent.

Part One: The MNP website and blog

The MNP website https://mathsnoproblem.com/ acts as an information point to help understand various elements of the MNP program, including an overview of the program,
testimonies, sales information, demos and training events. Of particular interest to this study were the following sections:

- **Maths Mastery Guide**
- **What is Maths Mastery**
- **What is Singapore Maths**
- **The CPA Approach**
- **Blog**
  https://mathsnoproblem.com/blog/

The above pages give insight into what MNP interpret as good practice, as well as linking to blog posts on specific academics who have influenced the MNP program, and detailed advice on how to teach. For the purpose of this research, blog posts hosted on the MNP website, which are authored by MNP staff and approved MNP lead school teachers, are assumed to be endorsed by MNP.

**Maths Mastery**

According to the MNP website, MNP promotes Maths Mastery, which it defines broadly as “an approach to teaching that gives pupils a deep, long-term, secure and adaptable understanding of mathematics with a uniform degree of understanding promoted across all pupils and actively discouraging streamed classes and groups based on perceived ability” (Maths No Problem, 2019b). MNP promotes all students going through mathematics materials in depth and at the same pace and the sharing of ideas amongst peers to solve problems using a variety of methods. Furthermore, the website cites Singapore Maths as both the original influence of Maths Mastery, and a foundation of MNP (Maths No Problem, 2019b). According to MNP, Singapore Maths places additional emphasis on problem solving, visualising maths, and a ‘minimal dependency’ on rote learning and procedural memorisation. Interestingly, the emphasis on problem solving in Singapore Maths is later
cited as being heavily influenced in Singapore by the Cockcroft Report published originally in the UK in 1982 (Fournier, 2022b) suggesting that the DfE are seeking to learn from countries that themselves learnt from England.

**Notable Expert Influences**

In the Maths Mastery Guide webpage, MNP cites five key ‘notable expert’ academic influences on their program: Jean Piaget, Zoltan Dienes, Lev Vygotsky, Richard Skemp and Jerome Bruner (Maths No Problem, 2019b). For the purpose of this research, focus will be maintained not on what these academics have written or whether their ideas are being interpreted accurately by MNP (which could easily unravel into another thesis), but instead, the focus will be on what ideas MNP specifically define as design principles used in the program which they attribute to each expert.

**Jean Piaget**

A specific idea attributed by MNP to Jean Piaget is the notion of students being given ample time to develop new ideas (Maths No Problem, 2019b), which manifests in MNP lessons as what they call an ‘anchor task’ - a single task that is used as the dominant feature of a lesson referred to within the textbooks as a lesson phase called ‘In Focus’. According to the MNP textbooks, ‘In Focus’ tasks are intended to be spent analysing and developing understanding of a key concept or idea with only a single question prompt. In a blog post about Piaget’s influence on MNP, Senior Copywriter at the company Chris Fournier writes that Piaget was an advocate of children constructing understanding of the world through active interaction and experimentation as opposed to didactic and rote learning methods that were dominant at the time (Fourier, 2022a). This has in turn influenced MNP to embed allowing ‘ample time’ for pupils to spend on a singular problem that encapsulates development of a key mathematical idea - the anchor task for each lesson. Maths, No Problem! series consultant Dr Yeap Ban Har describes Anchor Tasks as a lesson phase where “children work in groups on a single problem, surface what they know and the teacher then extends their understanding” (Ban Har, 2017). This phase covers multiple principles as we will discuss during this chapter. In this section, the principle of providing ‘ample time’ to develop new ideas, attributed to Piaget, is integrated into the In Focus task by allowing
pupils to spend time with a single problem rather than expecting them to work through several examples in a short period.

In addition, education consultant and MNP ambassador Currell (2018a) explains in her blog post for MNP that nurturing intrinsic motivation in students via journaling is also loosely attributed to Piaget, citing Piaget as a proponent of constructivist knowledge development, promoting motivation through internal reflection (manifesting as MNP journaling tasks), and peer collaboration on anchor tasks (Currell, 2018a). Currell discusses how Piaget’s ideas of knowledge acquisition using assimilation (relating new knowledge to existing schemas) and accommodation (adjusting existing schemas or creating entirely new ones where new knowledge contradicts or further develops understanding of what is already known) are also fundamental to the design of MNP discussion times and anchor tasks - allowing children time and space to perform their own assimilation and accommodation of new concepts and ideas rather than simply storing new knowledge as it is didactically delivered by the teacher.

Piaget’s influence on MNP appears limited to, or at least reduced to, two core ideas: an emphasis on spending more time on singular problems - a feature that is prominent in Singapore Maths lessons (Wong, 2009), and a further idea around a constructivist approach to knowledge acquisition which encourages play. Piaget’s constructivist views of knowledge acquisition are emphasised by MNP as being a key principle of their programme, which manifests explicitly in an emphasis on discussion, play with concrete models and visuals, and emphasis on exploration and multiple methods to reach answers to problems (Frier, 2022). Louise Freir blogs on the MNP site that Piaget’s influence is specifically that ‘play is integral to the development of intelligence in children’ (Freir, 2022). In this context, play is taken to mean exploration using concrete manipulatives, and discussion with peers. This process is described more broadly in Currell’s post titled ‘How Maths Mastery Fosters Intrinsic Motivation In Learners’ (2018), where she describes it as giving students the floor, and getting out of the way to give students more autonomy over their learning” (Currell, 2018c).

Interestingly, Piaget is not explicitly recognised by MNP as a contributor towards their advocated CPA model of teaching despite his ‘four stages of learning’ being generally acknowledged as a major influence on the model alongside Bruner’s work (Merttens, 2012).
It appears that, perhaps for simplicity, singular ideas are generally attributed to singular academics on the MNP site.

In summary, Jean Piaget, as a notable expert influence on MNP appears to be attributed to the following key principles of MNP design:

- Allow ample time for students to develop ideas.
- Allow for play in lessons to enable constructivist development of knowledge.

Zoltan Dienes

Hungarian mathematician Zoltan Dienes is the second cited influence on the MNP program and is accredited (by MNP) as promoting manipulatives, narrative structures, and games as effective ways to teach mathematics to primary age children (McIver, 2020). Dienes observed that through manipulative, games and narratives, young children were able to grasp mathematical concepts earlier and quicker than without them (Dienes, 1963). Dienes also invented mathematical tools referred to in modern day as Dienes Blocks to help children learn the underlying principles of some mathematical concepts - and these tools are used and promoted by MNP in their website and accompanying textbooks. McIver’s blog post on MNP’s site titled ‘Zoltan Dienes and teaching mathematics through games’ highlights that Dienes’ influence on MNP is again directly notable in the Anchor Task section of a lesson, in which students are encouraged to be ‘playful’ in exploring a mathematical idea or solution to a problem - although it is not clear from the blog post alone what ‘playful’ looks like in the classroom, or whether MNP is equating ‘learning through play’ with ‘peer discussion of open problems’ which appears somewhat removed from, say, gamification of concepts. Currell (2018b) sheds light on further areas of MNP that are directly influenced by Dienes, specifically the stories and pictures used across the textbooks. Chapters make use of the same familiar characters in various situations having to overcome problems in a narrative-based environment - which is a Dienes inspired strategy to embed narrative structures in children’s learning (Currell, 2018b). The difficulty in formalising what is meant specifically by ‘learning through play’ and ‘learning through games’ becomes simplified.
through Dienes’ own analysis of the comparatively useful subskills used when playing, which he layered into stages of learning as shown in Figure 5.1:

![Dienes' six stage theory of learning](image)

**Figure 5.1 Dienes' six stage theory of learning taken from Currell (2018b)**

It is through this lens that the idea of ‘learning through play’ in mathematics is perhaps clearer and more aligned with mathematics specifically, in that ideas are ‘played with’ initially, in order to familiarise oneself with materials through trial and error, before regularities and patterns of behaviour begin to expose themselves as ‘rules of the game’ - at which point, Currell (2018b) indicates that classroom discussions should begin between students, who then compare what they have discovered and articulate their discoveries before moving towards abstraction and developing pictorial representations of ideas, then forming a language to describe the ideas and concepts (this is listed as Symbolisation in Figure 5.1 above), then finally formalising the system by testing it against their perceived truths and theories. Similarly, McIver highlights that games promote trial and error, simplifying difficult tasks, looking for patterns, testing hypotheses, reasoning and proving / disproving ideas.

In summary, Zoltan Dienes, as a notable expert influence on MNP appears to be attributed to the following key principles of MNP design:

- Allow students to learn through play and exploration
- Use a variety of representations of mathematics including narrative structures and physical manipulatives

**Jerome Bruner**

Jerome Bruner was an American psychologist who made significant contributions towards cognitive psychology and cognitive learning theory. Bruner’s work included development of constructivist theory, which indicates that learning is an active process
where new knowledge, ideas and concepts are constructed by learners based upon current and past experiences and knowledge. In constructivist theory, learners select and transform information based upon their own existing schemas - and in doing so either create new schemas, or modify existing ones to accommodate new information. A simple example in mathematics could be that a learner develops their own interpretation of what a triangle is, based upon the information provided and from their own exploration of the shape. They may determine that all triangles have three sides, but may develop misconceptions in their initial schema such as deciding that the shape need not be *closed* or that *orientation* is important in defining the shape - but these misconceptions are later self-corrected by assimilating new information, thus their schema are modified to accommodate it. Bruner often wrote specifically about mathematics, and indeed, sometimes observed Dienes teaching (Bruner & Kenney, 1965). A specific example Bruner himself used when discussing his theory is outlined below:

In mathematical factoring, to start with an example, the concept of prime numbers appears to be more readily grasped when the child, through construction, discovers that certain handfuls of beans cannot be laid out in completed multiple rows and columns.

(Bruner & Kenny, pp51-52 1965)

This constructivist approach should not be misinterpreted as learning in the absence of a teacher, as Bruner, (and indeed MNP) was keen to point out, a good teacher must give students the information they need without organising it all for them. Bruner refers to this in his 1957 paper “going beyond the information given” as providing a ‘minimum set of propositions that will permit the largest reconstruction of unknowns’ (Bruner, 1957) and uses the term ‘scaffolding’ to describe this teacher role: the support needed from an experienced adult to assist in the natural development of a more inexperienced learner.

Whilst it may seem excessive to discuss Bruner’s work around the construction of knowledge, it is important in the context of this study, with regards specifically to research question 3: *How do teachers facilitate knowledge acquisition using DfE approved curriculum packages for primary mathematics in England?* If Bruner is indeed influential in the way in which MNP is designed and intended to be delivered, it is likely that the intended
curriculum package at least, incorporates and facilitates tasks and instructions to allow for pupils to construct their own understanding and knowledge of mathematics with the teacher acting as a facilitator and enabler rather than, say, assuming the role of an expert transmitting their own knowledge to the pupils. Indeed, Currell (2018a) explicitly states in a blog post titled ‘How to put Bruner’s key theories into teaching practice’ that pupil discoveries become more meaningful and deeply embedded, and that the role of the teacher is to facilitate experiences and provide the tools, task, questions, and opportunities for exploration rather than methods and procedures.

Currell (2018a) points out in her MNP blog post that the difficult part of teaching in this way is facilitating students constructing this knowledge instead of simply telling it to them, in a way that ensures that the correct interpretation of concepts is embedded – in other words, misconceptions are likely to arise during this process, but learners are guided towards understanding real truth and adapting their understanding (accommodating into their schema) as they progress throughout the lesson.

Bruner is also referenced by Dr Yeap Ban Har as developing what is referred to in Singapore Maths as the Concrete, Pictorial, Abstract (CPA) approach to learning (Bruner’s work refers specifically to ‘modes of representation’), a concept that is outlined explicitly as a core modelling principle of MNP’s program (Maths No Problem, 2019a) with Ban Har describing it as a way to connect children’s everyday real and imaginary worlds to mathematical ideas (Ban Har, 2021a; 2021b). Bruner did not use the terminology ‘Concrete, Pictorial, Abstract’, but instead introduced three key types of representation in learning: enactive (physical movement), iconic (image based) and symbolic - using letters, symbols and numbers. In addition, Maths, No Problem! explicitly includes ‘language’ in this category (Currell, 2018a).

What is the CPA model?

An example of the CPA progression, taken from MNP’s website, is shown below:
Here, multilink cubes are used to represent a number bond to five. Whilst this is necessarily pictorial in a written document, students would physically handle multilink cubes and construct them in a similar fashion to those shown in the diagram. This highlights the first of Yeap Ban Har’s indicators of advanced knowledge of a topic: showing a physical model. Following this, students would move towards pictorial representations of the same idea. In the diagram, this pictorial representation is specifically a bar model or part-part-whole model (MNP usually refer to them as bar models). Here, an important feature for this example is that the objects are no longer countable - you cannot, say, use a finger to count each segment and arrive at $3 + 2 = 5$, and so an element of abstraction is also introduced - whereas you can do that in the pictorial representation of the multilink cubes. This highlights that pictorially, there are a number of different ways to represent a concept, with differing layers of complexity and difficulty and this is arguably more of an issue than in, say, a concrete model. Similar pictorial models could be achieved with other tools such as a ‘ten-frame’ as shown below:
The ten-frame could be a physical one, or a pictorial one as shown above. Again, this is less abstract than the bar model in the first diagram, as objects are countable and no symbolism is used. Pictorial representations not only serve as the P in the CPA model, they also deliver on Yeap Ban Har’s second measure of ‘advanced understanding’ of a concept: *Draw something appropriate for problems* (Ban Har, 2017). Hence there is clear integration of elements of curriculum design and curriculum assessment across MNP. On this point, whilst the concept of CPA modelling may appear relatively simple, it’s clear that there is nuance regarding the suitability of ‘correct’ visual and concrete modelling tools, and indeed selecting less effective or even poor models at these stages in particular, can be detrimental.

Ma (2010) for example, highlighted that many American teachers in her comparative research study cited incorrect or poorly thought through models to convey mathematical ideas. There appears therefore to be a friction between allowing students the flexibility and autonomy to explore and play with mathematical ideas, whilst at the same time, ensuring that models and visual representations are high quality and mathematically sound, without risking misconceptions to develop through the choice of models used, or for understanding to be difficult to develop through poor or inconsistent modelling. The standardisation of models across MNP (for example, the specifically highlighted number bonds tools, bar models and Dienes blocks on their website) is perhaps a strategy to counteract any teacher use of inappropriate or less effective models and leans into a ‘teacher-proof’, rather than ‘curriculum proof’ approach to modelling which will be discussed further in the section ‘the intended teacher’ later in this chapter.

Finally the abstract representations replace visuals with symbols, numbers and letters. In the example in Figure 5.2, numbers are symbols (2, 3, 5) which are harder to conceptually understand than their concrete equivalents, and further abstract symbols are introduced (‘+’ and ‘=’), both of which are implied in the previous incarnations (concrete and pictorial representations), but are more explicit here. This leans into two of Yeap Ban Har’s measures of advanced understanding ‘explain the problem orally’ and ‘explain the problem in written form’ (Ban Har, 2022).
Bruner emphasised that a learner should move between (not necessarily sequentially) each type of representation, and only when they are able to work confidently across all three representations do they have true understanding of a concept (Bruner, 1966). Bruner acknowledges that each phase may be more or less suitable for different domains of knowledge. For example, a physical, ‘enactive’ representation of a law may be harder to conceptualise than, say, a symbolic representation. This is echoed in MNP by Dr Yeap Ban Har (2017) who states that a learner is ‘advanced’ in a concept when they can:

1. Show a physical model
2. Draw something appropriate for problems
3. Explain the problem orally
4. Explain it in written form
5. Find ways to challenge themselves

Point 1. alludes to the “enactive” representation, point 2. The “iconic” representation, and points 3 and 4 the “symbolic” representation. What is unclear however, is whether these representations should be directed by the teacher, or formulated by the child themselves. It is important to note that Bruner’s ideas came with specific mention of the sequential inference of the enactive, iconic and symbolic representations, pointing out that mathematical ideas or teaching may not be in the order of enactive, then iconic, then symbolic representation – because the CPA model that MNP uses frequently does infer that the steps are sequential (Maths No Problem, 2019c), stating “It involves moving from concrete materials, to pictorial representations, to abstract symbols and problems” which was confirmed in the research conducted for this thesis which did uncover some general trends within MNP’s content and teacher observations that the CPA model is predominantly taught in sequence (discussed in the next chapter). Indeed, blog posts from practitioners confirm the sequential use of the CPA model, such as Currell (2018a) stating:
“We began with multi-link cubes, progressed to the pictorial problems in the book and then on to questions with no visuals provided except numbers and the necessary mathematical symbols.”

Similarly, Pinnock (2022) mentions that we ‘start with’ real life objects, and ‘the pictorial part of the approach follows on from the concrete’, Dabell (2018) talks of the CPA ‘sequence’ in a post about CPA misconceptions and Dr Yeap Ban Har (2021a) refers to the CPA model as the use of concrete experiences to ‘move students to abstract learning’. It appears therefore, that MNP is not only promoting the explicit use of concrete, pictorial and abstract mathematical representations, but also promoting a movement through concrete, into pictorial and towards abstraction which should therefore be mirrored in the sequence of learning in the MNP textbooks. It is also worth noting that Ban Har specifically uses the term ‘concrete experience’ here, which is echoed in a blog post from Dabell who states “‘concrete’ doesn’t refer to just ‘concrete manipulative’ but also ‘concrete experiences’ through activities with suitable manipulatives” (Dabell, 2018).

Contrastingly though, some general misconceptions about CPA are highlighted by Dabell (2021) in his MNP blogpost, in which he directly addresses the linear progression model of CPA and states, in contrast to many of the other posts in the blog, that CPA need not be pursued in a linear fashion and may, in some circumstances, move from abstract to concrete for example. This evidences a friction between Bruner’s model which specifically emphasises that the model should not necessarily be taught in sequence, Ban Har’s specific wording about the CPA model being sequential, and endorsed blog posts giving somewhat mixed messages around its use. Considering how integral the CPA model is to MNP, this feels like an appropriate focus of analysis when studying the textbooks and workbooks for this research: how much the CPA model is mirrored in the books and whether it appears to confirm sequential progression as stated in the website.

In fact, as we will see in chapter 6, teachers using the MNP program were confident and comfortable to move between all three models interchangeably, but generally followed a model of introducing concepts as concrete examples, pictorial representations, then abstract mathematics.
In summary, Jerome Bruner, as a notable expert influence on MNP appears to be attributed to the following key principle of MNP design:

- The Concrete, Pictorial, Abstract approach to modelling and understanding mathematics

**Vygotsky**

Lev Vygotsky was a Soviet psychologist known for his concept of the zone of proximal development: the distance between what a student can do independently, and what they are able to do with support from a knowledgeable other. The lower limit of this zone would be the level a child reaches without a knowledgeable other, and the upper limit is the potential level a child can work to with the assistance of a knowledgeable other. Currell (2021) blogs about this for MNP, stating that in practice, the zone for proximal development manifests in MNP as designing lessons that fundamentally ask ‘what are my students ready to learn now?’, broadly matching student ‘readiness’ and pitching at a level that is above their current cognitive level so as to avoid ‘stagnation’, yet not so challenging that it is out of reach of the learners at their current level. In other words, the level of struggle must be carefully considered so that a balance is struck between challenge and accessibility. Vygotsky’s theory is that children learn best when they are just at the edge of their competency, highlighted in the figure 5.5 below.

![Zone of proximal development](image)

*Figure 5.5 A visual representation of Vygotsky’s Zone of Proximal Development taken from Wikimedia (2022)*
MNP interpret Vygotsky’s influence directly as “Vygotsky emphasises cooperative learning or collaborative structures during learning. His theories of proximal development and scaffolding show how students progress through interactions from the current to the potential area of development.” (Maths No Problem, 2019d). While the former point is discussed below, the latter is a significant challenge for this research in that whilst it is an explicit principle highlighted by MNP, it is not a particularly clear strategy to either identify and confirm in workbooks or textbooks, nor is it an easily observable pedagogical strategy or behaviour in the classroom. Indeed, there is a notable absence of blog posts and support pages regarding this specific influence (it is the only one without a specific blog post addressing how it manifests in the classroom). As such it does not feel useful without further interpretation, and consequently, in order to be transparent, I am taking the concept of ‘readiness’ in Currell’s (2021) blog post, that references the zone of proximal development, to support the notions of small learning steps and moving the whole class through the same content at the same pace. Both of these ideas are cited as elements of the mastery principles that MNP is built upon (Maths, No Problem, 2019b), and are more clearly observable design principles that arguably align closely with Currell’s interpretation of Vygotsky’s influence on the program.

An extension to this notion of working with a knowledgeable other, is Vygotsky’s argument that children learn better through interaction and discussion with knowledgeable others and peers who may share ideas and insights that help shape understanding. He emphasises social interaction and individual thinking as imperative for maximal learning (Shabani et al., 2010). which is noted in a blogpost by Bowles (2021) as a fundamental element of good teaching using the MNP program. Similarly, Currell (2021) emphasises how important Vygotsky’s thinking is in the design and embedding of discussion opportunities in MNP textbooks and lesson plans, highlighting that the ‘knowledgeable other’ is often another student who is further along with their learning, and can therefore enable rich insightful discussions that aid both learners to develop their understanding of a concept. The knowledgeable other (child) is able to develop their ability to explain concepts and problems (which fits with points 4 and 5 of Ban Har’s “advanced learning” criteria), whilst the other learner benefits from the social interactivity of speaking to a knowledgeable other. Again,
this strategy leans into the constructivist theory of learning and is therefore relevant for research question 3: students are constructing their own understanding through facilitation, rather than receiving knowledge from a teacher in a didactic manner. This facilitation in MNP is emphasised by Currell (2021) as being predominantly in the ‘anchor task’ at the start of the lesson, although she also acknowledges that facilitating meaningful and effective discussion can be difficult, but offers advice on how to foster the right environment in class by, for example, creating simple rules around how group discussions must be enacted, deciding upon minimum and maximum group sizes, and carefully constructing the learning environment to ensure collaborative work is easy to facilitate. Yeap Ban Har briefly describes the main influence of Vygotsky on MNP as being an emphasis on "cooperative learning or collaborative structures" (Ban Har, 2021a).

In summary, Lev Vygotsky, as a notable expert influence on MNP appears to be attributed to the following key principles of MNP design:

- Facilitate opportunities to learn from ‘knowledgeable others’ – both peers and teacher, in discussions such as those used during the Anchor Task in lessons.

The influence of Vygotsky’s zone of proximal development has been interpreted to inform the following key principles of MNP design:

- Small steps teaching
- Moving the whole class through the same content at the same pace

Richard Skemp

Richard Skemp was a renowned British mathematician and psychologist who’s work around mathematics education was held in high regard by peers such as Zoltan Dienes, Pierre Van Hiele and Anna Sfard - each of whom contributed towards “Intelligence, Learning and Understanding in Mathematics - A Tribute to Richard Skemp” (Tall & Thomas, 2002). Indeed Dienes writes of Skemp “his fairly recent book written for practising and future teachers is possibly the only ‘textbook’ that I could recommend as one for a course on
mathematics education” (Dienes, 2002). Skemp’s paper, ‘Instrumental and Relational Understanding’ focused on what he considered to be two distinct versions of teaching and understanding mathematics. The paper is specifically mentioned on the MNP website (Maths No Problem, 2019c). What Skemp called ‘instrumental’ understanding, is now sometimes referred to as *procedural knowledge*, that is, the ability to carry out a set of procedures or follow a set of rules to achieve a goal in maths. Rachel Helwig-Henseleit (2022), a MNP employee, refers to this type of knowledge as ‘rote learning’ in her blog post for MNP titled ‘Richard Skemp’s mark on maths mastery’. Whereas *relational understanding* refers to a deeper knowledge of underpinning structure and linkage between mathematical ideas.

These two ideas, Skemp argues, were until this point the *faux amis* of “mathematics”, in other words, two very different notions of what mathematics *means*. Skemp states that many teachers taught *instrumentally* rather than *relationally* although it is unclear how he evidenced this, and whilst the paper was written in 1977, it is worth noting that in England, Malcolm Swan (2006) made similar findings and, in 2012, Ofsted produced a report titled “Mathematics: Made to Measure” after observing over 1,200 secondary and 470 primary mathematics lessons across 320 schools which again confirmed a significant proportion of teachers focused predominantly on instrumental teaching, but should be developing relational understanding as well (Ofsted, 2012). It appears then, that MNP’s citing of Skemp’s work as a direct design influence correlates with the DfE’s desire for what Skemp refers to as relational teaching - a focus on why things work in mathematics to complement how to do them. Despite Skemp being keen to point out the importance of both instrumental and relational learning, several blog posts across the MNP website speak of ‘rote learning’ (equated to instrumental learning by Helwig-Henseleit ) and often in disparaging terms. McIver (2020) for example, refers to ‘the drudgery’ of rote learning, Fournier (2022) writes that rote learning risks creating learners with ‘empty shells’ and Pinnock (2022) claims “Rote learning will only allow pupils to imitate an adult’s methods in maths, which will result in them being unable to problem solve for themselves and transfer their understanding of mathematical concepts to new situations.”
It appears then, that while Skemp attempts to illuminate the need for both types of knowledge and move teachers towards embracing the use of them alongside each other, MNP blog posts often present instrumental knowledge in a more negative light. In a similar pattern, there is a lot of emphasis on constructivist approaches to knowledge acquisition, and didactic approaches are subtly discouraged which appears to present another dichotomous idealism: enquiry vs didactic teaching. In fact, this negative perception of the role of instrumental knowledge, which is subtle and predominantly displayed in blog posts rather than the more formal pages of the website, appeared to manifest in lessons as will be discussed in chapter 6 in more depth later. It was noted in several observations how teacher agency affected this particular area: teachers actively excluded, or seemingly demoted instrumental knowledge activities in favour of relational knowledge activities.

Interestingly, Helwig-Henseleit (2022) also highlights the risk of textbooks promoting instrumental knowledge when teachers seek to teach relational knowledge and cites concern over students wanting to learn relational knowledge when teachers teach instrumentally. It appears then, that the MNP textbooks and workbooks actively promote relational knowledge, however it is less clear whether instrumental knowledge is favoured in any capacity, or whether it is seen as unfavourable beyond subtle inferences within the MNP blog posts. This appears to be an issue that is addressed directly by MNP on their YouTube channel, where a short clip titled “The limits of teaching by enquiry” speaks directly about the dangers of assuming didactic teaching has no place in MNP approaches (Maths No Problem, 2022). The educator in the video, Adam Gifford, highlights the specific example of counting: presenting children with a randomly ordered set of numbers and expecting children to be about to ‘discover’ how to count in sequence. He calls the notion ‘ridiculous’ and affirms that there is indeed nuance and professional judgement to determine when and where to use didactic teaching. He states in the video that the key to good teaching is to ask ‘is this strategy going to benefit the child?’ And that this is what teachers should ask themselves when considering whether or not to use enquiry approaches or didactic teaching (Maths No Problem, 2022). The fact that the video was produced at all, alongside the disparaging comments from bloggers on the MNP site itself, seem to indicate that interpretation of the status of enquiry in relation to didactic teaching has proven challenging amongst teachers in schools. There is a more curious element to this discrepancy as well, in
that Bokhove (2021) discussed how textbooks in England fell out of favour for a prolonged period of time and were, at the time, sometimes cited as being the cause (or tool) of poor teaching. MNP, helped by endorsement from the DfE, uses textbooks as a dominant element of their lesson structure but is arguably working against the notion that textbooks are interlinked with poor teaching. Could it be then, that this apparent discomfort with didactic teaching in any form is in some way part of a wider discomfort with the optics of teaching with a textbook? Or might this simply be an over emphasis on one style of teaching over another, misinterpreted as a dichotomy of teaching styles?

In summary, Richard Skemp, as a notable expert influence on MNP appears to be attributed to the following key principles of MNP design:

- Focus on and develop both instrumental and relational understanding in mathematics

A brief summary of the 5 ‘Key Learning Theories’

MNP cites Singapore Maths as being a significant influence on the program, and Yeap Ban Har points explicitly to 5 ‘key learning theories’ that inform the MNP strategy. Each of these theories, he attributes to a different academic: Piaget, Skemp, Bruner, Vygotsky and Dienes. After analysing specific mentions of these academics across the MNP website, including various blog posts as well as dedicated feature pages, the 5 theories as stated by MNP (which may be better thought of as guiding principles) can be distilled into the following:

1. Children need time to accommodate new ideas - and construct their knowledge from the world around them.

2. Children need opportunities to play with mathematical ideas in an informal and exploratory way prior to any structured learning.

3. Learning is effective when cooperatively learning with knowledgeable others.
4. Relational understanding is emphasised alongside instrumental understanding.

5. Mathematics learning should utilise the Concrete-Pictorial-Abstract approach to develop understanding of concepts.

As we have seen from further analysis, I have identified several other *implicit* influences from each of these academics that have informed MNP’s guiding principles. These are:

1. Small steps teaching
2. Moving the whole class through the same content at the same pace
3. Use a variety of representations of mathematics including narrative structures and physical manipulatives

These principles primarily focus on how to teach (research question 1), rather than how children demonstrate their understanding (research question 3). However, Maths No Problem also emphasise their principles around identifying student readiness to progress, in the form of what they call ‘five core competencies’.

**MNP’s Five Core Competencies**

Several blog posts on the MNP site refer to their five core competencies: attributes that help learners develop deeper thinking (these are not to be confused with MNP’s ‘five learning theories’, or Ban Har’s ‘five indicators of advanced learning’). The competencies are listed as:

1. Visualisation

   Learners are expected to be able to communicate ‘how they know’ at all stages of explanation. Typically this may include drawing pictures or supporting ideas with a variety of reasoning strategies. Reasoning is expecting to go beyond instrumental understanding and demonstrate relational understanding.
2. Generalisation

Learners are expected to be able to ‘go further’ and use proof as a rationale. Advice offered here includes making ‘always, sometimes, never’ or ‘true / false’ statements with justification (Hopwood, 2022)

3. Communication

Learners are expected to answer in full sentences using mathematical language and terminology. For example, if asked ‘what is three add five?’ a child would be discouraged from providing just the answer ‘eight’ to a calculation verbally, but instead would be prompted to answer in a full sentence ‘three add five is equal to eight’.

4. Number sense

Learners develop flexibility and fluidity in the use of number. For example, understanding different representations of number, number symbols and vocabulary, the relationships between numbers and quantity, systematic counting and competence with simple number operations.

5. Metacognition

Help learners ‘think about how they are thinking’ in order to improve multi-step problem solving. Here, Alex Laurie (2020) elaborates on this competency in a blog post titled ‘how to develop learners’ metacognition skills with effective questioning’ and explains that MNP discourages the over-use of simple questions that require a yes/no or right/wrong answer, and instead encourage teachers to allow for opportunities for students to reason and reflect upon how they reached an answer, or how others have done so by asking peers to reflect upon how another member of the class has used a different method to their own, and why it makes sense. Laurie (2020) provides a number of explicit starting points for questions that encourage reflection, such as:

- What happens when we…
- How many ways can you find to…
- What is the same and what is different?
- What do you think comes next and why?
• What would happen if…
• Why do you think that?
• What made you decide to do it that way?

Furthermore general questioning strategies are discussed in the blogpost ‘how to improve your questioning skills’ by Adam Gifford (2020), in which he discusses a number of strategies that align with previously mentioned underpinning principles of MNP, such as supporting struggling students by asking them to model answers visually (see Figure 5.6),

![Figure 5.6](image)

Figure 5.6 an example of a ‘what do you notice’ task taken from Gifford (2020)

asking students ‘what do you notice?’ and prompting them to discuss their observations in groups, challenging students to come up with their own problems or further their knowledge by applying it to a new situation. For example, by asking ‘What would happen if you changed the numbers in the problem?’ or ‘if I find the solution, what other information can I find out?’.
In figure 5.7, prompts are notably open, and align with the 5 core competencies more generally, as well as appearing to fit with the learning theories discussed previously - promoting for example, collaboration, strong communication of ideas, construction of knowledge with the teacher in a facilitation role, visualisation of concepts and the CPA model.

Whilst the five competencies are mentioned only briefly in the MNP site, they do appear integral to the MNP strategy and will be reflected upon when analysing the textbooks, workbooks and lesson delivery. For example, the emphasis on open questioning and discouragement of closed response questions was easily identifiable in lesson observation analysis discussed in the next chapter. Similarly, expectations around enabling students to reason their answers and communicate in full sentences are relatively easy to identify in lessons and analyse.

**MNP Lesson Structures**

In this section I will explain the different phases of Maths, No Problem! lesson structures, and map against each phase the ways in which they do or do not align with the intended principles identified within the MNP website and blog posts discussed so far. This effectively
is the second of a three-point triangulation of the MNP principles and core competencies between:

1. What is stated in the website and blog posts
2. What is intended within each lesson phase and how that maps against the principles and core competencies identified in 1. above
3. What is contained within each phase across a sample of 2 textbooks and 2 workbooks and how that maps against the principles and core competencies identified in 1. above

This triangulation of sources enabled an interpretation of the intended curriculum of Maths, No Problem! that was robust enough to evidence how what is explicitly and implicitly said by MNP is (or is not) demonstrated in their resources that are used in the classroom.

Dr Yeap Ban Har describes the MNP textbook design being based upon a 3-part lesson:

1. The Anchor Task
2. Guided Practice
3. Independent Practice

However, when analysing the textbooks and workbooks, four phases of the lesson were more easily identified for analysis by their respective phase titles ‘In Focus’, ‘Let’s Learn’, ‘Guided Practice’ and ‘Independent Practice’. Each of these phases were included in every MNP lesson, with an additional two phases occasionally introduced called ‘Maths Journal’ and ‘Activity Time’.

Each phase, including the two less common phases, draws upon a combination of the principles outlined so far. A transposition of those principles onto each part of the lesson, with justification, is outlined below accompanied by real examples taken from MNP lesson plans:

Phase 1 “In Focus” (The Anchor Task)
At the start of an MNP lesson, an open task is introduced that incorporates a ‘real life’ scenario and involves recurring characters as shown in Figure 5.8

![Figure 5.8](image)

**Figure 5.8** An example of an In Focus task.

Here, there is a singular question or task to complete (in this case, “how much flour is left in each bag?”) and two of a set of ten recurring characters informally discuss how much flour they have used in a ‘real life’ problem – by which it is meant that the numbers and mathematics are introduced contextually through narrative.

This narrative approach aligns with principle 9: students should construct their knowledge from the world around them.

Lesson guidance provided to teachers reveals that whilst this task is singular in nature, the amount of prompting, querying and exploring facilitated by the teacher is quite extensive, and would take up a lot more time than the question by itself would suggest (Figure 5.9).

![Lesson Approach](image)

*Lesson Approach*

To begin this lesson, show pupils the In Focus task and ask them how we can attempt to solve the problem. What information do we know? What information is not in a helpful format? Why do we think that the children chose to remove quarters and eighths? Allow pupils time to discuss this idea.

Ask the class if they are able to find the answer by converting $\frac{1}{2}$ into quarters and eighths. Give them time to discuss this and ask them if they are able to prove their answers using a bar model or pictures. Is it possible? Then tell pupils you know of another way to solve the problem using both mixed numbers and improper fractions. How can this be done? Is it possible to do both? Work through Let’s Learn to show pupils the two methods. What is the difference between the methods? What is the same about them?

**Figure 5.9** The associated lesson guidance for the In Focus task in Figure 5.8
The amount of prompt questions, in addition to the instruction for teachers to “Allow pupils time to discuss this idea” both align with MNP’s assertion that students should be given time to discuss (design principle 1.) and explore a singular problem, and the encouragement to allow pupils to prove their answer (core competency 2. - generalisation) suggests an element of ‘playing’ with the problem (design principle 2.), and working collaboratively (design principle 3.).

Similarly, the encouraged use of “bar models or pictures” aligns both to the CPA model (principle 5.) and developing the “visualisation” competency (core competency 1.) of MNP’s 5 core competencies.

**Phase 2: “Let’s Learn”**

In the second phase of an MNP lesson, titled ‘Let’s Learn’, students are guided through new knowledge by the teacher. In this specific example, the teacher is expected to model the conversion of fractions so that their denominators are the same, in order to then subtract one from the other. Notably, two different methods are demonstrated, and both have associated visual representations as well as abstract notation. There are quite a few things happening here mathematically, and ample opportunities for a teacher to question and explore each method in detail.
Method 1 converts the fraction part of the mixed fraction whilst retaining the whole number, whilst method 2 converts the entirety of the mixed number into an improper fraction. Again, the lesson guidance instructs teachers to spend time asking prompts and facilitating discussion with the class, asking ‘what is the difference between the methods? What is the same about them?’ This again leans into a constructivist approach to teaching, and aligns with Vygotsky’s collaborative learning ideas, working with the knowledgeable other (both the teacher, and other students in discussions) to help students develop understanding, and includes a comparative element that could be ascribed to Dienes’ 3rd stage of learning discussed earlier and shown in Figure 5.1. Additionally, the emphasis on deconstructing what is happening mathematically appears aligned with Skemp’s concept of relational understanding, whilst also demonstrating the development of procedures in more abstract mathematics, and thus also nurturing instrumental understanding.

**Phase 3: “Guided Practice”**

Following on from the ‘Let’s Learn’ section of the lesson, students move into ‘Guided Practice’ (Figure 5.11), which differs slightly from ‘Let’s Learn’ in that it is generally...
introducing no new knowledge, but instead is creating an opportunity for students to reinforce the new learning with assistance from the teacher (knowledgeable other).

Figure 5.11  an example of the Guided Practice phase of an MNP lesson

Again, there is structural support in the form of visual representations of the mathematics (bar models), which can be viewed as scaffold which is gradually taken away - by the time students reach the second guided practice question, all imagery is removed and replaced with just the abstract maths - symbols, numbers, and letters. This fits both with Bruner’s scaffold approach, and further reinforces the CPA model of instruction - including, notably, the sequencing of moving from pictorial maths towards abstraction. Further scaffold is offered in the ‘differentiation’ notes section of the lesson plan (Figure 5.12), which state that struggling learners should be provided with pre-drawn bar models, and challenge is similarly included here as ‘ask students to prepare an explanation for the whole class as to why method 2 works’ which fits with Ban Har’s 3rd and 5th core competencies, namely ‘communication’ - communicating ideas in full sentences with mathematical language, and ‘metacognition’ - considering why things work. Furthermore, differentiation is not by task – students are expected to all complete the same tasks, which aligns with design principle 7. – pupils should move through the same content at the same pace.
Additional resources highlighted in the lesson plan material also support the ‘concrete’ element of CPA, suggesting teachers use ‘coloured strips of paper/card for cutting and folding’ and ‘access to a fraction wall’ (a physical set of strips representing various unit fractions) as seen in figure 5.13 below:

**Figure 5.13** an example of optional resources to support an MNP lesson taken from a lesson plan

**Phase 4: Independent Practice**

The fourth and final phase of a typical MNP lesson is ‘independent practice’, which allows for students to practice what they have learnt without direct teacher interaction or guidance. This practice is undertaken in provided workbooks that contain worksheets that correspond to the lesson they are working through, and includes questions in the style of those that students have already worked through in the lesson:
In figure 5.14, students are encouraged to work in the abstract (design principle 5.) to develop instrumental understanding (design principle 4.), although they are able to refer back to pictorial examples provided in the textbook for support. Assessment guidance for the lesson does not specify any parts of lessons at which assessment should be carried out, but instead asks prompt questions of what to look for in student understanding, for example ‘can pupils represent fractions using pictures? Can pupils use addition to check and verify subtraction sentences?’ These prompt questions are more flexible than, say, relying entirely on the independent practice to check understanding. Whilst I have demonstrated a single example lesson’s ability to map to different design principles and core competencies phase by phase, a full map of the lesson phases and their correlations is summarised shortly in table 5.1 after discussing the two main optional phases below. Furthermore, a full copy of this lesson plan can be seen in Appendix 1.
Additional Lesson Phase 1: ‘Activity Time’

This lesson phase appears very infrequently across the MNP textbooks, but provides students with an opportunity to work in groups to explore concepts or play games. Typically they allow for pupils to generate their own data (for example, by investigating the heights of their peers, or by selecting numbers from a selection of number card images). An example is shown below in Figure 5.15

![Activity Time](image)

Figure 5.15 An example of an ‘activity time’ lesson phase taken from a Year 1 MNP lesson.

Activity time phases promote children exploring ideas with their peers (design principle 2. and 3. – opportunities to play and learn effectively by cooperatively learning with others) using materials and resources around them (design principle 9. – constructing knowledge from the world around them) and helps develop student communication, visualisation and number sense (three core competencies). As with the previous lesson phases, there is good evidence to support the idea that each phase is carefully planned to align with both the explicit and implicit design principles of MNP, and to enable students to develop each of the five core competencies across a full lesson.

The fact that each phase incorporates a cluster of design principles and core competencies, but that no two phases develop the same cluster, highlights the importance of including each lesson phase as designed in order to capture all intended principles and competencies, and not risk under development of any specific aspects. For example, should a teacher...
decide to routinely improvise away from, say, the ‘In Focus’ task, then they risk undermining the principles of design of MNP by potentially not allowing for time to accommodate ideas, or play with ideas in an informal way with knowledgeable others.

**Additional Lesson Phase 2: ‘Maths Journal’**

Whilst not explicitly highlighted by Ban Har as a component of his three key parts of a lesson, journaling does form a significant element of MNP lessons, and is referred to both in blog posts and MNP lesson guidance as an opportunity to show understanding of any learning that has taken place. An example of a maths journaling task is shown below in Figure 5.16

![Maths Journal](image)

*Figure 5.16 An example of a ‘Maths Journal’ activity*

Ban Har does however, cite six key elements of a “Singapore Style” maths lesson in a keynote referenced in a Chris Fournier blog post (2022b)

Which refers to more general elements of lesson structure that include:

1. Play or exploration
2. Structured discussion
3. Practice (variation, not repetition)
4. Journal writing
5. Reading

6. Reflection

This additional list is already incorporated into the existing design principles, core competencies and lesson phases, and deemed superfluous, but it does indicate that maths journaling is perhaps more important than its irregular inclusion in MNP textbooks suggests (6 out of 462 tasks in a year 5 textbook were journal tasks, and 11 out of 271 tasks in a year 1 textbook). Indeed, it transpired that journaling was encouraged beyond the prescribed low frequency as presented in the MNP textbooks and workbooks in MNP training days and conferences, as well as in a blog post by Douglas (2021).

The journaling element of lessons is defined in the appendix of MNP textbooks as a reflective maths task in which students attempt to summarise their learning, which in turn generates another potential point of assessment for teachers. In her MNP blogpost titled ‘5 types of maths journals and how to use them’ Helly Douglas (2021) indicates that whilst journaling is a feature in MNP lessons, how they are used can be more discretionary and should be tailored to ‘what suits you’. Douglas advises that journals need not be marked, but should be regularly checked, and that they should be a planned part of lessons and allocated ample time to allow students a period of reflection upon their learning using writing, pictures and diagrams as appropriate. Douglas defines five types of journaling that students can do:

**Descriptive** - Students are asked to describe methods they have learned or explain a concept by (for example) writing instructions, writing a letter to another child, or annotating drawings.

**Evaluative** - Students are asked to choose a method they saw or used in a lesson and explain with justification how it was helpful to them. Prompts include ‘how can you show me that this method is most efficient?’, and ‘how do you know I’m wrong?’
**Creative** - Children are encouraged to develop models and stories to demonstrate their understanding. Prompts include ‘create their own number puzzle’ and ‘write a story to go with a calculation’.

**Investigative** - Children explore a new problem and record their findings. The new problem typically requires the same application of knowledge and concepts as used in other problems in the lesson.

**Formative** - Children write about what they have learned and understood in the lesson, including what they found challenging and why. Prompts include ‘what did you notice?’ And ‘what mistakes did you make?’ and ‘Does this remind you of any other work you have done?’

Each type of journal activity is subtly different, but all serve a similar purpose of requiring students to document a reflective activity about their learning independently. This appears to serve a purpose not only for the learner, but also an assessment opportunity for the teacher who, whilst being advised not to physically mark journals by Douglas, is provided with a window into how students think or how strong their understanding is around a topic recently taught. Interestingly the blog post appears aimed at teachers taking ownership of the journaling task concept and delivering their own versions of it, rather than delivering the prescribed irregular journaling tasks that are presented in the textbooks and workbooks.

Further to this, Alex Laurie (2021) writes in an MNP blogpost that journals must always include a date and title, but that the title itself must come from the learner rather than the teacher - as this in itself can provide useful insights into student thinking. For example, if a student writes ‘taking away logs’ as their title, this may, Laurie suggests, indicate that the child has understood the concept for the lesson but is not confident in using formal mathematical language. Similarly, if a child wrote ‘Subtract from 10’, it suggests more confidence with technical vocabulary.

Dabell’s blog post (2021) on self-assessment cites journaling as a natural differentiation tool and one of the most powerful ways to promote self and peer assessment, giving students the opportunities to articulate their understanding and sense their own strengths,
weaknesses and obstacles. He adds that the journal itself acts as a chronological portfolio of learning that captures previous difficulties and how they were overcome, giving students more confidence and resilience when handling new concepts that they might find challenging - hence the journal also contributes towards higher levels of motivation and a more positive and confident mindset around self-ability in mathematics. Finally, as Judy Hornigold (2021) points out in her blog post, journals allow students to capture their understanding of mathematical concepts in a way that makes sense to them, away from the constraints of the formalities and conventions of mathematical notation - which is particularly beneficial for younger students who may not have begun to use specific technical notation or vocabulary.

A summary of themes

A lot of information demonstrated predominantly across the extensive content available on the Maths, No Problem! website has been discussed. Table 5.1 shows a summary of MNP’s design principles, and how they are generally integrated into lesson phases although it should be noted that these are indicators only, and not a precisely prescribed design rule. For example, In Focus and Let’s Learn emphasise time being dedicated to them with relatively little maths to discuss, however it is feasible that teachers may allocate a proportionately large amount of time to other phases as well. Similarly, it is not an exact science to differentiate and separate out instrumental and relational understanding, but generally speaking, guided practice begins to integrate deeper understanding with procedures and algorithms, and independent practice focuses predominantly on the latter. Lesson phases are abbreviated (IF = In Focus, LL = Let’s Learn, GP = Guided Practice, IP = Independent Practice, AT = Activity Time and MJ = Maths Journal):
<table>
<thead>
<tr>
<th>Design Principle</th>
<th>IF</th>
<th>LL</th>
<th>GP</th>
<th>IP</th>
<th>AT</th>
<th>MJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children given time to accommodate new ideas</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td></td>
</tr>
<tr>
<td>Children given opportunities to play with mathematical ideas in an informal and exploratory way</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td></td>
</tr>
<tr>
<td>Cooperative learning with knowledgeable others.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td></td>
</tr>
<tr>
<td>Relational understanding is emphasised</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>Instrumental understanding is emphasised</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>The Concrete-Pictorial-Abstract approach is utilised</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>Teaching delivered in small steps</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>The whole class moves through the same content at the same pace</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>Knowledge constructed from the ‘real world’ eg narratives or data gathering</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>

Table 5.1: A summary of how MNP design principles are generally integrated into lesson phases

From analysis of the content on MNP’s website there are a number of clearly thought out and heavily structured components of MNP lessons. Each lesson follows the same familiar structure that emphasises relational understanding and thinking about mathematical concepts collaboratively with strong visual models to support through the Concrete, Pictorial, Abstract approach to introducing and scaffolding mathematical ideas. Independent practice is a relatively small component of lessons (it is the focus of just one lesson phase), but it is a key phase, nonetheless. Each phase does not appear to be weighted and seem to be equally valuable to MNP, however there are themes that spread across several phases, such as cooperative learning (In Focus / Let’s Learn / Guided Practice) and giving substantive time to accommodate ideas (demonstrated by MNP’s emphasis on working together as a group or class for three out of four of the main lesson phases).

There are no obvious suggested timings for each lesson phase, and as such this was a useful indicator of teacher agency in the lessons observed - in other words, how much time did each teacher ascribe to each phase of the structured lesson, and did this in any way give an
indication of perceived phase value (or value of specific design principles)? It is somewhat expected that within the constraints of fixed lesson times, general school structures such as break times, fixed obligations around literacy and numeracy policies, and the general messiness and unpredictability of children learning, lessons are unlikely to consistently run smoothly in terms of timings. In such circumstances, decisions around lesson components that get squeezed, and which ones are prioritised to maintain their length will give some indication of how a teacher perceives the value of each phase. Will there emerge a consistency amongst teachers around any prioritisation or timings in general? Considering that MNP lessons will be an embedded and familiar programme for teachers observed in this research (due to one of the features of research being that teachers have used the program for at least two years timings and priorities may have become general established practice based upon a belief system of perceived pedagogical value of each lesson component or whether any timings are based more around structural restrictions around the school day).

The analysis so far has been confined predominantly to outward facing materials, particularly the MNP website and endorsed blog posts. However, arguably the most important resources are the textbooks and workbooks themselves - do they align with, and communicate, the intentions outlined by the website? Do they speak to the emphasis on cooperative learning, the concrete pictorial abstract approach, play and exploration, and time to explore different methods and models to help build relational understanding of mathematics? Furthermore, how are tasks and practice structured for teacher and student alike in the textbooks, and how is independent practice structured for students in workbooks? What do these structures tell us about the intended curriculum, and what MNP values as good mathematics pedagogy and understanding? Are there contrasting messages between the books and the website posts? Perhaps most importantly, what do the MNP resources communicate or infer about what a maths teacher should be like? What do they not develop in teachers?

Part Two: Textbook and Workbook Analysis
As discussed in chapter 3, it was decided to use an adapted model of Berisha and Bytyqi’s (2020) Textbook Analysis Model, which itself is an amalgamation of Stein’s Mathematical Task Analysis Guide (TAG) (Stein et al., 1996), Zhu and Fan’s conceptual framework for task analysis (Zhu & Fan, 2006) and Gracin’s five-dimensional analysis of textbook exercises (Gracin, 2018). This tool analyses the following five dimensions of design:

1. **Contextual Features**
   This dimension will measure whether a task has fictitious or authentic application by specifically checking whether a task is:
   a) **Non-Applicative** - no connection with reality-related contexts (entirely abstract)
   b) **Fictitious application** - connected to reality-related contexts created by the author (e.g. narrative)
   c) **Authentic** application - utilising real-life data collected by students

2. **Forms of Presentation**
   This dimension will measure whether a task is presented in either:
   a) **Symbolic** form
   b) **Textual** form
   c) **Visual** form
   d) **Combined** choice of two or more of the above.

3. **Answer forms required**
   a) **Close-ended** tasks - have only one answer
   b) **Open-ended** tasks - Have several correct answers (this will include close-ended tasks where students are encouraged to find multiple approaches).
   c) **Multiple response** tasks - Respondents select only correct answers from the choices offered.

4. **Mathematical activity involved**
   a) **Representation and modelling** task - Requires the presentation of mathematical data in different forms or the translation of mathematical data from one representation to another
b) Calculation and operations task - Requires performance of mathematical operations, calculations, transformations, geometric constructions etc

c) Interpretation task - Requires recognition, reading and contextual interpretation of mathematical relations or data presented in different forms

d) Argumentation and reasoning task - Requires elaboration, description and stringing of the right arguments that lead to a conclusion

5. Level of cognitive demand

a) Memorisation task (Low Demand) - Involves the reproduction of previously learned rules, facts, formulae or definitions

b) Procedure with no connections (Low Demand) - Involves performance of general procedures and algorithms without making connection to underlying concepts or meaning

c) Procedure with connections (High Demand) - Involves performance of procedures and algorithms making connections to underlying concepts or meaning.

d) Exploratory task (renamed) (High Demand) - Involves complex and non-algorithmic thinking, exploration and understanding of mathematics concepts, processes or relationships.

Upon completion of the pilot study and developing a ‘coding agreement’ (utilising a second maths expert to cross reference codification), a few minor adjustments were made to the model which will be explained after demonstrating examples of how the data was codified (see below).
Examples of codification

Contextual Features

In figure 5.17, the In Focus task would be listed as *Fictitious* (F) in that it uses real data in a narrative format as a way to deliver the mathematical content. Despite it using realistic data, the fact that it does not require the students to produce their own data prevents this task from being identified as *Authentic* (A).

In figure 5.18, the In Focus task would be listed as *Fictitious* (F) in that it uses real data in a narrative format as a way to deliver the mathematical content. Despite it using realistic data, the fact that it does not require the students to produce their own data prevents this task from being identified as *Authentic* (A).
In figure 5.18, the task would be identified as Authentic (A) due to the fact that the data used for the task is created by the students themselves albeit within a few given constraints. Similarly, the Answer Form Required would be identified as Open since there is a range of possible responses to the task rather than one fixed ‘answer’.

Figure 5.19 An example of a ‘Non-Applicative’ task

Finally, the example in figure 5.19 would be identified as Non-Applicative (N) due to the question being presented in the absence of narrative or student-generated data.

Forms of Presentation

Figure 5.20 An example of a question (1) using symbolic, textual and visual presentation
In figure 5.20, question 1 would be listed as STV (Symbolic / Textual / Visual) format, because it includes symbolic representations of fractions (S) a text (T) introduction, and bar model visualisations to support the learner (V). Whereas question 2 would be recorded as Symbolic (S) because it is almost entirely (except for a single word) presented in mathematical symbols. Note that the category ‘combined’ was subdivided into combinations of S, T and V to better convey which forms of presentation were used in one of a few minor adaptations to the model (discussed in the next section).

**Answer Forms Required**
Closed-ended (C), Open-ended (O) and multiple response (M) task samples are provided below in figures 5.21, 5.22 and 5.23 respectively.

**Closed-ended (C):**

![Figure 5.21 An example of a ‘closed-ended’ question](image)

**Open-ended (O):**

![Figure 5.22 An example of an ‘open-ended’ question asking ‘Do you agree with Elliott?’](image)
Multiple response (M):

Figure 5.23 An example of a ‘multiple-response’ question

Mathematical Activity Involved

1. Representation and Modelling

Figure 5.24 is an example of what is categorised as a representation and modelling task (RM), in that it requires the student to work between two representations of a mathematical concept (subtraction specifically), working between the abstract (right) and pictorial (left).
2. Calculation and operations task (CO):

![Figure 5.25](image)

An example of a ‘calculation and operations’ task

Tasks that require the learner to perform a calculation to find an answer are recorded as CO like the one shown in figure 5.25.

3. Interpretation task (I):

![Figure 5.26](image)

An example of an ‘interpretation’ task

Figure 5.26 shows an example of a task categorised as an ‘interpretation’ task, because it requires students to recognise and interpret data.

4. Argumentation and reasoning task (AR):

![Figure 5.24](image)

An example of a ‘representation and modelling’ task

Figure 5.24 An example of a ‘representation and modelling’ task
Figure 5.27 An example of an ‘argumentation and reasoning’ task

Figure 5.27 shows an example of an argumentation and reasoning task, identified as it requires a description or justification for the answer provided.

Level of Cognitive Demand

1. Memorisation (M):

![Memorisation Example](image)

Figure 5.28 An example of a ‘memorisation’ task

Tasks labelled as ‘memorisation’ like the one shown in figure 5.28 are deemed to require simple recall of facts, definitions and formulae. In this example, students are required to recall from memory the look of a fifty-pound note, and monetary symbols.

2. Procedure:

![Procedure Example](image)

Figure 5.29 An example of a ‘procedure’ task

The decision was made to remove the categories ‘procedure with no connections’ and ‘procedure with connections’ and replace them both with a simpler ‘procedure’ due to the difficulty in determining what, in the context of each lesson, could be determined as ‘procedure with no connection’ considering that no lessons in the initial analysis of textbook 5A and workbook 5A were deemed to include questions that, when framed within the
lesson, ‘made no connection to underlying concepts or meaning’. Hence in the context of this study, tasks deemed as ‘procedural’ mean simply that they require a standard procedure or algorithm to solve such as the one shown in figure 5.29.

3. Exploratory task:

![Two examples of an ‘exploratory’ task](image)

**Figure 5.30** Two examples of an ‘exploratory’ task

The two examples shown in figure 5.30 above are categorised as ‘exploratory tasks’ due in part to their open response nature, but also in that they do not require ‘algorithmic thinking’ to develop understanding and make progress within the task.

Pilot Analysis

In order to enhance the validation of the application of the analysis tool, a brief pilot analysis of five MNP lessons across both the textbook and accompanying workbook was conducted by myself and, independently, a second maths expert guided by the same tools and explanations of task types provided by Berisha and Bytyqi (2020). Whilst the majority of tasks within those lessons were identically coded by both me and the independent ‘other’, there was significant discrepancy, and combined confusion over the interpretation criteria regarding the two categories titled ‘Procedures with connections’ and ‘Procedures without connections’. Specifically, it was difficult to determine if or when a question, sat within the context of an MNP lesson (as they all did), alongside the supportive mathematical diagrams and contextual narrative of each lesson, could be determined as a ‘procedure without
connection’ when the criteria for such categorisation states ‘focus students’ attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.’ Given that it appeared many if not all procedural questions presented in MNP sat within the framework of a well-structured lesson developing deeper understanding, the usefulness of these two categories was debated and subsequently altered (see ‘Adaptation 2’ below). A sample of the side-by-side comparison of the analysis can be found in appendix e.

Results Tables and Adaptations to the Model

<table>
<thead>
<tr>
<th>Question Type</th>
<th>NG</th>
<th>FA</th>
<th>SA</th>
<th>ST</th>
<th>SV</th>
<th>TV</th>
<th>TVV</th>
<th>STV</th>
<th>O</th>
<th>C</th>
<th>M</th>
<th>RM</th>
<th>CO</th>
<th>AR</th>
<th>I</th>
<th>E</th>
<th>P</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>IP</td>
<td>19</td>
<td>55</td>
<td>3</td>
<td>1</td>
<td>8</td>
<td>0</td>
<td>37</td>
<td>12</td>
<td>1</td>
<td>35</td>
<td>42</td>
<td>27</td>
<td>7</td>
<td>10</td>
<td>28</td>
<td>24</td>
<td>7</td>
<td>55</td>
</tr>
<tr>
<td>LL</td>
<td>38</td>
<td>156</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>0</td>
<td>24</td>
<td>4</td>
<td>25</td>
<td>194</td>
<td>39</td>
<td>155</td>
<td>0</td>
<td>14</td>
<td>116</td>
<td>33</td>
<td>31</td>
<td>41</td>
</tr>
<tr>
<td>GP</td>
<td>115</td>
<td>67</td>
<td>0</td>
<td>24</td>
<td>9</td>
<td>0</td>
<td>69</td>
<td>7</td>
<td>7</td>
<td>56</td>
<td>13</td>
<td>169</td>
<td>0</td>
<td>11</td>
<td>138</td>
<td>20</td>
<td>28</td>
<td>10</td>
</tr>
<tr>
<td>MW</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Mi</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>AT</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5A Textbook Total</td>
<td>166</td>
<td>283</td>
<td>13</td>
<td>21</td>
<td>19</td>
<td>0</td>
<td>113</td>
<td>24</td>
<td>23</td>
<td>242</td>
<td>106</td>
<td>156</td>
<td>0</td>
<td>38</td>
<td>279</td>
<td>75</td>
<td>70</td>
<td>121</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Question Type</th>
<th>NG</th>
<th>FA</th>
<th>SA</th>
<th>ST</th>
<th>SV</th>
<th>TV</th>
<th>TVV</th>
<th>STV</th>
<th>O</th>
<th>C</th>
<th>M</th>
<th>RM</th>
<th>CO</th>
<th>AR</th>
<th>I</th>
<th>E</th>
<th>P</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>WS</td>
<td>130</td>
<td>60</td>
<td>5</td>
<td>32</td>
<td>27</td>
<td>0</td>
<td>83</td>
<td>12</td>
<td>2</td>
<td>28</td>
<td>4</td>
<td>191</td>
<td>0</td>
<td>3</td>
<td>125</td>
<td>16</td>
<td>51</td>
<td>2</td>
</tr>
<tr>
<td>MW</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>MVR</td>
<td>28</td>
<td>17</td>
<td>0</td>
<td>5</td>
<td>15</td>
<td>0</td>
<td>21</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>35</td>
<td>10</td>
<td>1</td>
<td>38</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>RTV</td>
<td>32</td>
<td>14</td>
<td>0</td>
<td>11</td>
<td>6</td>
<td>0</td>
<td>22</td>
<td>2</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>46</td>
<td>0</td>
<td>3</td>
<td>33</td>
<td>5</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>REVW</td>
<td>3</td>
<td>8</td>
<td>0</td>
<td>2</td>
<td>5</td>
<td>0</td>
<td>6</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>16</td>
<td>0</td>
<td>0</td>
<td>12</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td>5A Workbook Total</td>
<td>109</td>
<td>103</td>
<td>6</td>
<td>51</td>
<td>64</td>
<td>0</td>
<td>133</td>
<td>19</td>
<td>2</td>
<td>39</td>
<td>6</td>
<td>292</td>
<td>10</td>
<td>7</td>
<td>211</td>
<td>29</td>
<td>61</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 5.2: Codified analysis of textbook 5A

<table>
<thead>
<tr>
<th>Question Type</th>
<th>NG</th>
<th>FA</th>
<th>SA</th>
<th>ST</th>
<th>SV</th>
<th>TV</th>
<th>TVV</th>
<th>STV</th>
<th>O</th>
<th>C</th>
<th>M</th>
<th>RM</th>
<th>CO</th>
<th>AR</th>
<th>I</th>
<th>E</th>
<th>P</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>WS</td>
<td>130</td>
<td>60</td>
<td>5</td>
<td>32</td>
<td>27</td>
<td>0</td>
<td>83</td>
<td>12</td>
<td>2</td>
<td>28</td>
<td>4</td>
<td>191</td>
<td>0</td>
<td>3</td>
<td>125</td>
<td>16</td>
<td>51</td>
<td>2</td>
</tr>
<tr>
<td>MW</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>MVR</td>
<td>28</td>
<td>17</td>
<td>0</td>
<td>5</td>
<td>15</td>
<td>0</td>
<td>21</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>35</td>
<td>10</td>
<td>1</td>
<td>38</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>RTV</td>
<td>32</td>
<td>14</td>
<td>0</td>
<td>11</td>
<td>6</td>
<td>0</td>
<td>22</td>
<td>2</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>46</td>
<td>0</td>
<td>3</td>
<td>33</td>
<td>5</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>REVW</td>
<td>3</td>
<td>8</td>
<td>0</td>
<td>2</td>
<td>5</td>
<td>0</td>
<td>6</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>16</td>
<td>0</td>
<td>0</td>
<td>12</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td>5A Workbook Total</td>
<td>109</td>
<td>103</td>
<td>6</td>
<td>51</td>
<td>64</td>
<td>0</td>
<td>133</td>
<td>19</td>
<td>2</td>
<td>39</td>
<td>6</td>
<td>292</td>
<td>10</td>
<td>7</td>
<td>211</td>
<td>29</td>
<td>61</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 5.3: Codified analysis of workbook 5A

<table>
<thead>
<tr>
<th>Question Type</th>
<th>NG</th>
<th>FA</th>
<th>SA</th>
<th>ST</th>
<th>SV</th>
<th>TV</th>
<th>TVV</th>
<th>STV</th>
<th>O</th>
<th>C</th>
<th>M</th>
<th>RM</th>
<th>CO</th>
<th>AR</th>
<th>I</th>
<th>E</th>
<th>P</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>IP</td>
<td>9</td>
<td>82</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>37</td>
<td>0</td>
<td>8</td>
<td>21</td>
<td>21</td>
<td>0</td>
<td>7</td>
<td>3</td>
<td>14</td>
<td>18</td>
<td>18</td>
<td>21</td>
</tr>
<tr>
<td>LL</td>
<td>29</td>
<td>59</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>44</td>
<td>4</td>
<td>35</td>
<td>13</td>
<td>76</td>
<td>0</td>
<td>8</td>
<td>35</td>
<td>14</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>GP</td>
<td>47</td>
<td>43</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>7</td>
<td>3</td>
<td>39</td>
<td>4</td>
<td>28</td>
<td>4</td>
<td>86</td>
<td>0</td>
<td>9</td>
<td>42</td>
<td>7</td>
<td>32</td>
<td>2</td>
</tr>
<tr>
<td>MW</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>8</td>
<td>0</td>
<td>1</td>
<td>9</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>Mi</td>
<td>3</td>
<td>1</td>
<td>7</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>0</td>
<td>1</td>
<td>11</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>AT</td>
<td>1</td>
<td>0</td>
<td>27</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>18</td>
<td>0</td>
<td>5</td>
<td>23</td>
<td>5</td>
<td>0</td>
<td>15</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>REVW</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1B Textbook Total</td>
<td>91</td>
<td>142</td>
<td>30</td>
<td>2</td>
<td>18</td>
<td>8</td>
<td>10</td>
<td>352</td>
<td>8</td>
<td>73</td>
<td>81</td>
<td>190</td>
<td>0</td>
<td>39</td>
<td>99</td>
<td>49</td>
<td>84</td>
<td>77</td>
</tr>
</tbody>
</table>

Table 5.4: Codified analysis of textbook 1B
Adaptation 1: Reducing the number of books

The original plan was to analyse a sample of 6 of the available 24 MNP books in this way - three pairs of Workbook / Textbooks from three different year groups, and each year group corresponding to the year groups that would be observed, due to the fact that at this stage it had been established that the opportunities and invitations to classes to be observed in schools would be for Year 1, 5, and 6 (this will be discussed in the enacted curriculum section of this research).

However, the time required to analyse just one book was much greater than anticipated, and upon concluding the codifying procedures for the fourth sample book, ‘Year 1B Workbook’, it was decided that the insights to be gained from this process would be sufficiently realised from a smaller sample of four books rather than six. The insights gained from further books being analysed seemed significantly imbalanced against the time and work commitments required to do it, and sampling saturation appeared to have been reached. This process and realisation aligns with what Hennink, Kaiser and Weber (2019) describe as a saturation assessment point where data begins to stabilise and fulfil already observed patterns of behaviour.

Adaptation 2: ‘Level of Cognitive Demand’ Data Field

In the initial design of the analysis tool, the ‘level of cognitive demand’ category listed two different types of procedural task: with and without connections to underlying concepts,

```
| Question Type | F | F | F | G | G | G | G | G | G | H | H | H | I | I | I | I | J | J | J |
| WS            | 58| 55| 0 | 0 | 3 | 0 | 13| 70| 0 | 27| 1 | 112| 0 | 13| 33| 23| 44| 1 | 99| 13|
| MW            | 4 | 6 | 1 | 0 | 1 | 0 | 1 | 9 | 0 | 9 | 2 | 9 | 0 | 0 | 4 | 6 | 1 | 8 | 3 | 0 |
| MI            | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 3 | 0 | 0 |
| REVW          | 53| 18| 0 | 0 | 4 | 0 | 19| 30| 0 | 18| 2 | 59| 10| 9 | 34| 8 | 20| 2 | 63| 6 |
| 1B Workbook Total | 120| 97| 1 | 0 | 9 | 0 | 39| 131| 0 | 45| 6 | 208| 10| 24| 75| 44| 77| 12| 187| 25 |
```

Table 5.5: Codified analysis of workbook 1B
however as previously stated, this proved difficult to differentiate between, in so much as no tasks were categorised as Procedural without Connection across the entirety of the first book (5ATB & 5AWB). Hence these two categories were merged and simplified to ‘Procedure’ which, according to the tool descriptors, would still be categorised as a ‘high demand’ task type.

Adaptation 3: ‘Forms of Presentation’ Data Field

After analysing textbook 5ATB, it became clear that the original options for responses (symbolic, textual, visual and combined) were unsuitably inaccurate, specifically the ‘combined’ field began to hide observed emerging patterns regarding the ways in which question sequences became more abstract and reduced the visual support but maintained textual and symbolic elements. As such, it was decided that combinations of forms of presentation could be more specifically entered into the tool, such as Symbolic / Textual (ST) and Textual / Visual (TV) and so additional input fields ST, SV, TV and STV replaced ‘Combined (C)’.

Analysis of the four MNP textbooks was divided into two sections: pattern analysis and intended curriculum analysis. The former sought to establish data patterns across various elements of the MNP book designs such as comparative patterns between Year 1 and Year 5 content, frequency of different question types and presentations and so forth. Intended curriculum analysis will reflect these patterns against the philosophical purposing and positioning of MNP’s pedagogical and mathematical principles as discussed and displayed across their website and lesson plans, in order to establish how well MNP’s textbooks support Ban Har’s principles of lesson design, and whether there is further evidence as to MNP’s vision of what a maths teacher is or should be. This data and analysis was then used to evidence reasoning to answer the following two research questions:

1. What strategies are emphasised and prioritised in curriculum packages endorsed by the DfE for primary mathematics in England?

2. How are textbooks used as teaching aids in DfE approved curriculum packages for primary mathematics in England?
Pattern analysis

Number of questions:

Across the Year 5A textbook and workbook (which equate to half of the full content of Year 5), students are exposed to a total of 770 maths questions – around 263 of these are intended to be studied with the teacher (“In Focus” and “Let’s Learn” questions specifically), with a further 182 ‘guided practice’ questions that would likely involve some teacher or peer support. This leaves around 320 questions intended to be carried out without much (or any) teacher support, 308 of these appearing in the student workbook, and the remaining questions being journaling or peer group activities from the textbook. Across Year 1 (5-6 year olds) and Year 5 (9-10 year olds), the number of questions Year 1 are exposed to is lower. The Year 1B textbook hosts nearly 200 fewer questions (see table 5.6), but also contains 23 fewer lessons which likely reflects the amount of time dedicated to maths across year groups (textbook 5B for example, which was not analysed, contains 71 lessons - suggesting the books for one year group are not designed to be used unequally across a year, but rather they are intended as literally two halves of a year’s worth of content). With this difference considered, the mean number of Year 1 questions per lesson is 11.8, whereas the mean number of Year 5 questions per lesson is 10.3, suggesting that Year 1 students tend to do more questions per lesson than Year 5. This is a surprising result considering the age and developmental differences between a Year 1 child and a Year 5 child, demonstrating that a differentiated number of questions across year groups was either not a design consideration, or one that is consciously dismissed.

<table>
<thead>
<tr>
<th>Book Code</th>
<th># Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>5ATB</td>
<td>462</td>
</tr>
<tr>
<td>5AWB</td>
<td>308</td>
</tr>
<tr>
<td>Total Y5</td>
<td>770</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Book Code</th>
<th># Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1BTB</td>
<td>271</td>
</tr>
<tr>
<td>1BWB</td>
<td>224</td>
</tr>
<tr>
<td>Total Y1</td>
<td>495</td>
</tr>
</tbody>
</table>

Table 5.6: A comparative summary of the number of questions per book
**Contextual Features:**
This dimension measures whether a task has fictitious or authentic application using the codes N (Non-Applicative), F (Fictitious) and A (Authentic)
From the summary table above, it becomes apparent that there is a high proportion of Fictitious (F) task types across both Year 1 (142 ~53%) and Year 5 (283 ~61%) textbooks (TB), whereas the student workbooks (WB) contain a high number of Non-Applicative (N) task types. Authentic task types feature much less across both books. A combined look at both textbook and workbook for each year group shows a similar balance of non-applicative task types, and fictitious task types, suggesting that the proportional balance of each of these task types effectively swap over from textbook to workbook, and from a lesson perspective, a student is roughly exposed to an equal amount of non-applicative and fictitious tasks.

Upon further inspection there are additional patterns to be found within each book across the MNP categories of tasks.

<table>
<thead>
<tr>
<th>Question Type</th>
<th>5ATB</th>
<th>1BTB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non Applicative</td>
<td>Fictitious</td>
</tr>
<tr>
<td>In Focus</td>
<td>16%</td>
<td>80%</td>
</tr>
<tr>
<td>Lets Learn</td>
<td>20%</td>
<td>80%</td>
</tr>
<tr>
<td>Guided Practice</td>
<td>63%</td>
<td>37%</td>
</tr>
<tr>
<td>Mind Workout</td>
<td>17%</td>
<td>50%</td>
</tr>
<tr>
<td>Maths Journal</td>
<td>17%</td>
<td>33%</td>
</tr>
<tr>
<td>Activity Time</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Table 5.7: A comparative summary of the contextual categories of questions per book

Table 5.8: A comparative summary of contextual question types between a Year 5 and Year 1 textbook
Within the textbooks, Fictitious contexts were most frequently presented for “In Focus” (80% of Year 5 and 75% of Year 1), and “Lets Learn” (80% of Year 5, and 66% of Year 1) for both year groups, whereas “Guided Practice” tasks were mostly Non-Applicative, with a particular emphasis on non-applicative guided practice in Year 5 (63% compared to 52% in Year 1). This supports what MNP say on their website regarding the purpose of this phase: that it is intended to allow time for pupils to play with ideas and explore concepts (design principles 1. and 2.). Authentic tasks - those in which data is generated or collected by students, were almost entirely reserved for ‘Mind Workouts’, ‘Maths Journal’ tasks and ‘Activity Time’ in both years. Each of these phases are non-standard phases (they appear infrequently throughout each book) but appear clearly designed to strategically introduce opportunities for collaborative exploratory tasks in Activity Time (design principles 1., 2., 3. and 9.) and to develop metacognition (an MNP core competency). ‘Activity Time’ tasks were more frequent in the Year 1 textbook, with 28 tasks across the book compared to just 5 in Year 5. This suggests a more play-orientated approach to Year 1 mathematics than in Year 5, where ideas and concepts are perhaps more established.

<table>
<thead>
<tr>
<th>Question Type</th>
<th>5AWB Contextual</th>
<th>1BWB Contextual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non Applicative</td>
<td>Fictitious</td>
</tr>
<tr>
<td>Worksheet</td>
<td>67%</td>
<td>31%</td>
</tr>
<tr>
<td>Mind Workout</td>
<td>17%</td>
<td>67%</td>
</tr>
<tr>
<td>Mid-Year Review</td>
<td>62%</td>
<td>38%</td>
</tr>
<tr>
<td>Revision</td>
<td>70%</td>
<td>30%</td>
</tr>
<tr>
<td>Review</td>
<td>50%</td>
<td>50%</td>
</tr>
</tbody>
</table>

Table 5.9: Analysis of the contextual features of question types in a Year 5 Student Workbook

With regards to contextual feature analysis, student workbooks present a different picture to teacher textbooks. Most tasks fall under the ‘worksheet’ (WS) category with a majority of these in both Year 5 and Year 1 being ‘Non Applicable’, however Year 1 is much more finely balanced between Non Applicable and Fictitious task types. It appears therefore, that whilst the narrative element of task presentation is dominant in the textbooks, a large proportion of workbook tasks in Year 1 also retain narrative as a way of delivering tasks whereas in Year...
5 narrative style questions are reduced for independent practice in favour of more abstract ‘non-applicative’ formats. Revision tasks (REVI) tend to be more abstract and Non-Applicative across both year groups, as do Mid Year Review (REVW) tasks. Both suggest that ‘end point’ learning - that is, where learners are expected to be in their learning for assessment periods, lies predominantly in the abstract part of the CPA model.

Forms of Presentation

Table 5.10: A summary of the analysis of the forms of presentation in questions in textbooks and workbooks

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Year 5</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SATB %</td>
<td>7%</td>
<td>4%</td>
<td>0%</td>
<td>24%</td>
<td>5%</td>
<td>7%</td>
<td>52%</td>
</tr>
<tr>
<td>SAWB %</td>
<td>17%</td>
<td>21%</td>
<td>0%</td>
<td>43%</td>
<td>6%</td>
<td>1%</td>
<td>13%</td>
</tr>
<tr>
<td>Total Y5</td>
<td>11%</td>
<td>11%</td>
<td>0%</td>
<td>32%</td>
<td>6%</td>
<td>5%</td>
<td>36%</td>
</tr>
<tr>
<td><strong>Year 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1BTB %</td>
<td>1%</td>
<td>7%</td>
<td>3%</td>
<td>4%</td>
<td>56%</td>
<td>3%</td>
<td>27%</td>
</tr>
<tr>
<td>1BWB %</td>
<td>0%</td>
<td>4%</td>
<td>0%</td>
<td>17%</td>
<td>58%</td>
<td>0%</td>
<td>20%</td>
</tr>
<tr>
<td>Total Y1</td>
<td>0%</td>
<td>5%</td>
<td>2%</td>
<td>10%</td>
<td>57%</td>
<td>2%</td>
<td>24%</td>
</tr>
</tbody>
</table>

Further evidence of Year 5 work becoming deliberately more abstract can be found in the ‘forms of presentation’ analysis in table 5.10. It’s clear that questions presented entirely symbolically in the first column (ie, using numbers and symbols only, with no or very little text to introduce them) are almost completely absent in Year 1 books (1BTB: 1%, 1BWB: 0%), whereas they form 7% and 17% in Year 5 textbook and workbooks respectively.

Table 5.11: A summary of the use of visuals in questions across textbooks and workbooks.
Furthermore, by examining the cumulative frequency of tasks that include visual elements as shown in the final column of Table 5.11, it becomes apparent that only 19% of questions in Year 5 independent workbooks have visual elements to support learners in them at all. This reduction in scaffold contrasts with the Year 1 workbook (1BWB) where 79% of questions continue to rely on any visual imagery. Similarly in the textbooks 5ATB and 1BTB, Year 5 integrate visuals for 65% of questions, whereas Year 1 integrate visuals for 89% of questions (see Figure 5.31 below).

**Figure 5.31:** The reduction of visual support from textbook to workbook in both Year 5 and Year 1

This demonstrates that teacher-led parts of lessons are also less reliant upon visuals in Year 5 compared to Year 1. For further clarity, visuals in the MNP textbooks and workbooks are in fact predominantly mathematical visual models such as bar models, number lines, dienes blocks, number bond diagrams and other conceptual structural support models, hence the reduction in these in Year 5 supports a move away from the concrete and pictorial support in the CPA model and towards abstraction. The data here also supports findings in the ‘contextual features’ analysis in that there appears to be a strategic move towards abstraction in the independent practice in workbooks for both year groups, and scaffold is reduced compared to what is present in the teacher-led textbooks.

As an overview of the combined textbook and workbook, a similar pattern occurs, with 54% of questions in Year 5 presented without visual support, compared to just 15% in Year 1. It
should be noted however, that visuals still play a significant role in Year 5 despite it being a reduction from Year 1. 52% of lessons presented in the Year 5 textbook are presented with a combination of visuals, symbols and text (compared to 27% in Year 1), suggesting a purposefully designed strategy to ensure students are still exposed to and accessing mathematical models introduced much earlier in their schooling. In contrast, Year 1 students are predominantly exposed to Textual / Visual (TV) questions in the textbook (56%), suggesting that early development of and under familiarity with symbolic mathematics is considered and catered for in the design of these lessons - abstract mathematics seems to incrementally feature more frequently as children grow older and their confidence and knowledge of mathematics grows.

![Table 5.12](image)

<table>
<thead>
<tr>
<th>Phase</th>
<th>SATB</th>
<th>IBT</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Focus</td>
<td>1%</td>
<td>4%</td>
</tr>
<tr>
<td>Lets Learn</td>
<td>3%</td>
<td>1%</td>
</tr>
<tr>
<td>Guided Practice</td>
<td>13%</td>
<td>5%</td>
</tr>
<tr>
<td>Mind Workout</td>
<td>0%</td>
<td>33%</td>
</tr>
<tr>
<td>Maths Journal</td>
<td>0%</td>
<td>17%</td>
</tr>
<tr>
<td>Activity Time</td>
<td>0%</td>
<td>60%</td>
</tr>
<tr>
<td>In Focus</td>
<td>2%</td>
<td>0%</td>
</tr>
<tr>
<td>Lets Learn</td>
<td>0%</td>
<td>2%</td>
</tr>
<tr>
<td>Guided Practice</td>
<td>1%</td>
<td>9%</td>
</tr>
<tr>
<td>Mind Workout</td>
<td>0%</td>
<td>9%</td>
</tr>
<tr>
<td>Maths Journal</td>
<td>0%</td>
<td>27%</td>
</tr>
<tr>
<td>Activity Time</td>
<td>0%</td>
<td>14%</td>
</tr>
</tbody>
</table>

Table 5.12: A summary of the question types in textbooks broken down by lesson phase

A more in-depth textbook analysis of the question types by lesson phase (see Table 5.12) within both textbooks reveals that lessons frequently start with text and visuals in Year 1 for In Focus tasks (88%), but include more examples of abstract symbols alongside those components in Year 5 (51%), and the visual support reduces in Year 5 lessons at the point of guided practice, whereas visual support is maintained in Year 1 throughout the textbook directed part of the lesson. In other words, mathematical scaffold is sustained for most of the Year 1 textbook lessons during the phase that is led by the teacher, whereas in Year 5

166
that scaffold is reduced at the point of guided practice. This can be visualised more clearly in figure 5.32 below:

![Comparison of Visuals use in lesson phases Y1/Y5](image)

**Figure 5.32:** The difference in ‘visuals’ support in Year 1 and Year 5 textbooks by lesson phase.

This trend continues into the independent practice section of the lesson too - a similar phase by phase analysis of task types in the student workbooks (see table 5.13 below) reveals that independent worksheets in Year 5 utilise visuals only 22% of the time - a further drop off from the guided practice part of the lesson, whereas in Year 1, visuals are still frequently utilised, identified in 86% of questions for independent practice.

![Table 5.13](image)

**Table 5.13:** A summary of the use of visuals in workbooks broken down by lesson phase
Figure 5.33: The reliance upon text interpretation in Year 1 and Year 5 textbooks by lesson phase

A further observation when analysing the forms of presentation in the MNP textbooks and workbooks is around the high proportion of text used. In both year groups, the use of text was identified in at least 80% (see figure 5.33 above) of the content in each lesson phase in textbooks, and in workbooks (not shown), where practice questions tended to be more abstract, the figure was still high at 50% (Year 1) and 52% (Year 5) respectively. This is interesting in that it supports the core competency of ‘communication’ and reading.

Answer Forms

<table>
<thead>
<tr>
<th>Lesson Phase</th>
<th>Y5</th>
<th>Y1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Open Questions</td>
<td>Closed Questions</td>
</tr>
<tr>
<td>In Focus</td>
<td>61%</td>
<td>39%</td>
</tr>
<tr>
<td>Lets Learn</td>
<td>20%</td>
<td>80%</td>
</tr>
<tr>
<td>Guided Practice</td>
<td>7%</td>
<td>93%</td>
</tr>
<tr>
<td>Mind Workout</td>
<td>33%</td>
<td>67%</td>
</tr>
<tr>
<td>Maths Journal</td>
<td>83%</td>
<td>17%</td>
</tr>
<tr>
<td>Activity Time</td>
<td>100%</td>
<td>0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Workbook</th>
<th>Y5</th>
<th>Y1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Open Questions</td>
<td>Closed Questions</td>
</tr>
<tr>
<td>Textbook</td>
<td>2%</td>
<td>98%</td>
</tr>
<tr>
<td>Mind Workout</td>
<td>33%</td>
<td>67%</td>
</tr>
<tr>
<td>Mid Year Review</td>
<td>0%</td>
<td>78%</td>
</tr>
<tr>
<td>Revision</td>
<td>0%</td>
<td>100%</td>
</tr>
<tr>
<td>Review</td>
<td>0%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 5.14: Answer Forms analysis across Year 1 and Year 5 textbooks and workbooks

Using table 5.14, it is clear that MNP does not favour multiple choice questions, which feature only as a small proportion of revision and review exercises rather than within regular
lesson components. It is unclear why MNP prefers not to use multiple choice questions, which are not referred to in the MNP website. However, there is an interesting balance between open and closed question types across both year groups, with plenty of opportunities for students to access open questions, which are often encouraged in the teacher notes to develop into exploration or answer justification or enable opportunities for group discussion tasks to share thoughts or alternative methods (as shown in figure 5.34 below).

![In Focus](image)

**Lesson Approach**

To begin this lesson, show pupils the In Focus task and allow them to discuss the important information in the question. Ask them if they are able to show what the question is suggesting using bar models. How can we show 1 whole minus \( \frac{1}{6} \)? What do we need to do to the whole amount? Once pupils have been able to represent the bar appropriately, ask them to think about all of the different ways Lulu and Amira can share \( \frac{5}{6} \). Allow them some time to work this out, using pictures and concrete materials.

*Figure 5.34: An open question from a Year 5 textbook with accompanying lesson notes for teachers*

Open questions are often presented within teacher-led phases such as In Focus and Let’s Learn tasks, suggesting the facilitation of open discussion and response is seen by MNP as integral to the teacher role, rather than being left entirely to students independently. Similarly, guided practice and independent tasks are almost exclusively closed tasks across both year groups - which further supports the notion of open tasks requiring more teacher facilitation. Furthermore Activity Time, Maths Journal and Mind Workout tasks, whilst infrequent, usually offer opportunities for students to explore a concept without the need
for a fixed, closed answer to the task. It seems these task types are specifically integrated to allow for more opportunities for exploration and play, which helps align some tasks with some of the MNP principles outlined in the MNP website.

Mathematical Activity

Table 5.15 shows that calculation and operation tasks dominate Year 5 textbook and workbook tasks - 64% of all tasks across the books are identified as CO, a combined average of 60% in textbooks and 69% in workbooks. It is not surprising that Interpretation tasks are much lower, as these kinds of tasks are arguably more specific to particular units of work, such as interpreting charts and graphs, interpreting tables of data, or telling the time. This is evidenced in the long form data collection that shows the makeup of mathematical activities at an individual lesson level. Related to this, there are more units of work related to data interpretation in the Year 1B books than there are in the Year 5A books, hence any conclusion around this, or indeed any other mathematical activity category, needs to be contextualised against what units are actually taught in that year half. A related challenge to this section of data is that how a teacher uses a particular task could change the category it is identified as. For example, if a task, as presented in the book, states ‘How many sweets does Cathy receive?’, this could be perceived as a calculation and operation task - a student works out the answer through calculation. However, if the teacher created an opportunity through this question to work together and come up with multiple approaches, or asked
them to justify their answer with a drawing or model, then the task context could justifiably alter the category from ‘Calculation and Operation’ to, say, ‘Representation and Modelling’ or Argumentation and Reasoning. This conundrum is in itself a good example of the importance of teacher agency - how exactly do teachers take some of these tasks and deliver them, and does that in fact change or enhance the purpose of them. Lesson plan guidance helps decipher the intent for some of these tasks (as seen in figure 5.14 above), as do the MNP principles outlined earlier, however, the way in which they are presented in the textbook and workbook in isolation is more subjective, and so some of those opportunities could feasibly be delivered instead as a more directed ‘calculation and operations’ task. As the tasks as written in the books do not specifically state that the activity should be delivered in a particular way (ie, they do not specifically say, for example, ‘discuss in groups, or ‘find alternative solutions’ or ‘draw a diagram to support your answer’), they are identified here as a best fit to how they are presented. With those cautions in mind, the data as presented indicates a wider spread of mathematical activity types across the Year 1 textbook and workbook, showing that 19% of tasks promote argument and reasoning, 33% of tasks test interpretation, and 13% of tasks specifically requiring representations and modelling, with a small majority of tasks (36%) being calculations and operations. Conversely, Year 5 lessons incorporate a much less balanced selection of task types, leaning much more into calculations and operations (64%). Again, this appears to support the conclusion that visual structures are becoming phased out by Year 5, with tasks asking students to use or translate from mathematical tools such as number lines, place value counters, bar models and number bond diagrams appearing less.

A further point of interest is that both the Year 1 and Year 5 textbooks and workbooks maintain a very similar balance of mathematical activities from one to the other. Specifically, the change in percentage for each task type from textbook to workbook in Year 1 is no more than 2%, and in Year 5 no more than 5%. This suggests that whilst we have seen that the workbook moves students to work more in abstraction, the workbook itself seems to play no significant strategic role in changing the mathematical activity types from teacher-led learning (textbooks) to independent practice (workbooks), but rather appears by design to reinforce the same activity types.
Table 5.16 shows the textbook analysis in more detail (specifically looking at lesson phases) and shows that Representation and Modelling in Year 5 is most frequent in ‘In Focus’ activities to introduce a lesson, and ‘Activity Time’ tasks which are interspersed infrequently across the textbooks. Activity time tasks appear more in Year 1 lessons, which accounts in part for the larger proportion of Representation and Modelling tasks across the book.

Argument and Reasoning tasks also appear a lot in the In Focus tasks across Year 1 and Year 5, which mounts further evidence to support the exploratory, open ended nature of lesson introductions, allowing time for discussion and play. Maths journals also focus heavily on both ‘representation and modelling’, and ‘argument and reasoning’ highlighting further support of the journals being a tool by which students are able to demonstrate relational understanding through explaining the ‘how’ or ‘why’ of mathematical concepts with opportunities for justification and pictorial modelling.

Independent practice (workbooks) tends to promote calculation and operations tasks and interpretation tasks predominantly (see table 5.17 below), however as previously mentioned, the latter appears to have more to do with specific units of work (ie, statistics units) than a preferred activity across all unit types.
Table 5.17: Mathematical Activity analysis across Year 1 and Year 5 textbooks and workbooks

Table 5.18 shows an overview of the interpreted level of cognitive demand across all four books, it is clear that memorisation tasks are unfavourable in the MNP program, and contextually, the larger proportion of memorisation tasks in Year 1 is attributed predominantly to a unit on recognising money rather than a general task approach used throughout the books. What is perhaps more interesting is that there are clear opportunities for exploration tasks throughout the textbooks for both Year 1 and Year 5, with over a quarter of tasks in each book being identified as exploratory (high demand), although notably this decreases significantly in both workbooks, indicating that independent work
with less teacher input is not considered a favourable environment for students to perform exploratory tasks.

As previously mentioned, the context in which procedures are conducted in lessons means that categorising any as ‘low demand’ (‘with no connections’) was difficult, resulting in a high proportion of questions being technically identified as ‘high demand’ (E and P categories). Whilst the problems with identifying tasks as ‘procedure’ were not fully resolved, and therefore conclusions regarding this category should be cautionary, it is evident at least that MNP does contain a high proportion of ‘high demand’ tasks for students to work through. A lesson phase level of analysis (see table 5.19) reveals further support of the books aligning closely with MNP’s intent in its design principles in that the In Focus tasks, Maths Journal and Activity Time all heavily focus on explorative tasks.

<table>
<thead>
<tr>
<th></th>
<th>Exploration</th>
<th>Procedure</th>
<th>Memorisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Focus</td>
<td>80%</td>
<td>20%</td>
<td>0%</td>
</tr>
<tr>
<td>Lets Learn</td>
<td>21%</td>
<td>79%</td>
<td>0%</td>
</tr>
<tr>
<td>Guided Practice</td>
<td>5%</td>
<td>94%</td>
<td>1%</td>
</tr>
<tr>
<td>Mind Workout</td>
<td>50%</td>
<td>50%</td>
<td>0%</td>
</tr>
<tr>
<td>Maths Journal</td>
<td>83%</td>
<td>0%</td>
<td>17%</td>
</tr>
<tr>
<td>Activity Time</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Table 5.19: Level of Cognitive Demand analysis of Year 1 and Year 5 textbooks at a lesson phase level

Textbook support of the design principles and core competencies of MNP

There are some explicitly defined intentions of the MNP curriculum that link to both pedagogy and lesson structure that MNP links to academic researchers and philosophies. Similarly there are some more implicit elements of the MNP design intent in the ways in which tasks are structured within the textbooks and workbooks. In this section I analyse whether, or to what extent, the workbooks and textbooks appear to align with the principles
outlined in the MNP website and blog posts, and how they support the five core competencies. This section is intended to shed light on the full extent of MNP’s intended curriculum and better understand how well the textbooks and workbooks support what is said about MNPs intent on their website. As such, should there be a misalignment between the two, or contradictions between messaging and deliverable resources (tasks, lesson content etc), then the intended curriculum risks becoming muddled which could unduly affect both interpretation and enactment of the MNP curriculum. Determination of the intended curriculum of MNP specifically addresses research question 1.

Key elements of the intended curriculum will therefore be directly compared against textbook and workbook content in this section. For recap, the eight design principles identified through the MNP website are categorised as follows:

1. Children need time to accommodate new ideas - and construct their knowledge from the world around them.
2. Children need opportunities to play with mathematical ideas in an informal and exploratory way prior to any structured learning.
3. Learning is effective when cooperatively learning with knowledgeable others.
4. Relational understanding is developed alongside instrumental understanding.
5. Mathematics learning should utilise the Concrete-Pictorial-Abstract approach to develop understanding of concepts.
6. Teaching progresses in small steps.
7. Moving the whole class through the same content at the same pace.
8. Use a variety of representations of mathematics including narrative structures and physical manipulatives.

Furthermore, the five core competencies outlined by Ban Har are:

1. Visualisation
2. Generalisation
3. Communication
4. Number sense
5. Metacognition
How these principles and competencies are supported within the textbooks and workbooks will be evaluated through revisiting the MNP lesson phases: In Focus, Let’s Learn, Guided Practice and Independent Practice and the less frequent phases: Maths Journal and Activity Time through the lens of the above analysis of the textbooks and workbooks.

Phase 1: “In Focus”

The ‘In Focus’ section of MNP textbooks is cited as being a key opportunity in MNP lessons to explore open problems through narratives involving recurring characters. The conducted analysis of MNP textbooks revealed that ‘In Focus’ tasks did predominantly feature Fictional tasks - meaning they utilised a narrative to create a fictional setting in which to deliver a mathematical problem - 80% of tasks in 5ATB and 76% of 1BTB respectively. Fictional tasks directly correspond to design principle 9. which specifically cites narrative structures as a way of enabling students to construct knowledge from the world around them, hence the textbook tasks support what is stated in the website and blog posts. Fictional In Focus tasks frequently utilised a set of recurring characters introducing or discussing problems, and most of those identified as Non-Applicative also tended to use the same characters to introduce prompt questions as well. The choice to use recurring characters again supports principle 9.

Furthermore, the majority of In Focus tasks were also classified in the analysis as Exploratory, and Open task types. This combination of categories (Fictional, Exploratory and Open) supports the claims by MNP that ‘In Focus’ is intended to enable and develop structured, cooperative discussion. Hence the task design and presentation in this section does appear to match with the intent statements made by MNP in their blog and website, specifically supporting principles 1. (children need time to accommodate new ideas), 3. (learning is effective when cooperatively learning with knowledgeable others), 4. (relational understanding is developed alongside instrumental understanding) and 9. However, it is important to note the role of teacher agency here, as many of the tasks for ‘In Focus’ were categorised as ‘closed’, meaning that there was one fixed closed answer required in order to complete the task being presented. As mentioned previously, this does not necessarily mean that the task would be delivered in a way that only allowed for singular responses, only that
it was presented in that way. For example, in Lesson 5 of the 1BTB, the In Focus task asks the question “What is the difference between the number of Elliott’s tennis balls and the number of Holly’s tennis balls?” . This is a closed question with only one possible correct answer, however the way in which it is delivered could make this task more open ended than presented (for example, ‘can you convince your partner this is true? Can you draw a diagram to support your answer? Can you find two ways to achieve the answer? Etc). Furthermore, the question could be posed and left to students to decide collaboratively together on tables for a longer period of time than the question alludes to. The decision making around this task is arguably rooted in task intent, which is not entirely clear from the textbook itself (In Focus general guidance within the textbook reads as ‘Includes questions related to various lesson objectives as an introductory activity for pupils’ which does not specifically encourage any of the design principles) , however the lesson plan guidance for In Focus tasks tends to offer far more details, specific questioning and prompts than the book itself.

For example, in Chapter 20 L3 in 1BTB, the In Focus task simply reads ‘How do we describe different ways to turn our bodies?’. However the lesson plan guidance adds a significant amount of further guidance and prompting that reshapes the intent of the question, asking, for example:

Ask them if they think they are able to turn their bodies like the characters in the picture. Ask them how they would describe the turns. Tell them your friend said all the characters started by facing forward. How did she know that?

(Lesson Plan 1B_20_3, Appendix b)

Hence the lesson plans, which are not included in the MNP textbooks or workbooks and require subscription to MNP’s annual subscription package, significantly augment the intent of the In Focus task, but without them, the task itself is at risk of being delivered incorrectly (that is, in a way that undermines MNP intent).

On a similar note, the notion of ‘play and exploration’ (principle 2.) is again less clear in the textbooks than it is within the lesson guidance itself. The guidance is far more suggestive in
terms of *how long* the ‘Let’s Learn’ cycle of an MNP lesson should take, with several thinking prompts for students, additional models to introduce, and generally what appears to be a highly structured and partially scripted dialogue, but with more open expectations for how students respond (there is limited guidance to suggest what to anticipate as a response, or how to guide response from students compared to the scripts provided for teachers). Hence the lesson plans appear to strongly support In Focus tasks as encouraging the design intent points 1., 2. and 3., whereas the textbooks alone do this less effectively or explicitly. How the ‘In Focus’ task is enacted in classrooms will be a particular point of interest based on these findings, as it appears to be an area that could feasibly be interpreted very differently depending on whether a teacher has access to and embraces the lesson plan guidance (which shapes the intent much more clearly), or whether they deliver tasks as written in the textbook alone (which risks delivering the tasks in a way that does not support author intent).

In conclusion, the In Focus element of MNP lesson design as presented in the textbooks appears to align closely with the intended MNP principles outlined in the MNP website, specifically evidencing design principles 1., 2., 3., 4. and 9. when supported by the lesson plans for delivery guidance. However, without that additional guidance, the purpose and perceived benefits to this particular lesson phase are at risk of being significantly muted or ignored due to the ways in which the task can be interpreted through the textbook alone, particularly with regards to the amount of discourse and time MNP appears to intend for the task to involve, none of which is indicated in the textbook.

If the In Focus phase is delivered as intended, then it also supports several core competencies: visualisation and communication (pupils discuss the problems with each other and are encouraged to support reasoning with pictures or models), and number sense (students are often encouraged in lesson plans to play with a mathematical idea using different representations), with opportunities for generalisation. However all of these competencies are again not explicitly developed in the textbook, but rather in the lesson plan itself.
Phase 2: “Let’s Learn”

The ‘Let’s Learn’ activities are described in the textbook guide notes (within the book itself) as ‘Introduces new concepts through a CPA approach (principle 5.) with the use of engaging pictures and manipulatives. Guided examples are provided for reinforcement’. Analysis of textbooks 1BTB and 5ATB highlight that visual support in this section was identified in 84% of examples, often supported with accompanying text and abstract symbols (69% of tasks), and in fact this visual support is a spike in Year 5, demonstrating that the Let’s Learn phase in this year group is recognisably a component with design features that include specific addition or inclusion of visuals to tasks:

![Comparison of Visuals use in lesson phases Y1/YS](image)

**Figure 5.35:** The inclusion of visuals by lesson phase in Year 1 and Year 5 textbooks

It was noted that no imagery used in any of the four books analysed was considered ‘unnecessary’ and all were considered to add to the relational understanding of either the problem being presented, or the mathematical concepts being developed. Even without the lesson plan guidance, the textbooks present ‘Lets Learn’ as a cooperative task between teacher and students (principle 3.) and provide imagery that supports the CPA approach (principle 5.), particularly the use of pictorial imagery, quite explicitly.

In a similar manner to the In Focus tasks, the use of manipulatives is sometimes inferred through pictures, but only explicitly instructed or suggested in the lesson plans. Using the textbooks without the lesson plans, teachers are left to their own intuition and agency as to when and where manipulatives are introduced and used, whereas the lesson plans themselves often explicitly encourage their use.
For example, in Lesson 15 Chapter 6 Year 5, Let’s Learn is presented by recurring characters as shown in figure 5.36:

![Figure 5.36: An example of a Let’s Learn activity that does not explicitly require manipulatives](image1)

There is no indication here that concrete manipulatives should be used to deliver any part of this lesson phase in the book itself, although there is a minor prompt in that the characters are talking about actual paper that they used. However, in the more detailed lesson guidance in the MNP lesson plans, both resources and opportunities for the use of manipulatives are listed as shown in figure 5.37:

![Figure 5.37: Manipulatives support highlighted in an MNP lesson plan for a Let’s Learn activity](image2)

**Resources**

- Circular representations of fractions (useful but not essential)
- Coloured strips of paper/card for cutting and folding (useful but not essential)
- Access to a fraction wall (useful but not essential)
Use of recurring characters in Let’s Learn helps build upon any narrative structures (principle 9.) set up in the In Focus section of the lesson, and typically, Let’s Learn simply builds structured exploration of the In Focus task with worked examples and, often, alternative approaches to the same problem, or problems that utilise the same numerical values throughout. As such, Let’s Learn problems tend to continue to use the narrative structures from phase 1 (80% of Year 5 tasks in Let’s Learn were identified as Fictional, and 66% in Year 1), further supporting principle 9.

Whilst Let’s Learn is usually structured using questions that can be answered by or with pupils, occasionally the ‘Lets Learn’ section is entirely worked (complete) solutions to problems, or indeed multiple methods to solve the exact same problem, demonstrating that the lesson phase is intended at least some of the time to be delivered more as cooperative learning with a knowledgeable other (in this phase, the role of ‘knowledgeable other’ is the teacher as opposed to peers in phase 1) rather than always providing questions to be solved by pupils. Again, the lesson plan guidance is more in depth and offers more insight into task intent. For example, in Chapter 6, Lesson 7 of 5ATB, Let’s Learn, there is a singular example with visual support to demonstrate how you can add 1/6 and 4/6 together.

![Figure 5.37: An example of a Let’s Learn activity](image)

However, the lesson plan guidance states “Use Let’s Learn to show pupils some of the ways to form the number pairs to 5/6”, demonstrating an emphasis on multiple methods in the task intent.

Within the textbook analysis, Let’s Learn was identified as having a significant majority of task answer forms as ‘closed’ rather than ‘open’, which suggests a steer towards directed
teaching in this phase to find solutions to problems rather than more discovery and play orientated methods identified in the ‘In Focus’ stage of learning. This arguably is beginning to introduce instrumental understanding alongside relational understanding (principle 4.). The problems in the ‘Let’s Learn’ stage are usually directly related to the problem in phase 1, often just posing alternative methods or furthering the problem itself. This appears to support principle 6. (small steps teaching).

In conclusion, the Let’s Learn element of MNP lesson design as presented in the textbooks also appears to align closely with the intended design principles outlined in the MNP website, which are again enhanced via the lesson plans for delivery guidance. Lesson guidance particularly enhances the promotion of discourse opportunities and multiple method exploration including the ‘concrete’ element of the CPA approach. Let’s Learn fulfils its task intent regarding the enhanced support of concepts using visuals, which in turn aligns itself with the CPA model of delivery, and it prompts teacher led learning akin to the ‘knowledgeable other’ philosophy highlighted in principle 3.

Arguably the same core competencies are also developed in this phase as in phase 1, with subtle shifts such as communication being more between teacher and pupils, and the pictures and models that develop understanding are explicitly provided in the textbook rather than being developed by pupils themselves.

**Phase 3: Guided Practice**

Textbook guidance states that Guided Practice “comprises questions for further consolidation and for the immediate evaluation of pupils’ learning”. This statement gives little information into the structure or general design principles of this lesson phase, however the analysis for this research reveals a few interesting points of note.

Firstly, there appears to be an intended increase in the frequency of entirely symbolic question types in Year 5, which moves up from 3% symbolic to 13% symbolic between the Let’s Learn phase of the lesson and Guided Practice, and ‘Symbolic / Textual’ tasks increase in frequency from 12% to 38% across the two phases.
For reference, symbolic question types rely almost entirely on symbols in their presentation eg “3 + 5 =” rather than combining the symbols with a more accessible form of presentation such as text or visual structures or models. This infers an expectation around increased confidence and competence with handling more abstract question types at this stage of development, which is notably absent in Year 1 where entirely symbolic questions remain very low in frequency in all three phases of In Focus, Let’s Learn and Guided Practice. Further evidence of this expectation of developing comfort with abstraction can be seen in the summary table of all instances of abstraction in question types (see table 5.20 below), this table includes instances where symbolic representations are not only presented in isolation, but also in combination with other formats such as text, visuals, or text and visuals.

When looking at this more holistic approach, it is still evident that there is a greater frequency of the presentation of abstract symbols in Year 5 than in Year 1 (91% in Guided Practice in Year 5, and 40% in Guided Practice in Year 1). It should be noted however, that this may, to some extent, also have a dependency on the curriculum content in each of those year groups. For example, the Year 5 curriculum coverage may by design incorporate more abstract concepts than in Year 1, hence the likelihood of more abstract presentation would be higher. Regardless, the contrast of 91% for Year 5 and 40% for Year 1 is significant and points towards a linear progression through the CPA approach (principle 5.) whether by design from MNP or by design from the DfE’s programme of study.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Symbolic only</th>
<th>5ATB</th>
<th>1BTB</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Focus</td>
<td>1%</td>
<td>2%</td>
<td></td>
</tr>
<tr>
<td>Let’s Learn</td>
<td>3%</td>
<td>0%</td>
<td></td>
</tr>
<tr>
<td>Guided Practice</td>
<td>13%</td>
<td>1%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Phase</th>
<th>Symbolic (all instances)</th>
<th>5ATB</th>
<th>1BTB</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Focus</td>
<td>78%</td>
<td>12%</td>
<td></td>
</tr>
<tr>
<td>Let’s Learn</td>
<td>97%</td>
<td>47%</td>
<td></td>
</tr>
<tr>
<td>Guided Practice</td>
<td>91%</td>
<td>40%</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.20 A summary of the use of symbolic representations in the first 3 lesson phases

This increase in abstraction also supports development of instrumental understanding (principle 4.) however it appears to be developed across year groups rather than being evident across lesson phases.
Another interesting finding during textbook analysis was that the Guided Practice phase of MNP lessons presents significantly fewer open response style questions to pupils, dropping between the In Focus and Let’s Learn phases, but also dropping again between the Let’s Learn and Guided Practice phases, which makes sense considering that by design the In Focus and Let’s Learn phases actively promote discussion, collaboration and exploration, whereas guided practice appears much more focused on developing procedural fluency
(principle 4.), again supported heavily by table 5.21 above which highlights the significant increase in ‘procedure’ questions asked in the Guided Practice section compared to the In Focus and Let’s Learn phases. Guided Practice is a section in MNP designed to utilise the teacher as a knowledgeable other (principle 3.) to assist with students while they attempt to accommodate new ideas (principle 1.) and appears to act as a bridging phase where an emphasis on relational understanding begins to shift towards instrumental understanding (principle 4.). In other words, the two previous phases strongly focus on deeper understanding of mathematical concepts and ideas, with heavy involvement of discourse and exploration, whereas during guided practice, emphasis is becoming refocused on being able to routinely solve similar problems, scaffolded primarily by the involvement of the teacher rather than the structure and presentation of the problems themselves.

### Phase 4: Independent Practice

During the fourth and final consistent lesson phase, Independent Practice, worksheets (WS) are completed for each lesson. Analysis in figure 5.38 below indicates how lessons converge towards closed question types for both Year 1 and Year 5, following a similar structure to the Guided Practice (GP) lesson phase, but with an intended reduction in teacher input (hence the use of ‘independent’ in the phase, evidenced also in no guidance or support for this part of the lesson in any of the MNP lesson plans seen).

<table>
<thead>
<tr>
<th>Phase</th>
<th>Open Qs</th>
<th>Procedural</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SATB</td>
<td>1BTB</td>
</tr>
<tr>
<td>In Focus</td>
<td>61%</td>
<td>50%</td>
</tr>
<tr>
<td>Lets Learn</td>
<td>20%</td>
<td>15%</td>
</tr>
<tr>
<td>Guided Practice</td>
<td>7%</td>
<td>4%</td>
</tr>
</tbody>
</table>

Table 5.21 A summary of the use of closed and procedural task types in the first 3 lesson phases
This phase of the lesson is defined in the textbooks as a specific opportunity to assess individual students, and is notable for its lack of open questions, and also high frequency of procedural question types. Combined, the two categories infer that independent practice is focused on developing procedural fluency, and therefore, instrumental understanding. In other words, the phase specifically has no teacher input, nor discussion, nor open ended enquiry. Instead it has closed questions requiring pupils to work through a process or calculation to arrive at a singular correct answer.

It appears then, that independent practice is focused predominantly on closed procedural questions which seems to further support the movement towards abstraction in the CPA approach, an observation that is strengthened by figures 5.40 and 5.41 below:
Each chart demonstrates that the worksheet (WS) activities in the Independent Practice phase of the lesson (the workbook) demonstrably reduce the frequency of support of visuals in year 5 (pictorial scaffold, usually by means of a mathematical model such as bar models or number lines) as shown in Figure 5.40, whilst increasing the number of tasks that use symbolic representations in the absence of visual support (Figure 5.41). Again, this phase of the lesson appears to support the linear progression of the CPA model (principle 5.), in that abstraction is more prominent both in the end phases of lessons, and at the end phases of primary school - older children appear to be exposed to more abstraction, and younger children are exposed to more visual scaffolding. Whilst this cannot be entirely conclusive.
without analysing the books for intermediate year groups, it does support the analysis of the MNP website and blog posts that pointed towards a linear progression through CPA.

**Additional Phases: Maths Journaling and Activity Time**

The two most frequently used phases that do not appear in every lesson are the Maths Journal, and Activity Time which make up roughly 1% of tasks in Year 5, and 14% of tasks in Year 1. These activity phases predominantly involved ‘authentic’ task types (tasks that involved data collection instigated by students) with 34/39 of the journal and activity time phases being identified as ‘authentic’ contexts in Year 1 (and only 4 other examples of tasks with authentic contexts out of 232 tasks from all other phases). See Table 5.22 for a concise summary.

![Table 5.22](image)

<table>
<thead>
<tr>
<th>Year 5</th>
<th>Maths Journal</th>
<th>50%</th>
<th>83%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activity Time</td>
<td>100%</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>Year 1</td>
<td>Maths Journal</td>
<td>64%</td>
<td>100%</td>
</tr>
<tr>
<td>Activity Time</td>
<td>96%</td>
<td>71%</td>
<td></td>
</tr>
</tbody>
</table>

**Table 5.22** A summary of authentic exploratory task types in journal and activity time phases

It appears evident then that both the maths journal and activity time phases are intended to specifically address certain MNP design principles. Both phases allow for students to explore ideas (principles 1. and 2.), and activity time is designed to explore ideas cooperatively with other students (principle 3.). Both contain a significant amount of authentic task types, which align with principle 9. ‘Students should construct their knowledge from the world around them’, and maths journals appear designed to specifically develop core competency 5: metacognition, by literally asking students to reflect upon what they know and how they know it is true. Indeed, the definition of a maths journal at the front of the textbooks states that its purpose is to ‘provide pupils with opportunities to show their understanding of the mathematical concepts learnt’, and activity time is defined as an opportunity to ‘explore mathematical concepts or to play games’.
Summary
A summary of the supporting evidence against each of the 9 identified design principles is shown in Table 5.23 below.

Key:
L Lesson Plan support
T Textbook and workbook support
W Website and Blog support

<table>
<thead>
<tr>
<th>Design Principle</th>
<th>In Focus</th>
<th>Let's Learn</th>
<th>Guided Practice</th>
<th>Indep. Practice</th>
<th>Activity Time</th>
<th>Maths Journal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Children given time to accommodate new ideas</td>
<td>LW</td>
<td>LTW</td>
<td></td>
<td></td>
<td></td>
<td>T</td>
</tr>
<tr>
<td>2. Children given opportunities to play with mathematical ideas in an informal and exploratory way</td>
<td>LW</td>
<td></td>
<td></td>
<td></td>
<td>TW</td>
<td></td>
</tr>
<tr>
<td>3. Cooperative learning with knowledgeable others.</td>
<td>LW</td>
<td>LW</td>
<td>LW</td>
<td></td>
<td>TW</td>
<td></td>
</tr>
<tr>
<td>4a. Relational understanding is emphasised</td>
<td>LW</td>
<td>LTW</td>
<td>W</td>
<td></td>
<td>W</td>
<td>TW</td>
</tr>
<tr>
<td>4b. Instrumental understanding is emphasised</td>
<td></td>
<td></td>
<td></td>
<td>TW</td>
<td>TW</td>
<td></td>
</tr>
<tr>
<td>5. The Concrete-Pictorial-Abstract approach is utilised</td>
<td>LTW</td>
<td>LTW</td>
<td>TW</td>
<td>TW</td>
<td>TW</td>
<td>W</td>
</tr>
<tr>
<td>6. Teaching delivered in small steps</td>
<td>LTW</td>
<td>LTW</td>
<td>TW</td>
<td>TW</td>
<td>W</td>
<td>W</td>
</tr>
<tr>
<td>7. The whole class moves through the same content at the same pace</td>
<td>LW</td>
<td>LW</td>
<td>LW</td>
<td>LW</td>
<td>LW</td>
<td>LW</td>
</tr>
<tr>
<td>8. Knowledge constructed from the ‘real world’ eg narratives or data gathering</td>
<td>TW</td>
<td>TW</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.23 A summary of the supporting evidence towards each identified design principle

From table 5.23, it appears that almost all design principles highlighted in the website are indeed supported and promoted through the textbooks and workbooks themselves. Certain
principles are more clearly defined and emphasised within lesson plans (such as design principles 1., 2. and 3.), however all but one design principle has clear supporting evidence.

**What does this tell us about the intended curriculum?**

Reflecting against two of the key research questions for this thesis, the evidence gathered can be used to help answer the following:

1. **What strategies are emphasised and prioritised in curriculum packages endorsed by the DfE for primary mathematics in England?**

2. **How are textbooks used as teaching aids in DfE approved curriculum packages for primary mathematics in England?**

   Research question 1) refers entirely to the *intended curriculum*, which has now been analysed and discussed more broadly, but here can be focused more clearly and specifically against the research question. 2) will be discussed here, but also revisited after the enacted curriculum, to compare both intent and actual use in the classroom. Similarly, both research questions will be discussed again in the conclusions section.

1. **What strategies are emphasised and prioritised in curriculum programs endorsed by the DfE for primary mathematics in England?**

   At the time of this study, only one primary mathematics curriculum program had been approved by the Department for Education, called ‘Maths, No Problem!’ (MNP) which has been the focus of this thesis. It should be noted that since this study was conducted, a second curriculum program, Power Maths, has also been approved, but is not part of this research. This study has analysed the content of the MNP website, which includes several pages of declarations of purpose and intent, and information regarding the structure and academic influences of the program. The website also includes over 200 blog posts from staff members of MNP and teaching ambassadors covering a range of advice and further information on the practical delivery and influences on MNP. Further to this, a sample from
the MNP package itself has been analysed, which comprises a set of twelve textbooks, twelve workbooks and detailed lesson plans. Analysis has shown that what is stated and promoted in the website materials is, largely, confirmed and supported with the textbooks and workbooks themselves, based on a sample of workbooks and textbooks in Year 1 and Year 5 respectively.

MNP state 5 key theories underpin their program, from which many of their teaching strategies are informed. Each one is listed below and discussed with the supporting evidence found through this analysis.

1. Children need time to accommodate new ideas and construct their knowledge from the world around them.

This statement, which MNP attribute to the work of Dienes, is strongly evidenced in MNP website, blog and lesson plan materials, with lesson phases (In Focus, and Let’s Learn) designed to incorporate this design principle specifically. Lessons are front loaded with discourse opportunities and facilitations that stay with a single problem or ‘Anchor Task’ posed in an opening lesson phase referred to as ‘In Focus’. In Focus is part of a multiple-phased lesson structure that is used for every single lesson in the MNP program, and always contains a single problem for whole class discussion or group work. Lesson guidance offers multiple prompts to prolong this section and facilitate students discovering and constructing understanding of both what the problem is asking mathematically, and what tools and knowledge they have that will help them uncover the solution. Often this is presented as an opportunity to attempt to solve the problem in multiple ways - again, a strategy to keep students focused on a single task explored in depth. This strategy is described by MNP themselves as ‘slow paced’ (Maths No Problem, 2019d), however it may be better understood as precisely focused in that whilst children are not being rushed into solving multiple questions early on in the lesson, they are being encouraged to work diligently and to think hard about mathematical structures and reasoning at all times during the lesson.

Similarly, analysis has demonstrated that a majority of In Focus tasks utilise narrative structures to introduce problems, which supports the strategy of ‘constructing knowledge
from the world around them’. This ranges from interpreting real-life data about country demographics to dividing cake for a group of friends, but rarely does MNP stray away from tangible recognisable scenarios to introduce mathematical problems in their In Focus tasks, and where they do, it tends to be for more abstract concepts (which are often introduced in MNP as games that children play or challenges that they give each other). In lesson plans, phrases such as ‘give pupils time to discuss’, and ‘give pupils time to read the table and discuss the pattern’ (Lesson Plan 5A_6_8, Appendix c; Lesson Plan 6B_9_5, Appendix d) are evident in the guidance for all fifteen of the In Focus phases of the lesson plans that were seen. Notably, there is risk of misinterpretation of the intent associated with the In Focus and Let’s Learn phases in that the focus on this design principle is particularly emphasised around the way in which the task is delivered, which is only explicit in the lesson plans and website but absent in the textbooks themselves.

Extracting tacit strategies from this principle, MNP explicitly facilitates an opening lesson phase that normalises a narrative driven single problem intended to be discussed and explored in depth to provide time for pupils to focus and think hard about one thing without the distraction of further problems that might risk focusing on procedures rather than concepts.

2. Children need opportunities to play with mathematical ideas in an informal and exploratory way prior to any structured learning.

This key theory is also heavily utilised in the In Focus lesson phase, with directed time in lesson guidance dedicated to allowing children to explore ideas for themselves with teachers acting as facilitators of discussion and idea generation rather than answer providers. A more directed, but still exploratory, lesson phase follows afterwards in the Let’s Learn phase which often explores several methods or scaffolded modelling of mathematical approaches to similar if not identical problems to the one outlined in the In Focus phase. The explicit strategy here can be summarised as facilitating opportunities for pupils to evolve their exploration and ideas, through the guidance of the teacher, into more formal methods and approaches to solving mathematical problems, with the use of scaffolding models and visuals for support.
3. Learning is effective when cooperatively learning with knowledgeable others.

This key theory is underpinned by the two previous strategies in that the In Focus task emphasises peer thinking and idea generation, thus the knowledgeable other in that phase is other pupils, then in the second lesson phase (Let’s Learn) there is a sharing of knowledgeable other roles between students and teacher, and finally in the third lesson phase, ‘guided practice’ the knowledgeable other role has fully shifted from peers to teacher. As an explicit strategy, this can be summarised as facilitating sequenced opportunities for pupils to learn from and generate ideas with their peers, followed by more direct collaborative guided instruction from their teacher.

4. Relational understanding is emphasised alongside instrumental understanding.

This principle is evidenced in MNP lesson plans and also underpinned by the structure and content of the textbooks and workbooks. Relational understanding is developed through narrative structures, concrete models and strong, consistent mathematical visual models such as bar models and number lines, to develop relational knowledge of mathematical concepts and ideas. A significant proportion of lesson time is spent on developing understanding of structures through the In Focus, Let’s Learn and Guided Practice lesson phases, whilst then moving strategically into more independent work that follows more procedural techniques to solving problems with less scaffold and more abstraction.

5. Mathematics learning should utilise the Concrete-Pictorial-Abstract approach to develop understanding of concepts.

This study has shown that MNP supports the CPA model not only in principle, but also in practice through the visuals utilised in their textbooks and workbooks. There appears to be greater support for pictorial representations in earlier year groups, with a gradual move towards abstraction both within lessons themselves (the initial phases of lessons tend to be
more scaffolded with pictorial models, and the end phases tend to be more abstract), but also across year groups - there is more abstraction in Year 5 than in Year 1. Whilst this appears to conclude that the CPA model is therefore sequential (i.e., pupils tend to move from concrete, to pictorial, to abstract). The CPA model is visibly supported and utilised across the MNP program both as a specified strategy listed in the books and website, and also as a visibly evidenced component in the textbooks and workbooks themselves. Again, there is some risk here in that the ‘concrete’ element of the CPA model is emphasised in the website materials and the lesson plans, but not in the textbooks themselves. As such, a teacher without subscription to the online support for the MNP program could miss the intended opportunities in lesson phases such as In Focus to allow for play and use of physical tools to explore concepts.

**Reading and Communication**

Within the five core competencies, there are references specifically to reading and communication (competency 3.) which is described as an expectation that learners are to answer problems in class in full sentences using mathematical language and terminology. Analysis of the workbooks and textbooks uncovered a high proportion of text in problems as can be seen in figure 5.42 below:

![Figure 5.42: Tasks using text in Year 5 and Year 1 by lesson phase](image)

Across each lesson phase, over 80% of questions were classified as including textual content in both year groups, including in the worksheets of the independent practice section, which appears in the student workbooks rather than the textbooks. Further research into this area would be interesting, to help determine, for example, the type and
quantity of text included in different questions and tasks, and the proportion of technical vocabulary used, neither of which is explicitly captured in this study. However, the data that has been captured indicates that pupils are frequently required to read and understand text and technical vocabulary, suggesting that a specific pedagogical strategy is to ensure that pupils are confident and competent with the use of mathematical vocabulary.

**What strategies are implied by MNP but not explicitly cited?**

Whilst a number of explicit strategies cited by MNP and supported by the content and structure of their lesson materials, there are some more subtle strategy inferences evident from the materials presented, and also by what is not present.

**Highly structured lessons delivered in small steps**

Almost every MNP lesson follows the exact same structure, which is prescriptive both in terms of lesson resources, but also scripted teacher dialogue, assessment prompts and in some instances, pre-emptive conversation chains between teacher and pupils. Deviation from the structure of 1) In Focus, 2) Let’s Learn, 3) Guided Practice, 4) Independent Practice is infrequent and involves adding further components rather than removing or rewriting the basic structure. In these instances, further lesson phases are introduced such as ‘Maths Journal’ which appears roughly once per unit, or ‘Activity Time’ (which is more frequent in Year 1 than in Year 5).

Whilst it might be tempting to assume this highly structured approach is typical, if not necessitated by the format it is presented in (ie, textbooks by design are ‘highly structured’), a brief analysis of another recently published mathematics textbook, Essential Maths (White & Rayner, 2020), highlighted a number of repeating section headings, but no clear consistency of structure across chapters. Further comparative research into the structural formatting of recent mathematics textbooks published in England would be of interest to develop this area of understanding, however it appears that the consistency of design within each lesson phase across units and year groups is purposeful and informed by the design principles. Furthermore, each lesson incrementally develops knowledge and understanding slowly, through introducing a problem, modelling several solution approaches and extensions to the problem, then finally students work through similar problems
independently. Lessons contain few learning points, and focus predominantly on developing relational understanding with instrumental understanding being developed only in the final phase of the lesson.

Furthermore, the apparent granular approach to developing understanding of mathematics does align with NCETM’s five big ideas of mastery (NCETM, 2017c) which states that small steps are ‘easier to take’ and recommends focusing on one key point for each lesson. It is perhaps significant that Debbie Morgan, Primary Director of NCETM, was one of the three members of the ‘expert group’ assigned to determine which textbook developers were of a high enough standard to be endorsed by the DfE (and MNP was the sole selection until late 2018). (NCETM, 2016a)

![The Five Big Ideas Behind Teaching for Mastery](image)

**Figure 5.43:** The Five Big Ideas of Maths Mastery as defined by the NCETM taken from NCETM (2017c)

**Moving the whole class through the same content at the same pace**

Throughout the MNP website and associated materials, there is no optionality or any clearly signposted extension exercises to further challenge pupils who may be finding content less challenging. This apparent lack of explicit differentiation is aligned with NCETM’s maths mastery guidance, which states that a key philosophy behind mastery is something akin to all children work together on the same content at the same time (Figure 5.43), however this has sometimes been misinterpreted as ‘no differentiation’ (NCETM, 2016b) when in reality it
is, in the case of MNP at least, more accurate that differentiation is more implicit than explicit - the differentiation comes as teacher strategies within the classroom that do not change the tasks that the pupils are attempting, and are more of a live intervention than a pre-planned redirection of a pupil's journey through tasks and objectives. These strategies are hidden away from the textbooks and workbooks, but are found as a specifically codified element of lesson plans as shown in the example in figure 5.44 below:

![Image of differentiation strategies](image)

**Figure 5.44:** Examples of differentiation in an MNP lesson plan

Whilst these codes are not present for every lesson, they do acknowledge and allow for additional in-class strategies to help to bring all students to the same minimum standard of learning for the lesson. It is interesting that advice in these sections appears to offer additional strategies or activities that complement the main task, but do not allow for pupils or teachers to change or ignore the main tasks of the lesson (historically this has often been referred to as ‘differentiation by task’ (McNamara & Moreton, 2016) - because this ensures, or at least, reduces the risk of, pupils falling behind or missing out on significant steps in their learning journey through the MNP program.
This minimum standard regarding completion of tasks or more broadly, meeting lesson objectives, is also explicitly highlighted in the lesson plans as a section headed ‘Non-Negotiable’ as shown in figure 5.45 below:

### Non-Negotiable

- Pupils can add fractions of the same denominator accurately.
- Pupils can find a common denominator between two fractions, when one of the fractions has the common denominator.

**Figure 5.45:** An example of ‘non-negotiable’ statements from an MNP lesson plan

This does invite an interesting question: what do teachers do in the classroom in instances where a pupil or pupils have not met the non-negotiable criteria? It is at best unlikely that in all circumstances, all learners would meet the non-negotiables all the time, regardless of context, background, or teacher. How then, do teachers account for those situations? Do they engage at all with this quandary, or do they press on against the philosophy of MNP either by choice or necessity due to, for example, timetable pressure to cover the curriculum? This appears to be a potentially recurring and likely situational issue that lacks guidance from MNP and would demand teacher agency to overcome.

**2. How are textbooks used as teaching aids in DfE approved curriculum programs for primary mathematics in England?**

By analysing the MNP website, a sample of lesson plans and a sample of the associated workbooks and textbooks, it has become clear that the intent for the usage of the books is that the textbooks are integral to the delivery of all maths lessons to not only enable discussion and learning new content, facilitated by a combination of the teacher and peers in a highly structured set of tasks (In Focus, Let’s Learn and Guided Practice), but that independent practice is also highly regulated and normalised across lessons using the pupil workbooks. Both books (textbook and workbook) are integral to a majority of activity time in lessons and include key content to access the lesson tasks and imagery used for pupils. Children break away from the usage of the books near the start of each lesson when they are encouraged in, for example, ‘In Focus’ activities to discuss ideas and approaches to
problems in groups. Despite this high dependence on the two books in each lesson, much of
the lesson structure is based around interaction, discussion, exploration and ‘play’ which is
merely given structure by the books rather than driven entirely by them. In order to perceive
this accurately, it may be more useful to imagine that the books do not work as well, or as
intended, in a solitary setting. The textbooks in particular, require an audience of peers and a
knowledgeable expert in order to be worked through as intended.

It is less clear how much the physical textbook is intended to be ‘in front of’ the pupils in
a literal sense in classes, or whether the intent is more for that content to be accessible to
the teacher, who facilitates much of the first three lesson phases. However, the implication
appears to be that pupils have access to the physical resource based upon the way in which
sales purchases are presented to schools (textbooks, as well as workbooks, are sold as class
sets). This question will be revisited in more depth following discussion of the enacted
curriculum in the next chapter.

What can we infer about the intended teacher?

Before we move to the penultimate chapter to discuss the enacted curriculum, there are
some worthy observations to discuss regarding the inferred intended teacher from the MNP
materials analysed. Within the literature review, there were cases of enactment of the
intended curriculum that were seemingly defined by the type of teacher involved in the
implementation of the curriculum, and the environmental influences of how teachers
adopted or adapted different curriculum materials. For example, Collopy (2003) noted that
teachers who were not confident in their ability, or their perception of their ability, to teach
well, were more accommodating and persuaded by the intent of the materials presented for
use in the classroom, and were perceived to engage with them more fully, and their
enactment was more closely aligned with the intent.

This sense of ‘buy in’ was also noted by Smagorinsky et al. (2002) who observed the
influence of peers as an integral element of how the intended curriculum was adopted, and
whether its influence or methods would be long lasting. Through the materials presented by
MNP, there are inferences towards what might be considered a model teacher, and
suggestions towards areas that MNP considers the responsibility of a teacher and a teacher’s agency to deliver effectively which could contribute towards understanding and answering research question 4 (‘How is teacher agency impacting upon the interpretation and delivery of MNP?’). Similarly, by contrast, the materials infer what MNP consider not to be the responsibility of a teacher either in any notable absence of guidance or reference, or in the explicit detailed guidance that appears to shift responsibility for decision making to MNP.

**Task design**

Whilst some support is offered in areas such as questioning and, in professional development materials, there is guidance in the use of the CPA model and associated tools such as the bar model and number bond diagrams, there is no clear support to explain why any practice tasks are designed in the way that they are. The purpose of each lesson phase is given attention and rationale as has already been previously discussed, but the content and any discussion or guidance to develop a teacher’s understanding of why a task is good, or what makes a good task is absent. This isn’t unusual, but it does seem to indicate that any teacher development or training designed by MNP is not focused on this area. This may perhaps be because designing tasks is not part of a teacher’s role in the implementation of MNP, however it could be argued that understanding the intent of a task is critical to being able to execute its purpose most efficiently – which would possibly reduce the risk associated with not including explicit guidance within the textbook around how to implement the ‘In Focus’ and ‘Let’s Learn’ lesson phases. A simple example (not taken from MNP) is highlighted in figure 5.46:

**Task 1: Calculating Areas**

Calculate the area of each shaded triangle.

a) ![Diagram a](image)

b) ![Diagram b](image)

c) ![Diagram c](image)
A task designer may have designed the above sequence of questions to give focus and attention to *structure* as well as to finding the answer (for example, a teacher may prompt ‘what happens if the same triangle is rotated?’ Or ‘what happens when all three sides of the triangle are provided? How do we decide what to use?’), however, without explicit guidance towards the *intent* of the task, a teacher who did not write the materials may simply run through it focusing only on the answers. There is guidance that adds clarity of task intent in the In Focus tasks, but it is implied rather than explicit, and therefore potentially easily missed. For example, in a lesson on finding the common denominator in fractions, the In Focus guidance reads:

Ask pupils if they can draw a picture to represent the bar of chocolate and colour in what the two friends said they ate. Would this be helpful? Would it allow us to see how much was eaten exactly? What makes adding thirds and ninths together challenging? Allow pupils to discuss and solve…

(Lesson Plan 5A_6_8, Appendix c)

My own interpretation of this task is that it appears to be designed explicitly to draw out the need to do something different, ie to change the denominators, so that the problem is more easily solved, and the diagrams encourage this ‘noticing’ by pictorial representations that utilise *size* as a visual scaffold. However, this may not be entirely clear to other teachers, indeed it may not be the exact intent (although it fits with the objective). The interpretation of task intent could feasibly be the difference between a teacher focusing on structures (a key principle of MNP), and another teacher focusing on procedures and answers. Thus, the task could be interpreted as focusing on relational understanding by one teacher and focusing on instrumental understanding by another.

*Agency*
It appears that a degree of agency is actively encouraged by MNP in some of the materials it produces to help tailor lessons towards the specific needs of an individual class. This is perhaps most explicit within the lesson plans provided, which often have specific headings called *Additional Activity* and *Variation*. Both of which offer alternative approaches as a way of tailoring to a teacher’s own pupils. As discussed previously, these are generally in addition to the main tasks rather than in replacement of them. For example, in Y6 Book 2 Ch9L5, the lesson plan offers the following additional activity:

During activity time, pupils work in pairs and take turns to make up a rule for the number crunching machine. Each pupil shows their partner how the machine works by giving them three examples. Then they try to guess the rule and use an algebraic expression to describe the rule. (Lesson Plan 6B_9_5, Appendix d)

This is a task not included in the textbook or workbook, and is provided as further challenge if required. Similarly, in the same lesson, under ‘variation’, it states:

Using ‘number crunching machines’ to determine rules by looking for patterns between the input and the output. Each machine shows three pairs of numbers. In each machine, the same three numbers are input. Example 1 (of 4): The rule is addition. It is possible to determine the rule from the first set, then use the others to check… (Lesson Plan 6B_9_5, Appendix d)

In this case, the task is not replaced, but altered to increase the difficulty whilst preserving its purpose. These opportunities for variance are designed for the teacher to determine whether they use them or not. Similarly, many of the blog posts hosted on the MNP website offer guidance on how a teacher might utilise technology to support the MNP program, or develop their own ideas for additional journaling tasks, or develop cross curricular links and opportunities from teaching maths to other subjects. Each of these is arguably an example of how MNP envisage teachers to use their own expertise, individual circumstances and, more broadly, their agency in general, to customise some elements of MNP to better suit
them. This approach aligns somewhat with what Taylor (2013) described as a shift towards a more enabling ‘curriculum proof teacher’ design rather than a restrictive ‘teacher-proof curriculum’ design, and as such the intended teacher is inferred to be one that has strong professional judgement (and subject knowledge) to assess and adjust elements of the MNP program to suit their own circumstances and environment. However, it is less clear whether MNP perceive part of their role as training and upskilling the teacher to become a ‘curriculum proof teacher’. For example, in their maths mastery guide, MNP state that many teachers are:

- ill-prepared to teach maths effectively
- Don’t have the necessary skills or resources
- Are generalists
- Lack confidence in their ability to teach maths
- Have a fixed mindset rather than a growth mindset
- Don’t accept that problem solving is at the heart of mathematics
- Transfer maths anxiety onto their pupils
- Cobble together free lessons from the internet

(Maths No Problem, 2019d)

This arguably frames MNP as a kind of problem solver viewing poor quality teaching as ‘the problem’ and their products and training as a solution. Hence whilst there are indications that MNP intends to produce a ‘curriculum proof teacher’ of sorts, it perhaps perceives the teacher starting their program as one in need of retraining and upskilling.
Chapter 6: The Enacted Curriculum

In this chapter the case studies of six different classes across four schools covering a total of eighteen lessons will be discussed and analysed. Initial participant schools were accessed via the Maths No Problem! Twitter account, which retweeted (a form of amplification of a social media post) school accounts sharing their work with MNP. Three schools identified in this way were approached via Twitter (Pilot School α, School A and School B), which in turn led to connections to School D (a local network connection to School A), and School C (a networked recommendation from School B). A summary of the pseudonymised participating teachers for reference in this chapter is in table 6.1 below.

<table>
<thead>
<tr>
<th>School</th>
<th>Region</th>
<th>Class</th>
<th>Teacher Code</th>
<th>Lesson Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>Cambridge</td>
<td>Year 1</td>
<td>α1</td>
<td>1B_20_3</td>
</tr>
<tr>
<td>A</td>
<td>Birmingham</td>
<td>Year 5</td>
<td>A5</td>
<td>5A_6_14b</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5A_6_15</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5A_6_15b</td>
</tr>
<tr>
<td>B</td>
<td>Selby</td>
<td>Year 1</td>
<td>B1</td>
<td>1A_9_2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1A_9_3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1A_9_4</td>
</tr>
<tr>
<td>B</td>
<td>Selby</td>
<td>Year 5</td>
<td>B5</td>
<td>5A_6_7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5A_6_8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5A_6_9</td>
</tr>
<tr>
<td>C</td>
<td>London</td>
<td>Year 6</td>
<td>C6α</td>
<td>6Bα_9_3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6Bα_9_4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6Bα_9_5</td>
</tr>
<tr>
<td>C</td>
<td>London</td>
<td>Year 6</td>
<td>C6β</td>
<td>6Bβ_9_3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6Bβ_9_4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6Bβ_9_5</td>
</tr>
<tr>
<td>D</td>
<td>Birmingham</td>
<td>Year 5</td>
<td>D5α</td>
<td>5B_8_1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5B_8_2</td>
</tr>
<tr>
<td>D</td>
<td>Birmingham</td>
<td>Year 5</td>
<td>D5β</td>
<td>5B_8_3</td>
</tr>
</tbody>
</table>

Table 6.1 School, Teacher and Lesson information
For each case study, each teacher was observed for three sequential maths lessons spanning three sequential days, except for School D, where a shared class was observed resulting in the third lesson (5B_8_3) being taught by a second teacher, Teacher D5β. It should also be noted that the teachers in London, Teacher C6α and Teacher C6β both taught the same lessons in the MNP program but to different Year 6 classes. Teacher codes indicate both the school and the year group taught (for example “B5” means “Teacher in School B teaching Year 5”) and lesson codes indicate the textbook, unit and lesson numbers (for example, “5A_6_7” means “Textbook 5A, Unit 6, Lesson 7”). The use of ‘b’ in 5A_6_14b, and 5A6_15b was used by the teacher of those lessons to indicate that the lesson had been adapted from MNP, but was not an MNP lesson - which will be discussed in more detail in Case Study A. Similarly, α and β are used to differentiate two classes using the same lessons in school C. Year groups and teachers were provided by the schools themselves and were not specifically requested.

A maximum of two teachers at the same school will be treated as a single case study, and focus will be given to a number of points of interest which are:

- The research questions
- The 8 design principles identified in the intended curriculum
- The 5 core competencies identified in the intended curriculum
- Positional design
- Response design
- Task design

However, case studies will be discussed in differing detail to cater for behavioural repetition and data saturation as ideas and themes are repeated in differing schools.

**Ethical approval and permissions**

Although schools were initially approached informally through social media or email, permissions, approval, assurances of non identifiability and an outline of the research intent
using university templates were sent to, and signed by, both the teachers who would be involved in the study, and headteachers of each school. Schools accessed via network connections from other schools (ie, Schools C and D) were informed of the connection in initial conversations - in other words, School D was aware that School A was involved in the study, and School C was aware that School A was involved in the study, but otherwise, schools were not provided with any specific identifiable data regarding other participants. No pupil data was collected other than occasional note taking regarding what some anonymised pupils said or wrote in their books during tasks.

The pilot study was discussed in the methodology chapter and will not be referred to here.

**Case Study 1: Teacher TA5 in School A**

**Information about this school**

The following information about School A is taken from their latest Ofsted report at the point of research collection. It is considered important only to help frame the situational setting of the school, and to highlight that the schools visited for this study were not demographically identical.

School A was a primary school based in Birmingham rated as ‘Outstanding’ by Ofsted and noted as a much larger than average-sized primary school with both the number of pupil premium and English as an additional language pupils considered higher than average. The school had an average number of pupils with special educational needs, and almost all pupils were from minority ethnic groups.

**Why this was considered a critical case**

School A have used the MNP program for four years prior to the DfE declaring it as an ‘approved’ textbook series. The school is accredited by MNP and Teacher A5 has taught using the program for three years. At the point of the research visit, all year groups in the school were using the program, and subject leads also ran regional mastery training that showcased MNP strategies and tools. Geographically, the sample of schools used for this
research is not confined to one localised area or authority, thus eliminating any inference of
localised influence within the study regarding, for example, whether schools have all drawn
from the same experiences or are sharing teachers across a multi-academy trust. Hence the
study can focus on teacher agency independent of geographical factors. These combined
factors helped ensure that Teacher A5 was experienced and familiar with the MNP program,
which was embedded in the school and with pupils.

As an accredited school, teachers at School A are deemed to be enthusiastic adopters,
and it should be noted that MNP formally recognises the school as one that has successfully
embedded the program.

Lessons observed

In School A, I was given access to three sequential lessons in Year 5 that took place on
three sequential working days, taken from Chapter 6 of textbook 5A, lessons 5A_6_14b,
5A_6_15 and 5A_6_15b respectively. Of these three lessons, only lesson 5A_6_15 was
directly from the MNP textbook. Both 5A_6_14b and 5A_6_15b were described by teacher
TA5 as adaptations of the existing lessons which were in effect additional lessons planned
based upon previous experience of teaching this unit, whereupon all teachers of the year
group had decided collectively that, informed by previous teaching experience, additional
lessons were required in this area to further support learning in this particular topic. Thus
before any lessons had been formally observed, it became clear that the staff at School A
had collectively added their own created lessons to the MNP program to, in their words,
better suit the needs of their own learners in their own school. This suggests that there
exists within the department an adapted sequence of lessons that differs from the MNP
provided scheme, although this was not seen. Indeed, it transpired through informal
interviews with Teacher A5 that several lessons (each labelled internally with ‘b’) were
integrated throughout the MNP program across year groups to accommodate what staff
considered to be areas that required prolonged focus to deepen learners’ understanding of
some topics. A sample of observation notes is provided as appendix a.

Lesson 5A_6_14b

In Focus
Despite this lesson not being a standard lesson listed in the MNP textbook, the lesson had been designed to emulate the same structures and format, starting with an In Focus task that was very similar to the one used in Lesson 14 in the textbook in line with what Brown (2009) refers to as ‘mutual adaptation’ of existing content – attempting to preserve intent when making changes. The In Focus problem in lesson 14 of the textbook had two MNP characters using differing amounts of flour from two identical bags each containing 2 1/2 lbs of flour. The question asks how much flour is left in each bag. In lesson 5A_6_14b, another character (taken from the MNP character set) had taken 3/4 kg of soil from a bag of 1 1/2 kg of soil, and pupils were again asked how much is left. This In Focus task emulates not only the lesson phase from MNP, but also the narrative element and recurring characters, thus adhering to design principle 8. associated with In Focus highlighted in the intended curriculum chapter. The teacher asked their pupils “What’s the calculation? How many ways can you represent it? How much soil is left?” Before presenting the same three questions on the board for reference. The teacher did not ask for answers, but instead instructed pupils to discuss these questions on their tables. It appeared that the positional design of these questions was to prompt pupils to explore their own thoughts and ideas, as MNP intended, and provide time and space to think, rather than force through a short, pressured deadline for a response from an individual. Despite ‘what’s the calculation?’ being a relatively closed question, the positional design of it was open and exploratory.

During this phase, teacher A5 walked the room and interacted with pupils using prompt questions and instructions such as ‘what are you doing?’, ‘why have you chosen multiplication?’ and ‘show me what you mean’. Each of these questions appeared designed to help pupils communicate their thinking rather than simply providing the answer to the problem, similar to the question prompts provided in the lesson plan for 5A_6_14. This again aligns with the intended purpose of MNP’s In Focus phase in that it is allowing space for freedom of thought (design principle 1) to share ideas and develop relational understanding of the problem with mathematical concepts (design principle 4).

In order to demonstrate to the teacher in response to being asked ‘show me what you mean’, a pupil instinctively drew a bar model to show to the teacher (without specific prompting). Similarly, other pupils had decided to use physical manipulatives provided on
desks in the form of equally sized paper strips which they folded into equal segments, again unprompted. These physical resources were part of a general pool of physical manipulatives found in the centre of each pupil table, which also contained dienes blocks and multilink cubes. This encouragement and emphasis on manipulatives aligns with the ‘concrete’ element of the CPA approach (principle 5.) Again, the lesson plan for 5A_6_14 included reference to ‘coloured strips of paper/card for cutting and folding’ in a list of ‘useful but not essential’ resources. This phase of the lesson lasted approximately 10 minutes, during which time several representations of the problem had been developed by pupils, including bar models, abstract fractions, circular fraction models, folded paper models and written text. All models that were mathematically correct received praise and occasionally prompts of ‘I want at least 2 different methods for this’ were provided by teacher A5.

As this lesson phase drew to a close, it appeared to be very much aligned to the intended purpose of MNP’s In Focus design. Pupils were given a single, narrative driven problem, which in turn they were allowed freedom to discuss in groups and enabled to explore the idea with a number of different representations and models that appeared consistent with the small number of flexible models encouraged by MNP to highlight connections across different topics and concepts. The teacher played the role of a facilitator and enabler rather than a provider of knowledge and answers, and they prompted thinking and flexibility over explanatory methods provided by pupils including concrete and pictorial representations. Overall, position design, response design and task design all appeared to align with the intent of MNP, and supported both the design principles, and enabling development of the core competencies of visualisation (1.) and communication (3.) in particular.

Let’s Learn
For this phase, the teacher brought attention back to themselves to discuss the problem presented on the board as a class, and demonstrated folding paper to highlight the equivalency of two quarters and a half, asking what they were representing with each fold in the paper (each bar in the physical bar model), then presented a slide with a larger model of the same divided bar, asking what pupils notice. When one pupil responded with “they are the same size”, the pupil was corrected to use the specific vocabulary “the bars are equal”,
and then used further technical vocabulary referring back to the problem and asking “can we call the original bag of soil our minuend?”, to which a pupil responded “what he uses is the subtrahend” which prompted the teacher to say “when subtracting fractions they should have the same…” and pupils responded with “noun”, then the sentence was repeated in full as a class.

In this short exchange, several interesting things occurred - firstly, there was a clear strategic focus both in positional and response design regarding the precision of language, and use of technical vocabulary which aligns with the MNP core competency of ‘communication’ (3.) as well as the lesson guidance notes for 5A_6_14 which specifically states that pupils can use the terms ‘numerator’ and ‘denominator’. However, the reference to fractions requiring the same noun when they are prepared to perform a subtraction operation is not mentioned in the plan. That language is specifically used by Ban Har in his video on the MNP YouTube channel when describing fraction operations in a keynote (Ban Har, 2020). This suggests that not only are the teachers in School A engaged with the MNP curriculum, they may also have gone beyond the provided guidance and sought or discovered further insights, possibly from MNP conferences, hinting at a broader community built or at least facilitated by MNP. The exchange also provides further insight into the positional design of questioning by Teacher A5.

Each question prompts more thinking, more explanation, more justification and more focus on precision of language seemingly to develop communication and relational understanding (design principle 4.). The live demonstrations were not only influenced by pupil thinking, but they anticipated it (with slides accurately predicting the models pupils may come up with) helping the lesson flow in the direction the teacher wanted to take it in, whilst being student led. Finally, the reciting and chanting of specific maths facts to aid committing them to memory was not a strategy specifically mentioned by MNP, however the same phrase was used several times after this occurrence in the lesson and appeared to be a purposefully planned component of teaching, demonstrating further willingness and agency to deviate from provided strategies.
A new slide was presented to the class, showing \( 1 \frac{1}{2} - \frac{6}{8} = ? \) and an empty bar model with no segments on it. Teacher A asked the class ‘how is this question different?’ and ‘if the denominator is bigger is it a bigger fraction?’. Both this question slide, and the one afterwards were confirmed by the teacher in a short interview to have been designed specifically to draw out misconceptions. After a brief discussion, the teacher modelled with the class how to solve the problem. The final slide before moving onto practice, stated “find the difference between \( 1 \frac{2}{3} \) and \( 1 \frac{8}{9} \).” Notably, the problems presented thus far had each been slightly different in their presentation. The first had narrative, the second was presented in abstract, and the third in words (“find the difference”). Again, there was specific purpose in the task design, in that the choice to use \( \frac{8}{9} \) as the second fraction was to try and “nudge pupils away from the bar model” (as clarified in interview) as 9 segments were trickier to draw. A cycle of discussion, prompts, models and explanations led by pupils continued as pupils worked through each slide, with a noticeable acknowledgement and praise for contribution from the teacher for all methods offered by pupils, regardless of their perceived efficacy (some methods were laborious or included some unnecessary steps to convert fractions more than once), leading to the final phase of the lesson, which started after 50 minutes of teaching. During this lesson, there was no clear ‘Guided Practice’ phase, nor was there an ‘independent practice’ phase. Instead, the teacher spent a large amount of time in ‘Let’s Learn’, and led or facilitated learning for the duration of the lesson (50 minutes) prior to the final phase, which was to write a maths journal.

By spending so much of the lesson in this phase, it appears the teacher was purposefully raising the status of design principles 1., allowing time for the development of new ideas, and 3., cooperative learning with a knowledgeable other. However, this caused a de-prioritisation of independent practice and the development of instrumental understanding (principle 4.). As such, there is seemingly a hierarchical teacher preference regarding each design principle and core competency that has developed with the teacher themselves – which became a more substantiated observation by the time all 3 lesson observations were concluded and indeed across different teachers in other schools as well.

**Maths Journal**

210
Whilst two stages of the MNP lesson structure were not included in this lesson, the Journal phase, which forms just one singular activity across a unit in the MNP texts rather than a lesson, was introduced as the final phase. Here, pupils were asked to write the lesson objective in their own words alongside the date, with a teacher prompt “what have we been learning?”. A second prompt was presented in the form of a pair of children and a bag of rice weighing 2 and \( \frac{3}{5} \) kg. No problem was presented, and instead pupils were encouraged to make a story (one pupil immediately asked “shall we create a number story?” suggesting this was a routine element of journal writing in this class. Once the story was created (which was individual work, not a group discussion), pupils were encouraged to represent the problem, and solve it. There was no explicit instruction regarding what the problem should be, although the inference was that it should follow the theme of the lesson, and no specific method or model was promoted by the teacher. Despite this, most pupils decided to use bar models for their way of representing the problem. The teacher walked the room and enquired with some pupils as to what problem they had decided upon and how they were modelling it as a number story. Journal entries were not discussed as a class, and the lesson ended around five minutes into the journal task time.

This lesson highlighted a key benefit to the methodology for this research, in that the lesson was not an MNP textbook lesson, but rather one written by the school in the style of an MNP lesson. This would have made interpretation of the enacted curriculum more difficult as a stand-alone lesson, and could potentially lead to an assumption that this was a typical lesson rather than a relatively unique addition to the existing scheme based upon perceived additional needs of pupils from previous teaching experiences. However, as this was one of a sequence of three lessons to be observed, a more accurate and rounded picture of the lesson structures and rationale was easier to identify.

Upon interviewing Teacher A5, it was stated that the lesson was purposefully focused on a more teacher-led approach to allow for more assessment for learning through live questioning of the whole class and emphasise some of the more difficult challenges around conceptual (relational) understanding of fraction subtraction with mixed numbers, which experience had taught the department was a particularly difficult concept for many pupils to grasp - but that the MNP program did not accommodate this perceived additional difficulty
and additional required emphasis. Whilst this could be perceived as a criticism of the programme, it could equally be viewed as a success of MNP in that the programme appeared to instil belief in the design principles and desire to achieve the 5 core competencies in the teacher to the extent that additional lessons were being designed collaboratively within the department to purposefully emulate Maths, No Problem! – hence whilst agency had affected the sequencing of lessons and phases within customised lessons, teacher intent appeared to be to preserve a shared responsibility with MNP around structure and design.

It was interesting to note that Teacher A5, or at least, the Year 5 teachers collectively, had decided to incorporate the Journal activity, and prolong the Let’s Learn phase of the lesson as priorities for what was arguably designed to be an *intervention* lesson to embed within the MNP program. This decision appeared to value teacher expertise and guided teaching over practice, which makes sense in that if pupils are struggling with the conceptual (relational) understanding, more teaching is needed from a knowledgeable other. By contrast, instrumental understanding is arguably more refined through practice, and it appears that the teachers viewed this as either secondary to development of relational understanding, or sequentially important to be *after* relational understanding is secured. Similarly, the journal activity is perhaps the most visual and explicit way for pupils to formally evidence their relational understanding of concepts to the teacher as it allows for an open approach free from rigid procedures and expectations around how work is presented. In fact, the favouring of journal writing and promoting its usefulness beyond the MNP recommended use of ‘once per unit’, became a theme across several case studies.

**Lesson 5A_6_15**

The second lesson from Teacher A was one taken directly from MNP, and followed the structure of In Focus, Let’s Learn and Guided Practice, but again the Independent Practice was replaced, and again it was a Journal task that was introduced despite it not being present in the MNP lesson.
Furthermore, the specific tasks in the MNP lesson had been adjusted to change the narrative and imagery whilst preserving the questions. The In Focus task is shown below in Figure 6.1

![Figure 6.1 The In Focus task for lesson 5A_6_15](image)

However, Teacher A5 presented a problem that instead showed a circular pizza, cut into four equal slices, with the characters using the same dialogue (except “used 3 times as much” was replaced with “ate 3 times as much”). This change meant that pupils were not exposed to the ‘four quadrants of a square’ representations highlighted in the textbook and instead focused predominantly on circular representations. After the lesson, Teacher A5 explained this change was about engagement, and attempting to ensure narratives were more in line with pupil experiences - the paper cutting was felt by the teacher to be too abstract. Interestingly, as MNP do not discuss task design in significant depth in their materials, it is difficult to know how important the square representations are for task intent, and Teacher A5 may not have considered this as an important detail in the lesson design, in part because it wasn’t communicated by MNP.

Further deviation from MNP took place in the Let’s Learn phase of the lesson, whereby the examples provided in the textbook were replaced by several models displaying questions...
or incorrect pupil responses that each appeared designed to draw out misconceptions in this topic. For example, a fully modelled abstract calculation $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{12}$ was presented and Teacher A5 asked “what have I done wrong? Talk to your partners”. This task is not in the MNP lesson, nor was the question on the next slide that read ‘is it possible to get an improper fraction when multiplying by fifths?’ Querying these decisions in an interview after the lesson, Teacher A5 explained that it was a purposeful decision to redesign elements of the lesson to address misconceptions that, in their words, “creep in early and are difficult to undo if they are not dealt with quickly”. Much of the lesson consisted of similar patterns of questioning to the previous lesson, with questions such as “how did you represent this?”, “what is the calculation here?”, “show me this in a different way”, “have you spotted a pattern?” And “what have you noticed? What is different and what has stayed the same?”.

Again, the positional design was not to expect closed short responses, but instead to promote deeper thinking about structure and relational understanding, with a focus on students pausing and constructing their own understanding of the mathematics with very little dictation of knowledge and facts from the teacher. This constructivist approach to knowledge acquisition aligns with the previously identified design principle 1. and speaks to the stated influence of Bruner on MNP. Divergencies were designed to faithfully uphold the principles and some structures of MNP. It appears then, that Teacher A5 delivered the intent of the MNP program, but was comfortable with deviation to suit learner needs, and the ‘buy in’ with the program is great enough that the core principles of MNP appear instilled in Teacher A5’s approach without the structural support of the textbook and workbooks when planning new lessons or content, or confidently adapting tasks or lesson phases.

The journal phase of the lesson again required pupils to decide what calculation is being represented and to form a number story, this time from a trio of bar models each representing $\frac{2}{5}$. The specific tasks relating to the journaling phases for both lessons were not taken from MNP, only the journaling concept itself. Notably, for two sequential lessons, independent practice was omitted suggesting that the teacher’s delivery of the MNP program seemed to downgrade the importance of the development of instrumental understanding in favour of more time and input around developing relational understanding
in the first two lesson phases, and metacognition (an MNP core competency) in the
journaling task, and thus improvising away from the intent of MNP regarding principle 4.,
 focusing instead on the more open, exploratory tasks with less abstraction that have been
shown in this research to dominate the In Focus and Let’s Learn lesson phases.

Lesson 5A_6_15b

As with lesson 14b, this lesson was designed by the teacher to emulate the MNP
textbook lessons. It was intended to help bridge a perceived gap in pupil knowledge and
pre-empt a historically challenging topic with pupils at School A by extending the amount of
time spent developing relational understanding. Again, the lesson began with an In Focus
narrative driven task, and the manipulatives were, for the third consecutive lesson,
organised at the centre of each pupil table for access should pupils wish to use them. Pupils
used a mix of representations when prompted to explore and explain to each other the
mathematics behind the narrative problem (“I ate three equal pieces of chocolate, Ravi ate
4x as much”). Observing three consecutive lessons enabled a sense of routine to be
captured, and it appeared that the In Focus task as an opening phase of the lesson was both
valued and fully embedded into practice, even when lessons deviated from the MNP
textbooks.

Teacher A5’s questions in this phase included “I’m looking for 2 methods at least”, “I’m
not bothered about the answer, I’m interested in how you got there” “what is each counter
worth in this representation?” and “which method is more efficient? Repeated addition or
multiplication?”. Thematically these questions fit with those in the previous two lessons,
 apart from the question around efficacy, which appears to be a direct developmental
increment from the first lesson observed, where efficacy was explicitly ignored. Similarly, it
appears to be part of a move in this lesson towards strategic development of instrumental
knowledge. In the Let’s Learn phase of the lesson, more abstraction was introduced, and
whilst bar models were present on the board, the majority of board work was in symbolic
representations of fractions (eg “5 x 1/9 = 5/9). Further into this lesson, efficacy was again a
point of discussion, with Teacher A5 asking “is multiplication the most efficient way to do
this? What other ways are there?”. Use of technical vocabulary was again evidently a priority, as observed in the following exchange:

A5: “How much is the ‘whole’ worth?”
Pupil: “8”
A5: “8 what?”
Pupil: “8 pieces”
A5: “8 equal pieces.”
Pupil: “The whole is worth 8 equal pieces”

The positional design of questions in this lesson appeared to focus heavily on the use of technical vocabulary, and on developing metacognition around reflecting upon the efficacy of various methods with a view to utilising the best procedure to perform calculations. Similarly, the task design appeared to strategically shift towards more exposure to abstract representations. Indeed, the third and final lesson observed closed with pupils working from the 5A Workbook (for the first time in 3 consecutive lessons) for around 15 minutes.

Case Study 1: A brief reflection against the Research Questions

Teacher A5 taught 3 consecutive lessons in the style of the MNP program albeit with some significant changes to structure in both the lesson that was cited as being directly taken from textbook 5A, and the two teacher created lessons that were designed to pre-empt pupils struggling with developing relational understanding regarding fraction operations.

RQ2: How are textbooks used as teaching aids in DfE approved curriculum programs for primary mathematics in England?

From this case study, the textbooks informed a majority of decisions made in the delivery of the lessons observed. Structures from the textbook were used as the basis for all lesson phases, and whilst the sequencing of those structures was not always observed, the integrity of each phase used was interpreted to be upheld or at least intended to be
maintained in any adaptations implemented. However, with the alteration of some tasks and introduction of new tasks within some phases, it is unclear as to whether task integrity was maintained, in part due to the lack of information and support from MNP in this area. The principles of the MNP program informed the ways in which Teacher A5 sequenced and delivered content, responded to questions, organised the classroom, and developed pupil competence and proficiency in mathematical ideas and concepts.

RQ3: 3. How do teachers facilitate knowledge acquisition using DfE approved teaching guides for primary mathematics in England?

From this case study, knowledge acquisition was primarily through the teacher as a facilitator of discussions, play and exploration in a culturally productive, safe and familiar environment to pupils where relational understanding appears to be emphasised and developed first, with the teacher willing to both add extra lessons and extend learning phases that focus on this type of understanding before developing instrumental understanding. Independent practice appeared to be less of a priority and took up only 15 minutes out of nearly three hours of lessons. Teacher A5 predicted and pre-empted misconceptions as well as likely models representing mathematical concepts that were developed initially by pupils, but fine-tuned by the teacher in collaboration with pupils. Pupils generally constructed their own knowledge and understanding under the guidance of the teacher in the In Focus and Let’s Learn phases of lessons, and these were usually extended to become the dominant feature of lessons, with practice being reduced or removed completely from lessons in the sequence observed.

**Position design**

As previously mentioned, the position design of the majority of teacher questions appeared to elicit deep thinking and elaboration of ideas and concepts to help pupils refine their relational understanding of mathematics, as well as work through misconceptions and promote a high standard of understanding and implementation of technical vocabulary in full sentences. Emphasis was given to thinking time, and lesson phases allowed for pupil discussions with no time pressure or rushing from the teacher. Pupils were rarely put on the spot to answer a question that did not involve sharing thinking (questions such as “what is
the answer to…” were much less common than questions akin to “how might…” or “why do you think that…”?) Both purposes align closely with the intended curriculum of MNP, which refers specifically to the development of relational understanding in principle 4. and vocabulary in their core competencies. Notably the time spent on singular problems, and the time given to pupils to both collaboratively explore ideas, and to think before responding to questions all appear to align with design principle 1., and the focus on only one small area of maths across all three lessons suggests that design principle 6. regarding small steps approach to content coverage was also adhered to.

Response Design

Elements of the lessons observed hinted at Teacher A5’s intent with regards to response design as well, with several corrections made to pupils who did not answer fully or with accurate vocabulary, as well as demonstrating slides that included diagrams that predicted mathematical models from the independent thinking that pupils were given the freedom to perform. Furthermore, misconceptions were explicitly planned for and prioritised, and likely misconceptions formed a strategic part of task design that was created independently of the MNP lesson guidance. Whilst these elements deviated from the MNP plans, they did shed light on the response design of Teacher A5’s lesson intent. Positive classroom culture appeared to be at the forefront of response design in Teacher A5’s lessons in that careful language was used to minimise risk of pupils feeling excluded or developing anxiety over getting answers right (Boaler & Dweck, 2016) and instead encouraged participation and the freedom to think about and generate ideas without fear of being wrong or pressure to think quickly. Similarly, live data formed from pupil participation and response in the form of mathematical modelling was used to inform discussion, leading to phases often requiring visibly improvised reactions to pupils, although some of this was aided by pre-empting responses during planning, evidenced by prepared slides that were close or identical to pupil ideas generated in the In Focus phase of some lessons.

Task design

Tasks were usually taken from MNP’s program or heavily influenced by it, mimicking the phases and style of questions such as narrative driven open questions and lengthy thinking time utilising multiple methods to convince others, including the teacher, that the maths to
solve a problem was the correct method or an accurate representation. Tasks in the first phases of the lesson (In Focus, Let’s Learn) were generally intended to develop relational understanding whereas journaling tasks were seen as assessment opportunities to test the depth of pupil understanding and their ability to communicate ideas and concepts. In the lessons observed, instrumental understanding was less of a focus, with both guided and independent practice tasks omitted completely from two of the three lessons observed. Tasks were given generous time to complete, in line with MNPs principles around allowing time for ideas to develop, but this came at the expense of some tasks towards the end of the lessons appearing condensed comparatively, such as maths journaling for around ten minutes, or independent practice for a similar amount of time, compared to around fifty minutes for the In Focus and Let’s Learn phases collectively.

Case Study 2: Teachers B1 and B5 in School B

Information about this school

School B was a primary school based in Selby rated as ‘Good’ by Ofsted and noted as an average sized school. The number of pupils with special educational needs was above average, and the number of pupils who joined or left the school part-way through their education was high. A large majority of pupils were of White British heritage.

For this case study, two teachers will be discussed, Teacher B1 teaching a Year 1 class, and Teacher B5 teaching a Year 5 class, both utilising the MNP program. The case study will occasionally refer to some elements of case study 1 where recurrence of themes or ideas is evident.

Why this school was considered a critical case

School B have also used the MNP program for four years, are accredited by MNP and Teacher B5 has attended several training sessions run by MNP and is considered the MNP lead in the school. All year groups use the MNP program, and the school is geographically and demographically different to school A. Teachers for both classes were experienced in the
use of MNP and had taught the same year groups in the previous academic year using the program (and so this was their second run through of the Year 5 and Year 1 programs respectively).

Lessons observed

Six lessons were observed in School B: 3 sequential Year 5 lessons taught by Teacher B5 (5A_6_7, 5A_6_8, 5A_6_9), and 3 sequential Year 1 lessons taught by Teacher B1 (1A_9_2, 1A_9_3, and 1A_9_4). Contrasting with School A, all lessons observed were lessons authored by MNP.

Year 5 Lessons

By coincidence, the lessons in this sequence came from the same year group and chapter as those observed in School A, albeit they were earlier lessons in the unit. Each lesson began with a starter unrelated to MNP - a Timestables Rockstars task (a popular gamification of times tables practice developed by Bruno Reddy, one of the three panel members that selected MNP as a DfE approved textbook series). In interviews, Teacher B5 admitted to feeling conflicted about the use of this program. The children enjoyed it and it did help enthuse them about developing procedural fluency (instrumental understanding) with times tables, but the program encourages and pressures pupils to recite answers at speed, which could be considered to oppose to the principles of MNP around pace and time allowed for thinking. It is interesting that Teacher B5 felt a degree of conflict about a program that develops instrumental understanding, which feeds into a wider narrative across the case studies of a sense of prioritisation of tasks that focus on relational understanding, and a lesser focus on development of instrumental understanding.

In Focus

All three of Teacher B5’s lessons, following on from the TTRockstars starting activity, faithfully reproduced the In Focus task from MNP’s textbook. The task presented in class was identical to the one in the textbook, both in terms of what was being asked, and the visuals being used. In each lesson, pupils were given around a minute or so to produce their first
thoughts on what to do with the problem, before the lesson developed into a prolonged
echange similar to those in School A, with time taken to allow for pupils to engage fully with
the problem and produce models. The teacher used similar questioning techniques to those
used in case study 1, designed to develop relational understanding of the mathematical
concepts. The teacher’s response to many pupils’ answers was to ask “do you agree?” to the
rest of the class, followed by requests to explain their agreement or disagreement fully. At
one point in lesson one Teacher B5 requested that pupils come up with “at least 3 ways” to
solve a particular problem, with each ‘way’ determined more by the model used rather than
the specific method. For example, a bar model, a concrete model, and abstract notation
would be acceptable, or three different pictorial models. As with lessons in the first case
study, the class setup included a selection of concrete manipulatives and paper strips at the
centre of each desk that were optional for pupils to use to help aid their thinking, and many
pupils chose to use these without prompting when in the In Focus phase of the lesson. This
appears to mirror what Lindorff et al. (2019) referred to in their research as ‘caddies of
manipulatives’ in their research into Singapore Maths style lessons in England. This phase
appeared to blur into the Let’s Learn phase of the lesson with no clear transition point,
which was noticed only by the models in the textbook appearing on the board as part of the
discussions around the In Focus problem. The models naturally arose from pupil ideas on
tables, and teacher prompts asking whether there were different ways to attempt the
problem.

One notable element of Teacher B5’s style of teaching that appeared not to be
influenced directly by MNP, was that Teacher B5 positioned themselves as someone who
didn’t know some of the mathematics or answers involved in the tasks presented. Questions
were often preceded by “I’m not sure how to do this”, or “I’m confused with this, can
anyone explain it to me?”, or “I haven’t been able to work this out yet, who can help me?”. This
positional design was discussed in interview, where Teacher B5 explained that they
found pupils more willing to engage and participate in lessons when they weren’t just
playing the role of the learner, but also the teacher, and that this shift in dynamic from
expert teaching novices to a more level playing field where all members of the classroom
were learning together was more productive. This approach extended to Teacher B5
sometimes framing problems from the perspective of the characters used in MNP, and
positioning the pupils in the class as experts who can help the character. For example, one statement from the teacher in their second lesson stated “Ravi got confused because he didn’t think he had 3/9, how much did he think? Who can help him?”. In the final lesson, a journal task was framed as writing a letter to an adult who was struggling with the concept the pupils had been learning in the lesson (the framing was not part of MNP’s lesson plan).

The sequence of lessons incorporated all 4 phases of MNP lessons each time (In Focus, Let’s Learn, Guided Practice and Independent Practice), and each phase closely matched the resources, questions and tasks provided both in the textbooks and workbooks, but also the lesson guidance within the lesson plans. However, no journal exercises were included in the delivered MNP lessons, yet the teacher incorporated journal tasks in two of the three lessons. In lesson 5A_6_8, journaling was used to capture the shared thinking and models discussed in the In Focus phase, and in lesson 5A_6_9, specific time was allocated to the whole class completing a journal activity at the end of the lesson. It appears that like School A, the journal task, which is a minimal element of the MNP program (it generally appears once per unit), has been enhanced by the teacher to become a more prominent feature of MNP lessons by taking the concept and integrating it into lessons more frequently.

When asked about the use of workbooks, Teacher B5 explained that MNP’s program encourages that workbooks are completed for homework in cases where the lesson time runs out, as it is an independent task and is designed not to require input from the teacher. On the latter point, Teacher B5 did request silence for the independent tasks and pupils were not allowed to converse with each other, hence the integrity of the task intent appeared to be valued (working independently), however Teacher B5 explained that pupils in their school need more support and the logistics of taking books home and bringing them back in (ie, they wouldn’t be regularly returned on time) meant that they did not do this. Instead (although not obviously as a result of this directly), maths classes were delivered in the mornings, and an afternoon slot was utilised three times a week to allow for pupils to complete their independent practice or receive additional support from the teacher should they have identified that anyone had not met the minimum expected progress requirements as cited in MNP’s lesson plans.
This explained how a school managed a conundrum identified earlier in this research regarding the MNP program, and mastery principles in general: where does intervention fit when time simply runs out in a lesson, in a program that insists that all pupils learn together and move on together (principle 7). In the case of School B, pastoral time at the end of the school day is regularly utilised to do targeted intervention with small groups who struggled in the lesson earlier in the day. This relatively immediate response, according to Teacher B5, was usually sufficient to bring pupils to a place where they would be able to access the next lesson to the preferred standard. This is an important detail, as it highlights how teacher agency has enabled the facilitation of a specific MNP principle which lacks guidance around the practicality of being a deliverable and achievable goal and appears to demonstrate that the teachers believe in the design principles to the extent that they are willing and able to alter timetable structures to accommodate them.

Whilst the three lessons observed appeared to faithfully follow the MNP program, albeit with a few additional features (TTRockstars, added journal activities and interventions), a further adaptation of the MNP program was the use of what Teacher B5 described as an ‘Anchor Chart’, which was an A1 piece of Flipchart paper that was gradually populated throughout the lesson with key information, mathematical models and vocabulary. The Anchor Chart was revisited as part of the lesson review, and then hung on a washing line that stretched across the classroom near the ceiling so as not to be in the way of the pupils. This in turn served as a learning journey for the pupils, as they could easily see the previous lesson, or lessons, to remind themselves of underlying concepts or other elements of other lessons. This physical representation of the building of knowledge was used every lesson, and the content of each chart was a mixture of planned words and models from Teacher B5, and improvised work that came from the pupils directly.

Two examples are shown in figures 6.2 and 6.3 below from both the Year 5 and Year 1 classes respectively:
Figure 6.2 An example of a washing line of Anchor Charts in Year 5 on display

Figure 6.3 Anchor charts in a Year 1 classroom

Year 1 Lessons

Teacher B1 identified themselves as being less experienced in MNP due to B5 being the ‘lead teacher’ for the program and therefore attending any external training events. The start of lesson 1A_9_2 involved pupils sat in a circle around the teacher rather than at desks like in all the previous observations, including the single Year 1 lesson observed in the pilot study. Rather than beginning with the In Focus task, Teacher B1 used the start of the lesson to discuss vocabulary with the pupils. Words like ‘tall’, ‘taller’, ‘tallest’, ‘long’, ‘longer’, ‘longest’ were discussed and used in sentences by the teacher and pupils, including several
full class recitals of the teacher using the words in sentences similar to the chanting style recall observed in case study 1. Following this, the teacher produced a tea towel and placed it in the centre of the circle that the pupils were sitting in, asking “how can I measure this tea towel?”. The discussion around this prompt involved suggestions such as “measure it with a ruler”, “a tape measure” and “count how many squares there are” (referring to the pattern of squares on the tea towel). In contrast, the lesson plan for 1A_9_2 from MNP reads:

To begin this lesson, ask pupils to look at a pencil and a crayon and ask them which one is longer. How can we measure? Refer to the picture in the In Focus task and ask pupils if there is anything in the picture that could help us compare their lengths.

(Lesson Plan 1A_20_3, Appendix b)

In the ‘Let’s Learn’ phase of the lesson plan, the activities show several objects and ask which are tall, tallest, short, shortest etc. It appears then, that the teacher made two significant decisions in altering the opening sequence in the lesson. Firstly, focus on vocabulary was introduced earlier, before any concrete objects were introduced or exploratory thinking was used, however, the open narrative driven task from the textbook for ‘In Focus’ was utilised afterwards. A significant adaptation was that the teacher decided first to focus on recapping key vocabulary from the previous lesson (according to the MNP textbook content) and also introduced a different task that was built around the principles of the In Focus phase using a tea towel to discuss how to measure it rather than a comparative task. This adaptation meant that the pupils did not focus on the contextual use of the revisited vocabulary (long, longer, longest etc) but instead on tools to measure with which appeared to be alteration that changed the intent of the task.

Next, the teacher introduced a box of wooden blocks and placed some to line up with the length of the tea towel. The blocks were not uniformly shaped, and so would not be a useful tool to fairly measure the tea towel (which was the purpose of the demonstration, but not shared). A pupil took a set of uniform blocks and placed them on the other side of the tea towel, and the teacher asked, “we have six blocks on one side, and five on the other - is the tea towel longer on one side than the other?”. Pupils were allowed five minutes to
discuss with their partners around the circle. Further prompts from the teacher followed: “what shape is the tea towel? Is it fair to use different sized blocks? Why?”. After some discussion, the term ‘non-standard unit of measure’ was introduced, followed by a demonstration using equal sized blocks to measure the tea towel, but with gaps between each block. The teacher asked “Is this fair? Why?”. The blocks were then pushed together but neither started nor finished at the corners of the tea towel. The teacher again asked about how fair this was and why. It is clear that the teacher demonstrations are purposefully planning their position design to uncover and dispel misconceptions around the ways in which measurement is fair and accurate, and introduced these misconceptions as models for discussion, the teacher aligned with MNP’s principles around the construction of knowledge and relational understanding by the pupil, and enabled by the teacher. In fact, some, but not all of these misconceptions, were highlighted in the MNP lesson plan. Key questions, observations and conclusions were written up by the teaching assistant onto an A2 paper ‘journal’ (the word used to describe it to the pupils), which was utilised in the same way as the Year 5 washing line Anchor Chart.

At this point, the teacher introduced the ‘In Focus’ task formally (midway through the lesson) which proceeded to discuss the merits of measuring both the pencil and pen using paper clips - as instructed in the MNP lesson plan. The lesson then proceeded to closely follow the MNP lesson plan and phases, although the guided practice section was skipped (due to the time used for non MNP lesson components earlier) in favour of preserving independent practice as the closing phase of the lesson.

A number of adjustments from the intended curriculum occurred in this lesson. The teacher made the decision to use time for recap of previous vocabulary, and introduce an entirely new task at the start to focus on misconceptions around measuring (which meant that a phase would need to be removed, or rushed through to finish the lesson on time - and so the guided practice phase was removed), before faithfully working through the MNP lesson plan phases and content, then making the decision to remove guided practice. It appears that Teacher B1 was at ease with adaptation and confident enough to develop new tasks that, like in School A, were designed to emulate the principles and phases of MNP’s provided lesson structure. Interestingly, the teacher chose to preserve the independent
practice and instead remove the guided practice phase unlike previous observations of other teachers.

Both the second and third lessons followed MNP’s program but again deviated in similar ways as the first lesson. Both In Focus tasks were adapted to replace the objects being discussed (intended to be a sofa in the second lesson, and a train in the third) with objects more relatable and obtainable for the teacher. In the second lesson, the teacher used objects available around the room to discuss measuring and comparative language around length and height, and in the third lesson the teacher used a picture of a sheep that was already in the classroom. In interview, the teacher explained that these were decisions made predominantly around practicality - they did not have toy trains or a sofa as physical representations. Again, this appears to evidence a willingness to deviate from MNP’s plans, but with the objective of preserving intent such as upholding principles around spending time to develop ideas and utilising relatable scenarios and familiar objects from the world around them (principle 1) were evident and actively enhanced by the teacher whilst embracing the ‘concrete’ element of the CPA approach (principle 5) by sourcing physical representations of objects and adhering to narratives (principle 8) to deliver content.

In the third lesson the train example was introduced later, pictorially. The second lesson again introduced a journal activity where one was not indicated in the MNP lesson plan, however the third lesson replicated the exact same phases as the plan. All three lessons utilised the Anchor Chart idea that was used in the Year 5 lessons, and questioning followed the same patterns as lesson 1 - focusing on exploring the depth of pupil understanding, multiple methods, sharing ideas and thoughts, and tackling misconceptions. Several questions were identical to those presented in the MNP lesson plans.

**Case Study 2: A brief reflection against the Research Questions**

Both teachers at School B taught three consecutive lessons following the prescribed sequence of lessons in the MNP textbooks rather than developing their own lessons in the style of MNP lessons as with School A. Both teachers were comfortable and confident in adapting tasks while attempting to preserve their intent, in order to better tailor context and
narrative to their own classes. Furthermore, both teachers again valued the journal task to the extent that it was utilised more frequently than MNP intends in its program. This happened in both School A and School B, seemingly independent of one another, but for similar reasons. Both schools valued the opportunities the journal gave pupils to convey their understanding of concepts with few constraints - pupils were free to draw pictures or models, write sentences, use examples or annotations, or different methods to those used by others in the class. This in turn allowed teachers to assess understanding in a relatively low stakes way routinely.

RQ2: How are textbooks used as teaching aids in DfE approved curriculum programs for primary mathematics in England?

From this case study, the textbooks were followed more rigidly than in School A, with all lessons observed mirroring lessons in the MNP textbook. Structures from the textbook were used as the basis for all lesson phases, although as with School A, independent practice was occasionally omitted in favour of maths journals being introduced. However, in one instance, independent practice was preserved, and the Guided Practice phase was removed instead. The narrative context of some In Focus tasks was altered in favour of utilising more familiar or accessible objects to allow for physical representations to be used in class. The principles of the MNP program informed the ways in which both teachers sequenced and delivered content, responded to questions, and developed pupil competence and proficiency in mathematical ideas and concepts. Generally, the teachers observed appeared to move between offloading MNP tasks as written, and taking a ‘shared responsibility’ towards adapting tasks whilst upholding the MNP principles and developing the 5 core competencies. Occasionally adaptation may have risked altering the original task intent as with case study 1, however this may be in part because MNP does not offer guidance or explanation as to the specific purpose of some tasks in their support documentation.

RQ3: 3. How do teachers facilitate knowledge acquisition using DfE approved teaching guides for primary mathematics in England?
Similar to School A, knowledge acquisition was primarily through the teacher as a facilitator of discussions, play and exploration in line with MNP’s principles 1 and 2. Relational understanding appears to be valued (or prioritised) more than instrumental understanding from the lessons observed, at least in terms of the time spent on lesson phases and the lesson phases that were chosen to be removed in favour of journal writing. Retrieval of knowledge to aid long term memory is facilitated through a school-led initiative independent of MNP’s program called Anchor Charts whereby lesson summaries are created throughout the lesson and displayed alongside previous anchor charts to highlight the learning journey over recent weeks, which is referred to during lesson time, particularly at the beginning of a lesson prior to introducing the In Focus task.

**Position Design**

The position design of teacher questions appeared to align with MNP principles, intending to help pupils develop deep understanding of mathematics through probing around mathematical connections, alternative methods, rigorous expectations of pupils with regards to justification of their answers and ideas. Teachers in School B often positioned themselves as non-experts who were learning with the pupils, and subsequently were asking pupils for advice or help with ideas they were having, which were sometimes purposefully erroneous or flawed, in order to indirectly test the knowledge and understanding of the pupil, or test misconceptions amongst the class, thus positioning teachers on equal footing with pupils, rather than being gatekeepers of knowledge. Similarly, both teachers referred to their pupils as ‘mathematicians’ collectively throughout the lessons. Headings for journals included ‘what have we been doing as mathematicians today?’ And ‘how does a mathematician measure using body parts?’. This positioning of pupils as experts, or as experts in progress, appears to be a strategic decision intended to boost confidence and dispel ideas of a mathematician being somehow elite or gifted, or that some children just ‘can’t do maths’ (Boaler & Dweck, 2016).

**Response Design**

Similar to School A, misconceptions were frequently planned for and incorporated into questioning and pupil discussion and models were displayed after discussions with pupils, which often predicted the outcomes of pupil activities in advance (they were made when
planning the lesson rather than during the implementation of it). As with School A, classroom culture appeared to be a key focus, specifically regarding keeping pupils confident and comfortable with participating and sharing ideas, through positive reinforcement of *contribution* rather than, for example, reserving attention and praise for correct answers. This aligns with motivational strategies highlighted by Peps McCrea in his book ‘Motivated Teaching (McCrea, 2020) that specifically support building in opportunities for students to experience success at a range of levels.

**Task Design**

Tasks for lessons were usually directly taken from MNP’s textbooks and delivered to pupils in a way as to allow space for thinking time and to develop ideas free from teacher pressure to give responses quickly. Ideas informed responses, and the cyclical back and forth between pupils and the teacher was facilitated by the teacher in order to enable whole class development of relational understanding. As with School A, preference for relational understanding appeared to often be at the detriment of instrumental understanding, as the independent practice phase of lessons was commonly replaced altogether with a journal task, or to allow for extended time in the other phases. In an interview, teacher 1B explained that they believed MNP placed less importance on procedures, but it appears that teachers in both School A and B reduce this emphasis even further by removing practice phases. However, in year 5 additional sessions were incorporated into the school week so that some pupils could focus more on independent practice.

**Case Study 3: Teachers C6α and C6β in School C**

Discussion of the final two cases will focus on unique distinguishable features compared to the previous case studies and significant differences and adaptations from the MNP program rather than a full description of each lesson to aid brevity and reduce data saturation.
Information about this school

School C was a primary school based in London rated as ‘Good’ by Ofsted and noted as a much larger than average sized primary school with a majority of pupils speaking English as an additional language. The proportion of disadvantaged pupils, and pupils with special educational needs, were both well above average. The proportion of pupils joining the school part-way through their education was higher than the national average.

This school offered the unique opportunity to observe two teachers teaching the same lesson to two different classes in the same year group. The lessons themselves were back-to-back on the school timetable. For this case study, teachers will not be separated out for discussion, but instead the identical lessons from MNP will be discussed together so as to be able to easily distinguish differences and similarities in the approaches of each teacher teaching the same content more easily. The school had been using MNP for four years and taught 90 minute maths lessons, which is half an hour longer than the other case studies.

Lesson 1:

Both teachers began the lesson with a prepared starting activity referred to as a ‘quick fire starter’ that was not part of the MNP program but was instead designed by the teachers collaboratively to revisit previously taught content and aide long term memory. Teacher C6α then spent around fifteen minutes unpicking misconceptions and testing depth of understanding through questioning in a model similar to that which is presented in MNP, ie, asking students to justify their answers and giving time for them to discuss how they might show someone else what the answer is using a model or diagram. Contrastingly, Teacher C6β read out the answers to the activity, except for one question, which when answered, asked the respondent directly “how do you know?”. In turn, this meant that the same activity took less than five minutes. The two slightly differing approaches recurred across all three lessons observed for each teacher - Teacher C6α seemed to interpret the task not only as a recap opportunity, but also as a task to be taught along the same themes as the In Focus and Let’s Learn phases of MNP lessons, unpicking pupil knowledge and attempting to develop or interpret the level of relational understanding the pupils had. This took time, in part because each question presented was on a different topic in maths, and in part due to the nature of
the style of questioning requiring pupils to pause and occasionally discuss their thoughts. Whereas Teacher C6β appeared to view the activity as a routine to remind pupils of previously taught content and diagnose instrumental understanding, but also did not treat the response design as a point of immediate intervention. Due to differing approaches to the ‘quick fire starter’ activity, In Focus began around ten minutes earlier in each lesson for Teacher C6β. However, both teachers approached the In Focus phase in similar ways, utilising the exact problem presented in the textbooks, and asking prompt questions including those provided in the lesson plan. One key difference that both teachers adopted, but that deviated from the textbooks, was that ideas and key learning points were summarised and captured on the board in real time and left there as the lesson progressed. The approach gradually created a storyboard of the lesson such that by the end phase, the board was mostly full of key knowledge and ideas akin to the ‘Anchor Chart’ used in School B as shown in figure 6.4 below.

![Figure 6.4](image)

*Figure 6.4 an example of a lesson storyboard created as the lesson progressed*

The photo in figure 6.4 above shows several phases of the lesson being captured at different times, and was used by each teacher to refer back to previous learning within the lesson. The approach appears similar to Bansho (board writing and organisation) techniques used in Japan, where a blackboard is typically gradually filled with representations and key ideas
from a lesson as it progresses (Tan et al., 2022) as shown in figure 6.5 below:

![An example of Bansho in a Japanese school](image1)

**Figure 6.5** an example of Bansho in a Japanese school

Alongside this, both teachers also created an Anchor Chart in the same way as School B, and occasionally used it as a facilitator for pupils to summarise their learning in a concise and precise way, asking questions such as “how can we describe this for the anchor chart?”. In interviews it transpired that this approach was developed in School C, and shared with School B in a Maths, No Problem! Network meeting of Accredited MNP schools. Both teachers created an Anchor Chart, an example of which can be seen in figure 6.6 below:

![An example of an Anchor Chart in School C](image2)

**Figure 6.6** an example of an Anchor Chart in School C
With these minor differences in the lesson design, the lesson phases for both teachers proceeded as per the MNP lesson plans, and all tasks were delivered without alteration for the In Focus, Let’s Learn and Guided Practice phases. However, both teachers introduced a Journal task where none had been suggested in the MNP plans, just like Schools A and B. Furthermore, due to a lack of time (this was the reason cited in interview), Teacher C6α’s pupils did not attempt the independent practice task, whilst Teacher C6β’s class did, as they had used less time for the starting activity. Once again, it appears that the independent practice phase of MNP’s lesson structure was the least valued or prioritised. To some extent this could be due to it being the last phase of the lesson, and time constraints squeeze the feasibility of doing this phase in real time during lessons, however that would not account for the fact that new phases were introduced and were delivered as well, thus making the time squeeze for independent practice even more difficult to manage. Similarly it is worth noting that lessons in this school were thirty minutes longer than in other case studies, and yet still the practice phase was not completed in this lesson. However, Teacher6α did ensure that independent practice was given time in the following two lessons, and explained in an interview that they had not anticipated needing to take as much time as they did to resolve misconceptions from the starter activity.

Lessons 2 & 3
Both remaining lessons followed similar patterns of behaviour and structure. Both teachers began their lessons with a starter designed collaboratively and independently of MNP, then began the MNP phases that were matched to the textbook both in terms of structure and tasks. Questions again focused heavily on explanatory responses from students (at one point Teacher C6α specifically stated “I’m getting a lot of single word answers but no-one is telling me why” – encapsulating this dominant expectation from the teacher). Both teachers completed all phases of the MNP lesson within the 90 minutes and included additional journal activities for both lessons. No significant adaptations or new phases were introduced apart from the additional journal activities.

Position design
Across the three lessons, it became clear that Teacher C6α positioned themselves similar to the teachers in School B, as a novice learning with the pupils rather than an expert teacher,
asking questions framed in ways such as “I agree, but I’m not sure why. Why do you think so?”, and “I forgot something here, I think. Can anyone help me?”. Further to this, some problems were presented utilising other teachers in the school as ‘non experts’ requiring help from the pupils as well, for example, in the third lesson, Teacher C6α said “Miss H thinks that the algebraic expression used by this machine is $y - 7$. Do you agree?” and “Miss got stuck at this part, why is it harder?”, followed by integrating the teacher character into the journal task for the class as someone who needs support in understanding the topic taught in that lesson. However, Teacher C6β did not frame questions or tasks in this way, and positioned themselves more as a facilitator than as a non-expert learning with pupils, asking questions such as “what do you think this number line is trying to show?” And “what do I mean when I say difference in maths?”. Whist both approaches differ slightly, their intent appears to still align with MNP’s principles and frequently focus on developing pupil explanations and relational understanding of mathematics.

**Response design**

Response design was similar to both previous case studies, with an emphasis on drawing out misconceptions and pre-empting mistakes or points of interest within the lesson. Responses were often captured on the board or the anchor chart to inform a lesson summary.

**Task design**

Tasks were delivered as intended by MNP, although it was notable that Teacher C6β asked significantly fewer questions during the In Focus and Let’s Learn lesson phases, appearing to only ask the prompts within the lesson plan rather than a more improvised approach, leading to those lesson phases being delivered in noticeably less time than Teacher C6α. Explanation for this difference would be purely speculative but it is perhaps significant that Teacher C6β was not the lead teacher for MNP, nor had they attended the external training for the program. However, a lead MNP teacher and a second teacher who had not attended the training were also observed in case study 2 and this difference was not evident.

**Summary**

Whilst both teachers adhered to the MNP program for all 3 of their lessons, albeit with the removal of independent practice in one lesson for one teacher, and the added journal
phases, one other difference was observed between the two teachers’ approaches regarding the amount of time spent on each phase. Typically, Teacher6α would spend longer in each phase both asking and allowing for responses, which in turn allowed for less time for pupils to work independently in their workbooks or on their journal activities. For example, in one lesson phase, they asked around 12 questions compared to just 4 from Teacher C6β, and the In Focus / Let’s Learn phases of the lesson finished at least ten minutes earlier for each lesson taught by Teacher C6β. It may be that Teacher C6β found independent practice to be a more important phase of the lesson, or that ‘buy in’ around the intent of MNP was less evident. For example, most prompt questions from Teacher C6β were taken directly from the MNP lesson plans, whereas Teacher C6α used additional questions in the style of those questions, preserving intent which may be linked to Teacher C6α being the lead teacher of MNP, attending several additional training sessions around the delivery of the program.

Case Study 4: Teachers D5α and D5β in School D

Information about this school

School D was a primary school based in Birmingham rated as ‘Good’ by Ofsted and noted as an average sized school with one third of pupils from minority ethnic backgrounds. Most pupils spoke English as their first language, and there was an above average number of pupils eligible for pupil premium.

In School D, teachers Teacher D5α and Teacher D5β taught part-time and shared their Year 5 class. As such, the first two lessons observed were taught by Teacher D5α, followed by the final lesson being delivered by Teacher D5β.

Why this school was considered a critical case

School D had taught the MNP program for over four years prior to MNP becoming a DfE approved textbook. All year groups were taught using the program, and the teachers observed had taught it prior to the academic year of observation. The school was not an accredited MNP school, but was recommended through School A as a school that had embedded the program.
Points of note

In the first observed lesson, a starting activity was introduced that did not follow the MNP program, called ‘Rolling Numbers’, (an idea popularised by King Solomon Academy and Bruno Reddy at http://rockingandrollingnumbers.com) that aims to build fluency in times tables through chanting and gamification of procedures. Following this, the teacher began the In Focus task for the lesson provided by MNP. The task was not adapted, and the teacher displayed the images provided by MNP. The teacher introduced the task by asking the question in the textbook and instructing the pupils to “represent this using your geoboards and numicon” (physical manipulatives placed at the centre of pupil tables).

Pupils were given around five minutes to discuss the problem, which was then discussed as a group with the teacher. The lesson then moved to the Let’s Learn phase, with one particular prompt question displayed in blue text rather than black, asking “how many was can you show this amount”. This was a theme across the three lessons. Questions in blue were considered by the teachers to be ‘depth questions’ and were developed by the teachers themselves, but agreed upon as a team and integrated into the slides by the teacher as a way of ensuring those questions were non-negotiable ones to include in the lesson. As pupils worked through the Let’s Learn phase of the lesson, the teacher walked the room asking questions such as “how can you prove this to me?” and “how can we say that amount as a fraction?”. The teacher also took photos of student work on their iPad which were then transferred onto the classroom board for discussion. During this phase the teacher deviated from the lesson slightly and focused on how students might be able to count the number of cookies shown in the array (see figure 6.7 below)
The class explored a few different ways of structuring arrays to count the cookies, and the teacher explored their specific use of the vocabulary multiplier and multiplicand when discussing the arrays. For example, if a student highlighted an array of 4 x 3 cookies as a helpful segment to count easily, the teacher would ask them what they saw as the multiplier and the multiplicand. Like School A, both teachers of this class emphasised the use of key vocabulary in sentences in terms of expectations around responses from students, but they also included sentence starters in their slides in a further adaptation to the MNP model. The guided practice phase was faithful to the MNP lesson plans and textbook, however the third question in this phase was not attempted, and instead the teacher gave pupils ten minutes to complete a journal task (again, not included in the MNP lesson plan).

As with the other case studies, the title of the journal was given to students to decide upon, and a word bank was provided of key terminology related to the lesson. Students were given a degree of autonomy in the journal activity, with prompts provided such as ‘explain in full sentences and use representations to support your reasoning’ - here the type of representations is a choice for the pupils, although the expectation that representations are used is not, and the use of ‘explain in full sentences’ again indicates that the use of technical vocabulary in context is a key principle of teaching maths in this class.

The journal activity was timed using a visible ten-minute timer on the board, and ambient music was played for the duration of the activity. Once it was completed, students
moved on to independent practice for the final ten minutes of the lesson, again with ambient music as the teacher walked the room and checked student work. Students in this class performed all maths on whiteboards except for the journal task, which the teacher explained was to help them feel comfortable with getting answers incorrect (mistakes were not permanently recorded and easily corrected).

Misconceptions again featured prominently and formed part of teacher questioning across all three lessons, and teachers included a ‘mistake of the week’ which they drew upon for discussion with the class, and which was a genuine mistake someone had made which the teacher deemed interesting enough to become a talking point for the class to help them overcome the misconception. Mistakes were, according to the teacher, not always a strategic decision to display based on a misconception and could instead simply be an error such as forgetting about a step in a procedure. All three lessons followed a similar format across both teachers. Lessons started with the rolling numbers activity, followed by the In Focus, Let’s Learn and Guided Practice phases from MNP, with no adaptation of the models, narratives or questions provided in the textbook, but with additional deeper understanding questions designed by the teachers, then a journal activity deviated from the MNP plans, followed by a return to the plan with the independent practice phase. Textbooks were not used by the children, and instead the teacher relied on the slides as the single point of presentation of the In Focus, Let’s Learn and Guided Practice phases.

A reflection of all four case studies framed against the research questions is covered in the next chapter.

**Innovation Configuration Map**

To summarise the interpreted alignment and deviation from the MNP program, and whether they overall appeared to uphold the integrity of MNP’s principles and core competencies, each lesson has been mapped onto an Innovation Configuration Map developed and presented by lesson phase as shown in tables 6.2, 6.3, 6.4 and 6.5 below.
Each category was constructed based upon the model examples provided by Huntley (2012), and are intended to visually demonstrate the spectrum of adaptation of the curriculum, with sections C and D deemed to include deviation from MNPs intent to the detriment of the program either by misinterpretation or removal of parts of the lesson’s designed sections, whereas section B is considered to be generally following the program as instructed, and section A captures where fidelity is considered to be strongly upheld even with adaptation, utilising the definition of ‘agency’ as the active, interpretive, and participatory role of teachers in shaping the curriculum based on their application of professional knowledge and judgement considering the needs of their students.
## In Focus Phase

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>Phase is included, in sequence and the principles of the phase are preserved: it is usually narrative driven, includes facilitated exploration of a concept or problem in partnership with knowledgeable others, and is given time for ideas to develop. Minimal adaptation or supplementation occurs, but where it does, it preserves or enhances the activity’s purpose based on application of professional knowledge and judgement considering the needs of students.</td>
<td>Phase is included, in sequence and the principles of the phase are mostly preserved: it is usually narrative driven, includes facilitated exploration of a concept or problem in partnership with knowledgeable others, but adaptation may be significant to the point that the purpose or design of the task may be compromised.</td>
<td>Phase is included, but possibly out of sequence and the principles of the phase are minimally preserved: for example, it is not narrative driven, or is directed by the teacher rather than given time to pupils to develop ideas with their peers, or the time spent in this phase is too short for ideas to develop independently of the teacher.</td>
<td>Phase is omitted or replaced with a different idea in its place that does not meet the same intended purpose.</td>
<td></td>
</tr>
</tbody>
</table>

5A_6_14b, 5B_6_7, 6Cα_9_3  
5A_6_15, 5B_6_8, 6Cα_9_4  
5A_6_15b, 5B_6_9, 6Cα_9_5  
1B_9_2, 5D_8_1  
1B_9_3, 6Cβ_9_5  
6Cβ_9_4  
6Cβ_9_3
### Let’s Learn Phase

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td><strong>B</strong></td>
<td><strong>C</strong></td>
<td><strong>D</strong></td>
</tr>
<tr>
<td>Phase is included, in sequence and the principles of the phase are preserved: it involves strong questioning that tests depth of knowledge, and a focus on relational understanding. It includes the questions and solution approaches provided, with minimal adaptation or supplementation occurring, but where it does, it preserves or enhances the activity’s purpose based on application of professional knowledge and judgement considering the needs of students.</td>
<td>Phase is included, in sequence and the principles of the phase are mostly preserved: it involves some questioning that tests depth of knowledge but may mix focus between relational and instrumental knowledge. It may include adaptations to either the questions asked, or the solutions provided, with some adaptation or supplementation which may have compromised task intent.</td>
<td>Phase is included, but possibly out of sequence and the principles of the phase are minimally preserved: Tasks minimally involve interaction with pupils, and questions tend to be surface level rather than those suggested in the lesson plan. Adaptations are likely to have compromised task intent.</td>
<td>Phase is omitted or replaced with a different idea in its place that does not meet the same intended purpose.</td>
</tr>
</tbody>
</table>

*Table 6.2 an IC Map of the In Focus lesson phase*
Table 6.3 an IC Map of the Let's Learn lesson phase

| 6Ca_9_3 | 1B_9_2 | 5D_8_1 | 5A_6_14b | 5A_6_15b |
| 6Ca_9_4 | 1B_9_3 | 5D_8_2 |          |          |
| 6Ca_9_5 | 1B_9_4 | 5D_8_3 |          |          |
| 5B_6_7  | 5A_6_15 | 6Cβ_9_3 |          |          |
| 5B_6_8  |          | 6Cβ_9_4 |          |          |
| 5B_6_9  |          | 6Cβ_9_5 |          |          |

**Guided Practice Phase**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase is included, in sequence and the principles of the phase are preserved: it involves strong questioning that tests depth of knowledge, and focuses predominantly on instrumental understanding using formal methods, as demonstrated in the textbook, with minimal adaptation or supplementation occurring, but where it does, it preserves or</td>
<td>Phase is included, in sequence and the principles of the phase are mostly preserved: it involves strong questioning that tests depth of knowledge, and focuses predominantly on instrumental understanding using formal methods, as demonstrated in the textbook, with adaptation or supplementation occurring that may have compromised task</td>
<td>Phase is included, but possibly out of sequence and the principles of the phase are minimally preserved: Tasks minimally involve interaction with pupils, and questions tend to be surface level rather than those suggested in the lesson plan. Adaptations are likely to have compromised task intent.</td>
<td>Phase is omitted or replaced with a different idea in its place that does not meet the same intended purpose.</td>
</tr>
</tbody>
</table>
enhances the activity’s purpose based on application of professional knowledge and judgement considering the needs of students.

<table>
<thead>
<tr>
<th>5B_6_7</th>
<th>5A_6_15</th>
<th>6Ca_9_4</th>
<th>5A_6_14b</th>
<th>6Ca_9_3</th>
<th>5A_6_15b</th>
</tr>
</thead>
<tbody>
<tr>
<td>5B_6_8</td>
<td>5D_8_1</td>
<td>6Ca_9_5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5B_6_9</td>
<td>5D_8_2</td>
<td>6Cβ_9_3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1B_9_2</td>
<td>5D_8_3</td>
<td>6Cβ_9_4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1B_9_3</td>
<td></td>
<td>6Cβ_9_5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1B_9_4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.4 an IC Map of the Guided Practice lesson phase
### Independent Practice Phase

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th></th>
<th>B</th>
<th></th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Phase is included in sequence and the principles of the phase are preserved: it involves opportunity to develop practice independently of teacher or peer guidance, and questions are identical or minimally adapted / supplemented from the workbook and adaptation preserves or enhances the activity’s purpose based on application of professional knowledge and judgement considering the needs of students.</td>
<td>Phase is included but may not be in sequence, but the principles of the phase are mostly preserved: it involves opportunity to develop practice usually independently of teacher or peer guidance, but questions may be significantly adapted / supplemented from the workbook, possibly compromising task intent.</td>
<td>Phase is omitted or replaced in the lesson with a different idea in place that does not meet the same intended purpose</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5B_6_9</td>
<td>6Cβ_9_4</td>
<td>5D_8_2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6Ca_9_5</td>
<td>1B_9_4</td>
<td>5D_8_3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6Ca_9_4</td>
<td>5A_6_15b</td>
<td>6Cβ_9_5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6Cβ_9_3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.5 an IC Map of the Independent Practice lesson phase
The Innovation Configuration maps add to an emerging picture around how teachers across the case studies appear to both interpret and adapt the MNP program. Firstly, the map in Table 6.5 clearly shows a pattern of behaviours around the use of independent practice which simply put is either conformed to, or as Brown (2009) phrased it, offloaded, or omitted altogether. Omissions linked either to time constraints (it is the last phase of the lesson) or a perception of it being less useful than, say journaling – which was not an MNP phase in any lesson plan for the observed lessons, but was elevated in importance by all teachers and incorporated into almost every lesson as an adaptation, often at the cost of omitting independent practice. This finding appears to align somewhat with Smagorinsky’s research that determined that teachers tended to prioritise tasks that they thought students would enjoy more (Smagorinksy et al., 2002) at the detriment of those they did not. However, what is particularly interesting is that teachers favoured tasks that included more discourse and open ended activities, and what Leinhardt and Steele referred to as ‘instructional dialogues’ (Leinhardt & Steele, 2005), which is the opposite finding of many studies discussed in the literature review in chapter 3 (Wood et al., 2001; Leinhardt & Steele, 2005; Sherin, 2002; Van Steenbrugge & Ryve, 2018), where teachers often disregarded those activity types in favour of more practice driven, or quasi-deductive (Leshota, 2020) style tasks.

What is also clearer, is that teachers generally were preserving the integrity of each lesson phase, even when creating their own lessons. The only significant deviation that was interpreted to compromise phase integrity were occasional In Focus tasks that were delivered in a very short time span and asking few questions with little time to respond, thus not allowing for ‘time to develop ideas’ (principle 1) which was interpreted as a key purpose of that phase. However, even in those instances, the phase was delivered, and perhaps would be more aligned with MNP principles had more guidance for the phase been incorporated into the textbooks themselves rather than only in the lesson plan guidance. Generally, content fidelity (McNaught et al, 2010) - how much of the MNP content is actually taught, was found to be high, with the exception of independent practice.

A reflection of how this aligns with the research questions will take place in the next chapter.
Limitations of the Innovation Configuration Map

Whilst mapping observational data to the ICM helped to visualise the frequency with which teachers across the case studies aligned closely with the MNP program and intent, the degree to which they added or adapted components of each section of the lesson was difficult to differentiate. For example, in the ‘Let’s Learn’ phase of the lesson, many observations adhered fully to the MNP task, however there were variations in this category that may have benefitted from further subdivisions. Some teachers chose to adapt Let’s Learn to suit their learners, and some spent more time questioning and allowing for activities than others. Examples such as these could in hindsight have helped further differentiate category A to better identify ‘active designers’ (Remillard, 2005) and widen the spectrum measures to better cater for the fact that these case studies are observing enthusiastic adopters of the MNP program as a specific type of teacher adopting prescribed curriculum materials. This is further supported by the lack of lessons categorised in the middle of the spectrum. In addition to this, whilst some lesson components were omitted across several lessons taught by different teachers, they were in fact replaced with components that despite not appearing in the lesson plans, are endorsed and MNP designed phases of lessons rather than entirely constructed by teachers and unaffiliated with the MNP program in any way. As such, one could argue that despite the intended phase being omitted, the Innovation Configuration Map fails to capture the possibility that the omission of the phase and subsequent replacement of it with a different MNP phase could demonstrate strong ‘active design’ despite it being categorised at the extreme right side of the enactment spectrum. Indeed, the removal of independent practice in lessons could be argued to align even more closely with lesson styles in Shanghai, where independent practice is often used only as a homework task to ensure that time with a teacher is spent developing relational understanding (Boylan et al., 2019)

Chapter 7: Conclusion

The aims of this thesis were to:

1. Identify specific pedagogical approaches in curriculum packages endorsed by the Department for Education for primary mathematics in England.

3. Determine how the teacher agency of enthusiastic adopters affects the enactment of the intended curriculum provided by DfE endorsed curriculum packages for primary mathematics in England.

Each of these aims were matched to the research questions below, and each aim has been met in this research. Specifically, research aim 1 has been met through answering research question 1, aim 2 has been met in research question 2 and 3, and aim 3 has been met in research question 4.

Research Question 1

What strategies are emphasised and prioritised in curriculum packages endorsed by the DfE for primary mathematics in England?

From this research I have identified that the curriculum package Maths, No Problem! utilises 8 principles of design that inform the ways in which MNP shape pedagogy in the classroom. The first five of these are explicitly mentioned on the MNP website, and the latter three are implicit across the website, blog posts and lesson plans.

1. Children need time to accommodate new ideas - and construct their knowledge from the world around them.

2. Children need opportunities to play with mathematical ideas in an informal and exploratory way prior to any structured learning.

3. Learning is effective when cooperatively learning with knowledgeable others.

4. Relational understanding is emphasised alongside instrumental understanding.
5. Mathematics learning should utilise the Concrete-Pictorial-Abstract approach to develop understanding of concepts.

6. New learning should be presented in small steps.

7. The whole class should move through the same content at the same pace.

8. Mathematics should be presented in a variety of ways including narrative structures and physical manipulatives.

A thorough examination of the textbooks, lesson plans, and workbooks has substantiated that these principles are not solely mentioned or implied on the website and its corresponding blog posts, but rather seamlessly woven into the tasks and phases of lessons provided to teachers. Moreover, the guidance provided within the lesson plans explicitly instructs teachers to engage in tasks and pose questions that are in harmony with these principles. Lesson phases are designed to specifically address or augment specific principles – for example, the initial lesson phase, In Focus, is designed to integrate principles 1., 2. and 8.

Lesson plans were found to be particularly instrumental in promoting task intent and design principles 1., 2. and 5. (specifically the use of concrete manipulatives, which are addressed relatively lightly in the textbooks by themselves). Principles 1. and 2. both rely on pupils being given time (to develop ideas and play, respectively) which is not explicit in the textbooks, and the ‘concrete’ element of the CPA approach in principle 5. is not referenced in most activities in the textbooks.

In addition, the pedagogical approaches embedded in the MNP program are intended to allow for a degree of improvisation and agency by the teachers delivering the package, as demonstrated within lesson plans that allow for multiple variations of activities, blog posts that point to ways to adapt and enhance lesson phases, and the openness inherent within the In Focus task that requires teacher response design to pick up and develop ideas produced by the pupils themselves. As such, it is my conclusion that the Maths, No Problem!
A curriculum package is designed to be what Ball and Cohen (1996) referred to as a ‘jointly constructed’ curriculum, anticipating delivery affordances that Brown refers to as ‘shared responsibility’ (Brown, 2009) and therefore MNP can be perceived as a package that is teacher enabling rather than teacher-proof (Taylor, 2013). It should be noted however, that this research is focused on a teacher’s fidelity with the MNP materials rather than the efficacy of the materials themselves, and as such, the suitability or effectiveness of MNPs design principles identified above are not a focus of this study. Furthermore, the scope of this study incorporated only a limited analysis of the longitudinal elements of curriculum design in the MNP textbook and workbooks - ie the ways in which MNP structures the sequencing of tasks across lessons within a unit or units, and as such there are likely to be further elements of MNP curriculum materials that support or do not support the implementation of the identified design principles not captured in this study.

This research question has met the objective of Research Aim 1: Identify specific pedagogical approaches in curriculum packages endorsed by the Department for Education for primary mathematics in England.

Research Question 2

How are textbooks used as teaching aids in DfE approved curriculum packages for primary mathematics in England?

This research has shown that textbooks and workbooks are integral to the MNP approach to teaching mathematics. Textbooks are used to present three of the four standard phases of MNP lessons (In Focus, Let’s Learn and Guided Practice), as well as additional phases that are used less frequently (Activity Time, Maths Journals and Mind Workouts), and workbooks are designed for pupils to complete independent practice for the final lesson phase. The books offer a highly structured approach to teaching, and provide prescribed tasks to develop relational and instrumental understanding which, whilst they arguably cannot truly be developed independently of one another, do appear to be somewhat sequential in focus such that In Focus and Let’s Learn phases focus on relational understanding, whilst the
Guided Practice phase appears to act as a bridge towards developing instrumental knowledge in the Independent Practice phase in the workbooks.

Textbooks and workbooks together accommodate for all tasks within a lesson, and therefore do not require teachers to design tasks themselves. Similarly, the lesson plans that accompany the lesson offer a number of questions to ask pupils which could be perceived as requiring that teachers do not need to design questions either. However, based upon evidence presented in MNP blog posts emphasising ways in which teachers could adapt and modify lessons to suit their learners, and features of the lesson plans that offer variation prompts for teachers, the textbook architecture appears to be built with teacher agency in mind, and generally seems to be more teacher-enabling than teacher-proof in approach (Taylor, 2013) and placing the teacher as an ‘active designer’ (Remillard, 2005) of elements of the lesson. In addition, the first two phases of MNP lessons require a lot of teacher input as pupils are given opportunities to explore mathematics on their own, which would likely result in teachers being required to effectively improvise and adapt to what is happening in front of them – which again supports a need for teachers taking an active role to assist in meaning-making. Whilst position design can be framed by MNP guidance, it seems more difficult to accurately or effectively plan response design within resources. As such, it appears that a wider goal of MNP is to instil an ethos based around its design principles that actively relies on teacher agency to enhance it. However, as noted in more detail in research question 4, teachers in the case studies for this research often did not use the workbooks in particular as intended and opted instead to incorporate additional customised tasks based upon the maths journal activities in the textbook, to the detriment of the workbook exercises which were often not completed – which in turn under emphasised instrumental understanding.

Research Question 3

How do teachers facilitate knowledge acquisition using DfE approved curriculum packages for primary mathematics in England?

This research has shown that MNP’s design principles are informed by a constructivist approach to knowledge acquisition influenced by MNP’s interpretations of the work of
Bruner and Piaget which manifests through the design of the first two lesson phases, In Focus and Let’s Learn, and the guidance in lesson plans and blog posts around the position and response design of (and to) questions in the classroom. Specifically, In Focus is generally designed to allow pupils to explore and try out ideas to formulate their own conjectures and proofs of what is and is not true in mathematics given a simple prompt from the textbook or teacher. Hence the role of the teacher, particularly in these two phases, is that of a facilitator of enquiry and hard thinking about a specific aspect of mathematics. The second phase, Let’s Learn, is intended in part to ensure that ideas generated from In Focus are aligned to what is true, and what the teacher wants pupils to learn. This is supported by the position, response and task design emphasis on making pupils aware of misconceptions, actively discussing them, and proving they are not true. Hence MNP appears to have a strong sense of balance between allowing for pupils to follow their own enquiry to develop a personalised learning journey, whilst carefully guiding them to ensure that they arrive at the desired outcome. Again, it is important to note that this approach is neither endorsed nor discouraged but merely observed in this research. Similarly, discourse between teacher and pupil, and pupil to pupil is emphasised and encouraged in what MNP refer to as learning from a knowledgeable other influenced by MNP’s interpretations of the work of Vygotsky – and features prominently in three of the four typical MNP lesson phases with In Focus and Let’s Learn being largely led and informed by pupils (and peer ‘knowledgeable others’), and Guided Practice positioning the teacher as the knowledgeable other more dominantly.

Development of knowledge acquisition in MNP lessons tends to be sequential (as dictated by the lesson phases) such that development of relational understanding of a concept begins in the early lesson phases where position design strongly encourages teachers to ask pupils to explain why things work, why answers are true and how concepts are connected to other areas of maths, and response design preempts misconceptions, challenges brief answers that lack explanation or reasoning, and upholds high expectations for the use of technical mathematical vocabulary, whereas developing understanding and fluency of procedures and algorithms appears in the latter phases where pupils practice calculations more frequently. However, again it should be noted that when teachers enacted MNP in the case studies, instrumental understanding in the form of independent practice was allocated little or no lesson time for many lessons observed. Finally, knowledge acquisition is also
developed through the Concrete, Pictorial Abstract method of developing understanding of mathematical concepts, and is again inferred to be a sequential process moving from concrete mathematical objects and models towards abstraction which generally dominates independent practice, and gradually appears to become more prominent as pupils progress through Year 1 to Year 5 – although this may be in part due to the construction of the curriculum itself as well as a specific MNP strategy, and textbooks were not analysed in Year 2, 3 or 4. Finally, it should again be noted that the ways in which MNP develop knowledge were not analysed in this study at a level in which the development of individual concepts over time (such as multiplicative reasoning or the development of knowledge of addition over time) could be commented upon effectively.

Combined, research questions 2 and 3 have met the objective of Research Aim 3: Determine the structure and formats of practice underpinning curriculum packages endorsed by the DfE for primary mathematics in England.

Research Question 4

How is the teacher agency of enthusiastic adopters impacting upon the interpretation and delivery of the intended curriculum in DfE approved curriculum packages for primary mathematics in England?

This research has demonstrated through a series of case studies that the teachers observed using MNP largely upheld the integrity and intent of the MNP curriculum. The lesson phases ‘In Focus’ ‘Let’s Learn’ and ‘Guided Practice’ were frequently adhered to and delivered as written in the textbooks, with position, response and task design aligned to the 8 design principles in most cases. As demonstrated in the Innovation Configuration Map in the last chapter, teachers were comfortable with making their own adaptations of some tasks to better reflect the environment they were teaching (by for example, replacing the subject or object involved in an In Focus task to one that was readily available to demonstrate as a physical object or subject that was more personal to their class), and some teachers designed their own lessons to offer more support or to allocate more time to particular topics. In hindsight, further categorisation of the ICM around teacher adaptation
and adoption of materials may have helped capture and differentiate teachers who appeared to make more significant adjustments to the program whilst maintaining MNP’s design principles.

This generally placed each lesson phase within what Brown (2009) referred to as a space of ‘shared responsibility’ within a continuum of MNP curricular affordances, compared to ‘offloading’ (delivering tasks exactly as written) and ‘improvising’ (delivering different tasks away from the prescribed ones in the MNP package). The lessons teachers designed themselves were imitations of the MNP structure and phases, with tasks that were modelled after the tasks presented in each MNP phase, suggesting that teacher trust in textbook integrity was high, as was their understanding and interpretation of MNP’s intent. Whilst the reasons why teacher enactment of MNP appeared to be closely aligned to MNP intent are not a focus of this study, it appears that MNP’s design embraces teacher agency to some extent, and within blog posts and lesson plans there appears to be acknowledgement that agency is important in order to better tailor lessons to suit a teacher’s own learners. Furthermore, the open ended ‘In Focus’ and ‘Let’s Learn’ phases of the lesson appear designed to embrace teacher agency and allow for productive conversations and exploration of mathematics, which therefore rely heavily on teacher agency. The case studies in this research indicated that this level of agency was indeed apparent and utilised to tailor elements of the lesson and personalise lessons towards (or in response to) the students in front of them.

This shared responsibility highlighted that teachers tended to flow between offloading MNP content and mutually adapting it which perhaps helped to maintain a sense of autonomy and agency whilst still preserving MNP intent. By observing enthusiastic adopters who had been using the MNP program for two or more years, particular developments were identified that would most likely not be evident without this case study constraint. Specifically, teachers had developed their own systems and lesson features that had become embedded into the MNP lesson phase structure such as Anchor Charts and initiatives to refer to pupils as ‘mathematicians’ whilst positioning the teacher as a learner rather than an expert when framing questions and problems. Most notably, the more frequent use of maths journaling had been adopted by teachers across several of the schools, which
appeared to have developed independently in at least one of the schools, and shared more widely through networking amongst MNP schools via MNP conferences and network meetings associated with some schools having ‘accredited status’. It appears that networking across schools was a catalyst for some of the more significant adaptations to the MNP program, which is notable for its potential impact in general with enthusiastic adopters of curriculum reform and would be worthy of further studies with this category of teachers.

It appears MNP supports extended use of journals, as demonstrated in a blog post titled ‘5 types of maths journal and how to use them’ (Douglas, 2021), which points to a more autonomous approach to developing journaling than the prescribed journal activities supplied in the textbooks themselves. However, as previously discussed, the additional focus on maths journaling appeared to be at the cost of independent practice which was often dropped by teachers across the case studies in favour of journaling. This resulted in pupils spending only around 10-15 minutes relatively infrequently on independent practice thereby substantially shifting the balance between developing relational understanding and instrumental understanding of mathematics. Whilst this appears to be an example of agency improvising away from the intent of MNP, it should be noted that there are inferences within blog posts on the MNP site that instrumental understanding is less important or less favourable, calling it, for example, ‘drudgery’ (McIver, 2020) that risks creating ‘empty shells” (Fournier, 2022). Furthermore a video response to frequently asked questions points towards a more widespread misunderstanding around direct instruction and enquiry (Maths No Problem, 2022). Considering these attitudes, and the reduction in independent practice, it appears that there is perhaps an inherent legacy conflict regarding the perception of more traditional teaching models of direct instruction and independent practice that manifests in teachers demoting practice and direct instruction rather than seeing them as equally important elements as enquiry and relational understanding as highlighted originally by Skemp who popularised both terms, which are cited in the MNP design principles as ‘Relational understanding is emphasised alongside instrumental understanding.’ with no clear prioritisation or demotion of either.

The focus on enthusiastic adopters is also likely to have dramatically reduced the potential influence of several factors that may contribute to how teachers took shared responsibility or offloaded the curriculum materials. The teachers participating in this
research were already in what were considered optimal conditions to use the program effectively, and subscribed to it before any Governmental incentivisation. Hence it was already embedded and thus circumvented any issues or conflicts associated with new learning and adjustment from prior teaching styles or beliefs, or from any sense of the program being imposed or directed from others, suggesting a predisposition less affected by external pressures, identity conflicts, or the challenges of significant pedagogical shifts that may have occurred as they began learning how to use the materials. These factors inevitably affect the ways in which teachers adopt materials as discussed in the literature review and should not be overlooked when assessing the utilisation of the materials in this study or the wider implications of curriculum reform in general, which has a well documented and difficult history as highlighted in the introduction section of this thesis. Indeed, in June 2023 an article in The Conversation (Marks, 2023) pointed to a broader context that suggests that some teachers adopting the MNP program after the DfE incentive was announced encountered a range of negative experiences when adopting MNP specifically, and 37% of schools surveyed that initially adopted the MNP program had since dropped it completely, with a further 24% reducing their use of the materials in some way (Marks et al., 2023) (for example, not renewing their online subscription or not replacing workbooks). Such experiences could range from ideological dissonance and professional identity crises to material and administrative or financial hurdles, potentially impeding the effective assimilation and implementation of these programs. For example, the MNP program is only partially funded by the DfE, and costs are annual for schools that continue to fully subscribe to and renew books and online services - factors that dissuaded some teachers from adopting the scheme (Marks et al., 2023).

This contrast highlights the importance of considering teacher attitudes, previous experiences, and the sociopolitical context in the adoption and success of educational programs. It suggests that while agency and familiarity can significantly enhance program adoption and adaptation, external pressures and the nature of the introduction of such programs can equally serve as barriers to their success. Therefore, any analysis of curriculum adoption and teacher adaptation must account for these multifaceted influences to fully understand the dynamics at play. However, this research has provided new insights into a specific set of circumstances where teachers were found to largely be faithful to both the
curriculum package and its design principles to the extent that they gained confidence to
enhance the program with meaningful active design of new or adapted components to
lessons.

In this thesis, it has been demonstrated that 'enthusiastic adopters' of a mathematics
curriculum package can actively utilise their agency to enhance the program, adhering
closely to its integrity and principles while remaining faithful to the source materials.

Research question 4 has met the objective of **Research Aim 2**: Determine how the teacher
agency of enthusiastic adopters affects the enactment of the intended curriculum provided
by DfE endorsed curriculum packages for primary mathematics in England.

**Contribution to new knowledge**

This research has provided the first detailed analysis of the Maths, No Problem! curriculum
package, which is significant as it is the first DfE endorsed and partially subsidised maths
package for primary schools in England. In order to become a DfE endorsed set of textbooks,
the series needed to ‘reflect a teaching for mastery approach’ (NCETM 2017b) and as the
only series to become endorsed after review, the design principles identified in this research
arguably give a strong insight into what the DfE perceives ‘mastery teaching’ to be and shed
light on the role of teacher agency on enacting the curriculum when the program is
established in schools. By extension, the five core competencies that MNP use as
assessment criteria give insight into what the DfE endorse as the aims of mastery teaching.

Furthermore, this research has demonstrated that teacher autonomy has a significant role in
the enactment of some phases of the MNP program which appears designed to be
enhanced by agency in some lesson phases – which suggests the first DfE approved
textbooks are designed to be teacher-enabling rather than teacher-proof, despite them
clearly promoting a specific style of teaching. Hence the MNP curriculum package is perhaps
best seen as a strategy from the DfE to endorse a style of teaching, but not to remove
agency from teachers entirely. The research holds currency in that Maths, No Problem! is
still a partially funded and endorsed DfE approved curriculum package and is one of only
two present day packages that the DfE approves.
The research contributes more widely to debate regarding what ‘maths mastery’ means, and towards textbook research in general regarding the intended and enacted curriculum and textbook structure and design, offering some insight into where enactment and intent can align after several years of using a curriculum package, specifically through the lens of enthusiastic adopters.

Limitations of this study

Whilst I have made efforts to maintain validity throughout this study, it should again be noted that there are examples of personal interpretation taking place throughout the gathering and analysis of data. Determining the enacted curriculum through interviews and observations involved personal interpretation of what teachers were doing, emphasising, and intending, as well as interpreting what was important to note and focus on. Where possible, the accuracy of interpretation was strengthened and adapted through teacher interviews to clarify elements of their pedagogy or decision making, however it is acknowledged that my own personal experiences and positioning within the study will have influenced my decision making. I am an experienced secondary maths educator and have trained primary teachers in mathematics. Furthermore, I am a maths education author and a relatively well-known member of the mathematics education community. These factors may have influenced the classroom dynamics during observations in ways that would not be evident to me. Similarly, my very presence in observations may well have influenced how teachers and pupils behaved (Hesse-Biber & Leavy, 2006), however this is an acknowledged risk within case study research and observations (Hesse-Biber & Leavy, 2006; Flyvbjerg, 2006; Denzin & Lincoln, 2008a; Creswell & Creswell; 2018).

It is also acknowledged that whilst a large amount of data was available for this research, due to the nature of the MNP subscription payment structure, materials behind paywalls were either difficult or not possible to obtain, which may have shed more light on, or proved false, some interpretations made in this research regarding the intended curriculum. Finally, this research is sensitive to the time in which it was conducted. There are now two DfE endorsed curriculum packages, and MNP have recently released an updated second edition of their resources which I have not seen, but may address some of the findings here around, for example, extended use of journaling and reduction in independent practice.
opportunities. As previously mentioned, this study focuses on enthusiastic adopters of a prescribed curriculum and caution should be taken over making wider conclusions as to the impact and effect of a prescribed curriculum package on teachers who do not fit into this category.

Reflecting on the choice of methodology

1. The intended curriculum

The decision to analyse the website, blog posts, textbooks and workbooks helped develop specific design principles that shaped my interpretation of what the intent of MNP was. The website was freely accessible and included clear intent statements that were complemented and elaborated upon within the numerous blog posts which helped develop a more detailed understanding of what the principles were expected to look like in the classroom with regards to teacher actions, questioning styles and lesson phase structures. Further insights could have been gained from the YouTube videos on the MNP YouTube channel, but only a small number of these were utilised in the research as links from the main website. However, the workload implications of cross referencing from another source material such as the YouTube videos alongside reaching a likely point of data saturation suggest that the data sources utilised in depth were enough to formulate a valid and reliably robust interpretation of MNP’s curriculum package intent.

Whilst the initial methodology highlighted an in-person training event hosted by MNP as a possible source of information, the event itself (which was attended) was deemed not to add any new insights into the curriculum package intent beyond what was already established from the website and blog, and so was not utilised or referenced in the analysis in chapter 5.

Textbooks and workbooks were also relatively easy to access and offered further insights into more subtle design elements of MNP around task design and structure, which in turn helped develop an understanding of how well the written intent of MNP was supported by the materials used in the classroom. The tools utilised for textbook analysis were customised to focus more acutely on specific elements of task design that would support interpreting the extent to which tasks upheld the design principles outlined in the MNP website. Without
this customisation, the analysis risked producing superfluous data that gave no insight into answering the research questions regarding uncovering the curriculum package intent.

Despite this, there were problems with analysing two aspects of task design using the five-dimensional analysis tool adapted by Berisha and Bytyqi (2020), namely the ‘mathematical activities involved’ category for tasks gave little reliable insight due to the category being so contextually linked to the topics being taught which in turn was dictated by the DfE Programme of Study, not MNP. Second to this, analysis of the ‘level of cognitive demand’ was incredibly difficult to interpret accurately as highlighted by the process of developing a coding agreement with a second expert - and as such, became a second category of the five dimensional analysis tool that seemed to offer little insight other than whether a task was procedural or not, which was not as useful as intended. Despite these minor setbacks, the intended curriculum analysis did successfully produce a set of design principles that were shown to be supported by the tasks presented in the textbooks and workbooks, which in turn provided a strong, clear foundation to draw a comparative analysis from to determine how the enacted curriculum differed adhered to or adapted from it.

2. The enacted curriculum

Observations of teachers were incredibly useful data sources to capture how teachers taught, with a minimum amount of interpretations needing to take place – in other words, data was captured first hand, and could be interpreted live in the moment rather than collecting data through interviews about how a teacher taught, which would rely on both memory, and the teacher’s own interpretation of how they taught which could be unreliable. Access was relatively easy to facilitate, and three sequential lessons was justified early in the research when, for example, School A taught lessons outside of the MNP program which would potentially render the school visit and data collection unreliable had it been the only lesson captured. Observation techniques focused heavily on what was said between teacher and pupils, with descriptions of what was presented on the board and what pupils did during exploratory tasks on their tables. This data proved invaluable to discern position design, response design and task design – all of which helped interpret the intent of what the teacher was doing in the classroom beyond simply describing their
actions. This in turn helped frame whether actions that deviated from the MNP lesson plans were intended to uphold the integrity of MNP design principles or deviate from them.

Teacher interviews gave minor support and reinforced some interpretative decisions made by me when observing live, but overall the interviews were not a strong data source to gain insight into teacher intent due in part to the lack of opportunities to speak to teachers within a short time frame of them teaching each lesson, and the lack of time available during interviews within a busy school day. Longer, more dedicated and rigorous interviews may have alleviated this, but would have placed additional burden on each teacher and would not necessarily have overcome issues around availability immediately after the lessons observed. Finally, a data source that would possibly have been very useful for this research was the support website and associated materials that require a login and annual subscription to access. Behind this paywall the lesson plans are stored, but also several additional support materials on how to teach. I was fortunate enough to be provided with the lesson plans for the lessons I observed, but there may have been further insights to be gained regarding the intended delivery of MNP from this source which was unfortunately inaccessible to me.

Further research opportunities

This research uncovered several interesting areas that would benefit further study to develop further understanding around the effect of agency upon the prescribed Maths, No Problem! curriculum package. Aside from repetition of this research in similar settings to observe a broader sample space, three areas of focus would complement this research:

1. Adjustment of experience for critical cases

For this research, critical cases were defined to be teachers that were considered the best placed to deliver MNP in optimum conditions – most notably, ensuring that teachers had already used the plan for a few years and that they had therefore adopted the package prior to it becoming endorsed by the DfE. A good complement to this would be to look at case studies where teachers were enthusiastic adopters, but had not utilised the program before and so could be classed as novices at using the program and look specifically at whether or how their initial interpretation of it changes over time in a
more longitudinal study. This may offer insights into the journey towards alignment or misalignment with a prescribed curriculum and could potentially give some indications of further factors that may contribute towards the successful adoption of a curriculum package with regards to preserving intent.

After this research began, a second curriculum package was approved and endorsed by the DfE called ‘Power Maths’, published by Pearson and written by Tony Stanef, a director of White Rose Maths. A comparative study of the intended and enacted curriculum packages of Power Maths and Maths, No Problem! would potentially offer insights into the effectiveness of each company’s accompanying teacher materials and training in supporting teachers to enact their curriculum as intended. As with focus area 1. above, both foci would develop understanding of the underpinning factors that may affect the adoption and adaptation of a prescribed curriculum and help determine how best to maximise the likelihood of aligning intent and enactment, which was not a key focus of this study, but peripheral to it.
### References

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Title</th>
<th>Year</th>
<th>Pages</th>
<th>Publisher/Website</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atkins, L., &amp; Wallace, S.</td>
<td>Qualitative research in education.</td>
<td>2012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ban Har, Y.</td>
<td>The five proven learning theories.</td>
<td>2021a</td>
<td></td>
<td>Maths – No Problem! <a href="https://mathsnoproblem.com/blog/teaching-maths-mastery/5-proven-learning-theories/">https://mathsnoproblem.com/blog/teaching-maths-mastery/5-proven-learning-theories/</a></td>
</tr>
<tr>
<td>Ban Har, Y.</td>
<td>Determining if Learners are Truly Advanced.</td>
<td>2022</td>
<td></td>
<td>Maths – No Problem! <a href="https://www.youtube.com/watch?v=R1kdc5FqGCQ">https://www.youtube.com/watch?v=R1kdc5FqGCQ</a></td>
</tr>
<tr>
<td>Author(s)</td>
<td>Title</td>
<td>Details</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------</td>
<td>-------</td>
<td>---------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bloom, B. S.</td>
<td>Learning for mastery.</td>
<td>Evaluation Comment(UCLA-CSIEP), 1(2), 1–12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bobbett, J. J.</td>
<td>School culture, teacher efficacy, and decision-making in demonstrably effective and ineffective schools.</td>
<td>Louisiana State University and Agricultural &amp; Mechanical College.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bowies, K.</td>
<td><em>Choose your words and your manipulatives carefully — two keys for maths success</em>.</td>
<td>Maths – No Problem! <a href="https://mathsnoproblem.com/blog/teaching-practice/choose-your-manipulatives-carefully">https://mathsnoproblem.com/blog/teaching-practice/choose-your-manipulatives-carefully</a></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boylan, M., Wolstenholme, C., Demack, S., Maxwell, B., Jay, T., Adams, G., &amp; Reaney, S.</td>
<td>Longitudinal evaluation of the Mathematics Teacher Exchange: China-England - Final Report.</td>
<td><a href="https://assets.publishing.service.gov.uk/media/5c49b38340f0b61717193d2d/MTE_main_report.pdf">https://assets.publishing.service.gov.uk/media/5c49b38340f0b61717193d2d/MTE_main_report.pdf</a></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


Denzin, N. K., & Lincoln, Y. S. (2008b). The landscape of qualitative research (3rd ed.). SAGE.


<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Title</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>DfE. (2016)</td>
<td>South Asian Method of Teaching Maths to be Rolled Out In Schools</td>
<td><a href="http://www.gov.uk">www.gov.uk</a></td>
</tr>
<tr>
<td>Didau, D. (2016)</td>
<td>Why “mastery learning” may prove to be a bad idea</td>
<td><a href="http://www.learningspy.co.uk">www.learningspy.co.uk</a></td>
</tr>
<tr>
<td>Ding, M. (2016)</td>
<td>Opportunities to learn: Inverse relations in US and Chinese textbooks</td>
<td>Mathematical Thinking and Learning, 18(1), 45-68</td>
</tr>
<tr>
<td>Douglas, H. (2021)</td>
<td>5 types of maths journals and how to use them</td>
<td>Maths – No Problem!</td>
</tr>
<tr>
<td>EEF. (2021)</td>
<td>Mastery Learning</td>
<td>The Education Endowment Fund</td>
</tr>
<tr>
<td>Eisenmann, T., &amp; Even, R. (2008)</td>
<td>Similarities and differences in the types of algebraic activities in two classes taught by the same teacher. Mathematics teachers at work: Connecting curriculum materials and classroom instruction</td>
<td>(pp. 152-170)</td>
</tr>
<tr>
<td>Eisenmann, T., &amp; Even, R. (2011)</td>
<td>enacted types of algebraic activity in different classes taught by the same teacher.</td>
<td>International Journal of Science and Mathematics Education, 9(4), 867-891</td>
</tr>
</tbody>
</table>


Fourier, C. (2022b). Advancing all learners. ‘It can be achieved. It has been achieved.’ (Part II). Maths – No Problem! https://mathsnoproblem.com/blog/teaching-maths-mastery/advancing-all-learners-it-can-be-achieved-it-has-been-achieved-part-ii/


Fullan, F. (1982). The meaning of educational change, Toronto Ontario, Canada: The Ontario Institute for Studies in Education


<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Title</th>
<th>URL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>Title</td>
<td>Source</td>
</tr>
<tr>
<td>--------------------------------------------------------------------------</td>
<td>----------------------------------------------------------------------</td>
<td>-----------------------------------------------------------------------</td>
</tr>
<tr>
<td>International Journal of Market Research 2010 52:3, 293-308</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

270
<table>
<thead>
<tr>
<th>Reference</th>
<th>Title/Author</th>
<th>Source/URL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merttens, R. (2015a).</td>
<td>Why are we blindly following the Chinese approach to teaching maths?</td>
<td>The Guardian</td>
</tr>
<tr>
<td>Author</td>
<td>Title</td>
<td>URL</td>
</tr>
<tr>
<td>---------------------</td>
<td>----------------------------------------------------------------------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>NCETM (2016b)</td>
<td>Meeting the needs of all without ability setting. NCETM.</td>
<td><a href="https://www.ncetm.org.uk/classroom-resources/cs-meeting-the-needs-of-all-without-ability-setting/">https://www.ncetm.org.uk/classroom-resources/cs-meeting-the-needs-of-all-without-ability-setting/</a></td>
</tr>
<tr>
<td>NCETM. (2017c)</td>
<td>Five Big Ideas in Teaching for Mastery. NCETM.</td>
<td><a href="https://www.ncetm.org.uk/teaching-for-mastery/mastery-explained/five-big-ideas-in-teaching-for-mastery/">https://www.ncetm.org.uk/teaching-for-mastery/mastery-explained/five-big-ideas-in-teaching-for-mastery/</a></td>
</tr>
<tr>
<td>Author(s)</td>
<td>Title</td>
<td>Source</td>
</tr>
<tr>
<td>-----------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>Rogers, B.</td>
<td>How to manage children's challenging behaviour.</td>
<td>Paul Chapman.</td>
</tr>
</tbody>
</table>


Tall, D., & Thomas, M. (2002). Intelligence, Learning and Understanding in Mathematics.


Walker, B. F. 1976. Curriculum evolution as portrayed through old textbooks. Terre Haute, IN: Indiana State University, School of Education.


<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Title</th>
<th>Publication Details</th>
</tr>
</thead>
</table>
Appendices

Appendix a: A Sample of Lesson Observation Notes

---

### Lesson Observation Notes

- **Instructor:** observed lesson on numeracy skills.
  - **Focus:** basic arithmetic operations.
  - **Notes:**
    - Highlighted key points for discussion:
      - "What is the total number of students in class today?"
      - "What is the average age of students?"
      - "What is the mode of test scores?"
    - Discussed strategies:
      - "Use visual aids to teach concepts." (visual model diagram)
      - "Encourage students to work in pairs for collaborative learning." (pair work)
    - Addressed common mistakes:
      - "Some students struggle with fractions; plan additional practice sessions.
      - "Encourage students to check their work for accuracy." (error checking)

---

278
Appendix b: Lesson Plan for 1B_20_3

Lesson Objective
To be able to understand how to make turns using mathematical language and connecting this knowledge to time.

Lesson Approach
To begin this lesson, show pupils the picture of the In Focus task. Ask them if they think they are able to turn their bodies like the characters in the picture. Ask them how they would describe the turns. Tell them your friend said all the characters started by facing forward. How did she know that? What are the arrows telling us? Ask them how they would describe the turn of a specific character, e.g. Elliott. He was facing us, but now his back is facing us. What happened? Amira turns all the way around and is now facing us again. What kind of turn did she make? Encourage pupils to talk and discuss the different turns. Also allow them to stand up and turn around themselves.

Show pupils an analogue clock. Ask them how Elliott’s turn and half past the hour are similar. Prompt them by helping them make the connection between the half turn and half past the hour. Ask them to pretend they are minute or hour hands and have turned on the spot. What would this look like on a clock? Use the clock to show them. Then work through Let’s Learn to consolidate the learning and allow pupils to practise using the terms ‘a whole turn’, ‘a half turn’, ‘a quarter turn’ and ‘a three-quarter turn’ in context.

During Guided Practice, pupils are describing how the hour hand and characters have turned. It is important they see how a clock turning and a person turning are similar.

Additional Activity
After the In Focus task, ask pupils to make groups of three. Ask one pupil to lead by telling the others to make quarter, half, three-quarter or whole turns. When one of them makes three mistakes, one swaps places with the leader and the task is repeated.

A variation: one pupil makes one of the turns and the other two state what turn he has made.

Misconceptions
Pupils think that the characters turn according to the arrows from the position shown in the picture.
Formative Assessment

- Pupils can show a physical example of a whole turn.
- Pupils can show a physical example of a half turn.
- Pupils can show a physical example of a quarter turn.
- Pupils can show a physical example of a three-quarter turn.
- Pupils can use the terms 'clockwise' and 'anticlockwise'.
- Pupils can show a physical example of a 'clockwise' and an 'anticlockwise' turn.
- Pupils can use a clock to show turns.
- Pupils can identify turns in pictures.

Non-Negotiable

- Pupils can physically demonstrate whole, half, quarter and three-quarter turns.
- Pupils can physically demonstrate a clockwise and an anticlockwise turn.

Variation

Example 1: Describing turns by looking at the face of a clock; noticing that the hour hand moves while the minute hand does not.
Example 2: Describing turns looking at pictures of children.

National Curriculum

Describe position, direction and movement, including whole, half, quarter and three-quarter turns. Tell the time to the hour and half past the hour and draw the hands on a clock face to show these times.

Resources

- Vocabulary cards: 'whole turn', 'half turn', 'quarter turn', 'three-quarter turn' (between two)

Assessment

A1 Can pupils use and understand the terms 'quarter turn', 'half turn', 'three-quarter turn' and 'whole turn'? Can pupils use and understand the terms clockwise and anticlockwise turns?
Differentiation

D1 For struggling learners, this needs to be a kinaesthetic exercise. Help them make the turns by asking them where they should be facing: use the walls in the classroom to help (4 walls: each is a quarter turn).

D2 For advanced learners, begin embedding the concept of clockwise and anticlockwise. Ask them to respond to questions on this when working with them, e.g. looking at Guided Practice 2(d). Ravi could have turned three-quarters clockwise OR a quarter anticlockwise.

Appendix c Lesson Plan 5A_6_8

Lesson Objective
To be able to add unlike fractions by finding a common denominator using pictorial methods.

Lesson Approach

To begin this lesson, show pupils the In Focus task and ask them to identify the important information. Ask them how much of the chocolate is being eaten all together. Tell them that it is quite a simple equation:

\[ \frac{1}{3} + \frac{1}{9} = \frac{2}{9} \]

Is this true? Is there a mistake? Allow them some time to discuss and prove the answer incorrect.

Ask pupils if they can draw a picture to represent the bar of chocolate and colour in what the two friends said they ate. Would this be helpful? Would it allow us to see how much was eaten exactly? What makes adding thirds and ninths together challenging? Allow pupils to discuss and solve. Ask them if they can draw a new chocolate bar, this time with 10 pieces in total. Tell them that three friends share it. The first friend eats \( \frac{1}{2} \), the second eats \( \frac{3}{10} \) and the third eats \( \frac{3}{5} \). Is this possible? Can we show this in our picture? Give pupils time to discuss and represent the amounts. Can we shade in fifths on a picture that is divided into tenths? What do we need to do?

Use Let's Learn to check pupils' understanding and clarify any misconceptions.

During Guided Practice, pupils are adding like and unlike fractions where the common denominator is in the group of fractions.

Misconceptions

Pupils add both the numerators and denominators to create a new fraction. Pupils do not find a common denominator when adding unlike fractions.
Formative Assessment

Pupils can divide and share concrete materials.
Pupils can represent fractions as a bar model.
Pupils can use the terms 'numerator' and 'denominator'.
Pupils can read a fraction written in words and numbers.
Pupils can recognise a fraction represented as a picture.
Pupils can add fractions with the same denominator.
Pupils can find common denominators.
Pupils can make equivalent fractions.
Pupils can use a number square to find common multiples.

Non-Negotiable

Pupils can add fractions of the same denominator accurately.
Pupils can find a common denominator between two fractions, when one of the fractions has the common denominator.

Variation

Example 1: Adding two fractions where one of the fractions has the common denominator.
Example 2: Adding three fractions where one of the fractions has the common denominator.
Example 3: Creating fractions that add to 1.

National Curriculum

Add and subtract fractions with the same denominator and denominators that are multiples of the same number.

Resources

- Circular representations of fractions (useful but not essential)
- Coloured strips of paper/card for cutting and folding (useful but not essential)
- Access to a fraction wall (useful but not essential)

Assessment

A1 Can pupils use pictorial representation to show fractions? Can pupils shade in unlike fractions on a pictorial representation (shade fifths in a model drawn for tenths)? Can pupils find common denominators? Can pupils explain why we cannot add unlike fractions?
Differentiation

**D1** For struggling learners, the pictorial representation is designed to support their learning. They need to understand that we cannot add fractions that do not have the same name. Provide them with examples, such as: 1 ruler and 1 rubber is equal to what? Is this possible? What do we have to do if we want to add items that are different? We need a common noun (name).

**D2** For advanced learners, ask them to create number stories for the Guided Practice questions.
Appendix d: Lesson Plan 6B_9_5

Lesson Objective
To be able to express a missing number algebraically.

Inside this Guide

Lesson Approach

To begin this lesson, show pupils the In Focus task. Give them time to read the table and discuss the pattern. Tell them your friend says the machine halves the number. Why might he think that? Could he be correct? Encourage pupils to challenge the idea using data from the table.

Can we determine the rule from only one pair of numbers? Why not? Give pupils time to determine the rule and check it, then use it to find the missing numbers in the table. Can we express the rule using a symbol? What if the number that goes in is \( x \)?

During Guided Practice, pupils are recognising rules, which include multiplication and writing algebraic expressions. Ask them to use \( y \) as the number put in and express the output using \( y \) in an algebraic expression for all the examples. Guide pupils on how to write an algebraic expression involving division.

Additional Activity

During Activity Time, pupils work in pairs and take turns to make up a rule for the number crunching machine. Each pupil shows their partner how the machine works by giving them three examples. Then they try to guess the rule and use an algebraic expression to describe the rule.

Misconceptions

Pupils are unable to express an unknown number using a symbol.
Pupils struggle to understand that the value of a letter can change, depending on the context.

Formative Assessment

Pupils can identify the rule by noticing the pattern in the table.
Pupils can show what is happening inside the machine using concrete materials, pictorial representation or algebraic expression.
Pupils can explain the misconception in Let’s Learn 1.
Pupils can write simple algebraic expressions using the 4 operations.
Non-Negotiable

Pupils can express an unknown number using a simple algebraic expression involving any of the 4 operations.

Variation

Using 'number crunching machines' to determine rules by looking for patterns between the input and the output. Each machine shows three pairs of numbers. In each machine, the same three numbers are input.

Example 1: The rule is addition. It is possible to determine the rule from the first set, then use the others to check.
Example 2: The rule is subtraction. It is possible to determine the rule from the first set, then use the others to check.
Example 3: The rule is multiplication. It is not possible to determine the rule from the first set (it could be \( \times 3 \) or \( +20 \)).
Example 4: The rule is division. It is not possible to determine the rule from the first set (it could be \( \div 2 \) or \( -5 \)).

National Curriculum

Express missing number problems algebraically. Use simple formulae.
Appendix e: A Sample taken from a Coding Agreement exercise

<table>
<thead>
<tr>
<th>Back</th>
<th>Chapter</th>
<th>Lesson</th>
<th>Question</th>
<th>Section</th>
<th>Contextual</th>
<th>Line</th>
<th>Presentable</th>
<th>Side</th>
<th>Sense Form</th>
<th>Laid</th>
<th>Numerical Activity</th>
<th>Capital Extent</th>
<th>Side</th>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>B7TB</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>IF</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>TV</td>
<td>TV</td>
<td>C</td>
<td>C</td>
<td>CO</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>B7TB</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>LL</td>
<td>F</td>
<td>F</td>
<td>STV</td>
<td>STV</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>CO</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>B7TB</td>
<td>10</td>
<td>1</td>
<td>2</td>
<td>LL</td>
<td>F</td>
<td>F</td>
<td>STV</td>
<td>STV</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>CO</td>
<td>C</td>
<td>PC</td>
</tr>
<tr>
<td>B7TB</td>
<td>10</td>
<td>1</td>
<td>2</td>
<td>AT</td>
<td>A</td>
<td>A</td>
<td>STV</td>
<td>STV</td>
<td>O</td>
<td>O</td>
<td>C</td>
<td>CO</td>
<td>C</td>
<td>E</td>
</tr>
<tr>
<td>B7TB</td>
<td>10</td>
<td>1</td>
<td>2</td>
<td>GP</td>
<td>F</td>
<td>F</td>
<td>STV</td>
<td>STV</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>PC</td>
<td>C</td>
<td>E</td>
</tr>
<tr>
<td>B7TB</td>
<td>10</td>
<td>1</td>
<td>2</td>
<td>GP</td>
<td>F</td>
<td>F</td>
<td>STV</td>
<td>STV</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>PC</td>
<td>C</td>
<td>PC</td>
</tr>
<tr>
<td>B7TB</td>
<td>10</td>
<td>1</td>
<td>2</td>
<td>GP</td>
<td>F</td>
<td>F</td>
<td>STV</td>
<td>STV</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>PC</td>
<td>C</td>
<td>PC</td>
</tr>
<tr>
<td>B7TB</td>
<td>10</td>
<td>2</td>
<td>1</td>
<td>IF</td>
<td>N</td>
<td>N</td>
<td>TV</td>
<td>TV</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>CO</td>
<td>C</td>
<td>E</td>
</tr>
<tr>
<td>B7TB</td>
<td>10</td>
<td>2</td>
<td>1</td>
<td>LL</td>
<td>N</td>
<td>N</td>
<td>SV</td>
<td>SV</td>
<td>O</td>
<td>O</td>
<td>C</td>
<td>CO</td>
<td>C</td>
<td>E</td>
</tr>
<tr>
<td>B7TB</td>
<td>10</td>
<td>2</td>
<td>2</td>
<td>LL</td>
<td>N</td>
<td>N</td>
<td>SV</td>
<td>SV</td>
<td>O</td>
<td>O</td>
<td>C</td>
<td>CO</td>
<td>C</td>
<td>E</td>
</tr>
<tr>
<td>B7TB</td>
<td>10</td>
<td>2</td>
<td>3</td>
<td>LL</td>
<td>N</td>
<td>N</td>
<td>SV</td>
<td>SV</td>
<td>O</td>
<td>O</td>
<td>C</td>
<td>CO</td>
<td>C</td>
<td>E</td>
</tr>
<tr>
<td>B7TB</td>
<td>10</td>
<td>2</td>
<td>3</td>
<td>GP</td>
<td>N</td>
<td>N</td>
<td>SV</td>
<td>SV</td>
<td>O</td>
<td>O</td>
<td>C</td>
<td>CO</td>
<td>C</td>
<td>E</td>
</tr>
</tbody>
</table>
Appendix f: Sample Participant Consent Form

University of Huddersfield  
School of Education and Professional Development

Participant Consent Form (E4)

Title of Research Study: An Evaluation of the Perspectives of Staff Working with Children with Additional Communication Needs Regarding the Impact of Taught Social Skills Lessons.

Name of Researcher: Ed Southall

Participant Identifier Number: 297B

I confirm that I have read and understood the participant information sheet related to this research, and have had the opportunity to ask questions.

I understand that my participation is voluntary and that I am free to withdraw at any time without giving any reason.

I understand that all my responses will be anonymised.

I give permission for members of the research team to have access to my anonymised responses.

I agree to take part in the above study

Name of Participant: ...

Signature of Participant: ...

Date: 14.2.18

Name of Researcher: Ed Southall

Signature of Researcher:

Date: 14th February 2018